## Measuring Angles



Fig 1: Islamic Astronomers of the Middle Ages in an Observatory in Istanbul.


Fig 2: Tycho Brahe in Denmark. His observations were later used by Kepler.

Have you ever thought how we determine the positions of stars, or the sizes of anything? If you want to know the size of Jupiter (or even just the moon), you can hardly fly to it with your ruler in your hand and measure it. However, the moon has a definite size in the sky. And what's the size of the entire sky? This seems a ridiculous question, however if we think in terms of angles it gets easier. One whole circle has 360 degrees. A huge cloud that occupies about one $10^{\text {th }}$ of the sky then has a diameter of $180^{\circ} / 10$, i.e., $18^{\circ}$. Similarly, you can measure the diameter of the moon, which turns out to be about $0.5^{\circ}$. So in Astronomy we always think in angles - i.e., in "angular sizes." Thousands of years ago we observed and measured angles in the sky, and today we still do the same - and ironically we still use pretty much the same methods. In this Lab you will learn the simplest and most basic method of how to measure angles.

## Part I

There are several methods of measuring angles and rather sophisticated instruments have been designed. Yet it is surprising how few people actually know how to determine angles with their own body parts... Maybe it is not quite as accurate, but at least it provides rather good estimates. This is how you do it stretch out your arm. Your fist will cover an angle of about $10^{\circ}$, your index finder $1^{\circ}$, and you whole hand about $20^{\circ}$.


Let's check if this hypothesis. What is the angular size of your finger?
a) Extend your arm in front of you. Measure the distance between your eye and the tip of your index finger.
b) Measure the width of your index finger (across your finger nail).
c) Calculate the angular size of your finger in degrees. Consult the TOOLKIT to figure out which trigonometric function to use. Write the trigonometric function into the box. SHOW your calculation; ZERO points otherwise!

$\qquad$
$\qquad$
$\qquad$
$\qquad$
d) Rewrite your answer using the rules of significant figures
e) How could you determine the uncertainty in the angular size? Make a suggestion.
$\qquad$
$\qquad$
Your estimated uncertainty (guess this) in the width of your finger is
f) Rewrite your answer using significant figures and error estimates

|  | degrees |
| :---: | :---: |
| $\pm$ | degrees | (Your answer should have the form of $13.4 \pm 0.5$ or so.)

f) Does the One-Degree-per-Finger Rule apply to you?
(In Astronomy we often use estimates. If your value falls within 30\% of the expected value, your estimate is considered reasonable.)

## Part II I

## Determining the Height of Gibbs Lab using your Fingers



Fig 3: Woodcut from 1533: surveyors are using a cross-staff to measure angles.
Whether you use your fingers, or a cross-staff like in the above picture, the basic method is the same. While the measurements with the cross-staff are more accurate, your finger is a hell of a lot 'handier.'
a) From Prospect Street, how many finger widths is Gibbs Lab? Estimate the accuracy of your measurement. (Your answer should have the form of $13.4 \pm 0.5$ or so.)

Go roughly halfway to the campus center. How many finger widths is the campus center at half that distance?

Go roughly quarter the way to the campus center.
b) What is the angular size of the campus center from building 4N?

What is the angular size at half the distance to the campus center?
What is the angular size at quarter the distance to the campus center?

| $\pm$ | fingers |
| :---: | :---: |
| $\pm$ | fingers |
| $\pm$ | fingers |
| $\pm$ | degrees |
| $\pm$ | degrees |
| $\pm$ | degrees |

c) Explain in words how angular size and distance are related
d) Abbreviate what you said (e.g., angular size increases/decreases as distance increases/decreases)
e) Insert three points into the plot below.


Angular Size (in degrees)


Is this plot consistent with your statement in (d)?
f) Now figure out the exact relationship between the angular size, the linear size and the distance.
g) Given the distance to Gibbs Lab (from Prospect Street) is 110 meters calculate physical height in meters? Show your calculation.
$\qquad$
$\qquad$
$\qquad$
e) Estimate the uncertainty in that measurement.
f) The height of the campus center is

|  | meters |
| :---: | :---: |
| $\pm \quad$ meters |  |

g) Let's check if this is a good method. Make an educated guess (in meters) of the height of each floor of Gibbs Lab.
h) Multiply by the number of floors to calculate the physical height
$\qquad$
$\qquad$ $\pm$ meters
i) Are these two values consistent? (i.e., within $30 \%$ of each other?) $\qquad$
j) Do you think this is a good method to determine the height of Gibbs Lab? Would you use this method in Astronomy? EXPLAIN.

## Part IIII

## Do AT Sunset

There is a rule of thumb (no pun intended) that the sun moves "one finger every ten minutes." Your assignment is to check this statement.

## Observations

a) Find a spot where you can watch the Sun. Bring a watch and write down the time from the watch.
b) Using your fingers determine the height of the Sun above the horizon. $\qquad$
c) Estimate the uncertainty (fractions of fingers are okay)
d) This corresponds to $\ldots \pm \ldots$ degrees:

|  | degrees |
| :---: | :---: |

e) Predict the time the sun will set according to the 10 min per finger rule.
f) Come back at Sunset; record the time
g) Was it close to your prediction?
(A measurement is generally regarded as "reasonable" if it falls within one third of the predicted value. For example if the predicted value is 100 , any measurements within 60 and 130 are considered reasonable.)

## Theory

h) Calculate the theoretical value, i.e., determine how many minutes it takes the sun to move $1^{\circ}$. (Hint: The Sun takes 24 hours to move around the sky. This corresponds to $360^{\circ}$. So you need to figure out how many minutes it takes the Sun to move $1^{\circ}$.) SHOW calculation!
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
i) Is the scheme of " 10 minutes per finger" reasonable? (A measurement is reasonable if it falls within one third of the predicted value.)
k) If it is not reasonable suggest a different wording

## Part IV

## DO AT Night

Go outside and look at the night sky and try to find the constellations below (look towards the Southwest, all the way up to the Zenith). You'll see Deneb and Vega above you (Vega in you Zenith), and Altair a little further South. These are the three brightest stars in the summer sky and are often referred to as the Summer Triangle. Use your fist and fingers to determine the size of the Summer Triangle. How far is Vega from Deneb, Deneb from Altair, and Altair from Vega? Use your fist and fingers to determine the distances.

Deneb is in the constellation Cygnus (or Swan), which is sometimes called the Northern Cross. The star at the other end of the cross is Alberio, a rather spectacular double star. A little further to the south you'll see Altair in Aquila, west of that you'll see Hercules, and further in the Southwest will be Ophichus, the Serpent Holder. Scorpio (below Serpens) will be on the Southwest Horizon, but you might only see Scorpio's brightest Antares, which will be a little above the Horizon.


