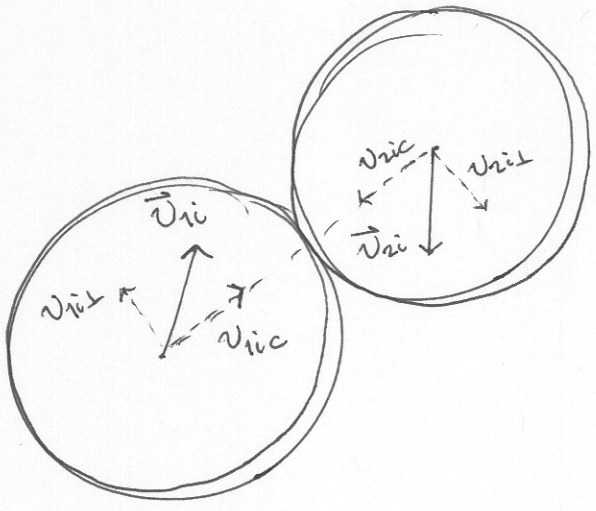
Elastic collisions in two dimensions:



The forces act along the center line, so in the perpendicular direction the velocities don't change:

$$v_{1f\perp} = v_{1i\perp}$$
$$v_{2f\perp} = v_{2i\perp}$$

In the centerline direction:

conservation of momentum: $v_{1fc} - v_{1ic} = -v_{2fc} + v_{2ic}$ (1)

conservation of energy: $\frac{1}{2}m v_{1fc}^2 - \frac{1}{2}m v_{1ic}^2 = -\frac{1}{2}m v_{2fc}^2 + \frac{1}{2}m v_{2ic}^2$ (2)

dividing (2)/(1):

$$\frac{v_{1fc}^2 - v_{1c}^2}{v_{1fc} - v_{1c}} = \frac{v_{2ic}^2 - v_{2fc}^2}{v_{2ic} - v_{2fc}}$$

$$\frac{(v_{1fc} - v_{1c})(v_{1fc} + v_{1c})}{v_{1fc} - v_{1c}} = \frac{(v_{2ic} - v_{2fc})(v_{2ic} + v_{2fc})}{v_{2ic} - v_{2fc}}$$

$$v_{1fc} + v_{1ic} = v_{2ic} + v_{2fc} \Rightarrow v_{2fc} = v_{1fc} + v_{1ic} - v_{2ic} \quad (3)$$

(3) into (1):

$$v_{1fc} - v_{1ic} = -v_{1fc} - v_{1ic} + v_{2ic} + v_{2ic}$$
$$2v_{1fc} = 2v_{2ic} \Rightarrow \boxed{v_{1fc} = v_{2ic}}$$

and from (3):

$$v_{2fc} = v_{2ic} + v_{1ic} - v_{2ic} \Rightarrow \boxed{v_{2fc} = v_{1ic}}$$

so $\vec{v}_{1f} = \vec{v}_{1fc} + \vec{v}_{1f\perp} = \vec{v}_{2ic} + \vec{v}_{1i\perp}$

$\vec{v}_{2f} = \vec{v}_{2fc} + v_{2f\perp} = \vec{v}_{1ic} + \vec{v}_{2i\perp}$

the unit vector along the centerline is $\hat{d}_{12} = \dfrac{(x_2-x_1, y_2-y_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} = \dfrac{\vec{d}_{12}}{\|d_{12}\|}$

so the components of the initial velocities are:

$$\vec{v}_{1ic} = (\vec{v}_{1i}\cdot\hat{d}_{12})\,\hat{d}_{12} \qquad \vec{v}_{2ic} = (\vec{v}_{2i}\cdot\hat{d}_{12})\,\hat{d}_{12}$$

$$\vec{v}_{1i\perp} = \vec{v}_{1i} - \vec{v}_{1ic} \qquad \vec{v}_{2i\perp} = \vec{v}_{2i} - \vec{v}_{2ic}$$

then

$$\vec{v}_{1f} = (\vec{v}_{2i}\cdot\hat{d}_{12})\,\hat{d}_{12} + \vec{v}_{1i} - (\vec{v}_{1i}\cdot\hat{d}_{12})\,\hat{d}_{12} =$$

$$= \left[(\vec{v}_{2i} - \vec{v}_{1i})\cdot\hat{d}_{12}\right]\hat{d}_{12} + \vec{v}_{1i}$$

$$\vec{v}_{2f} = (\vec{v}_{1i}\cdot\hat{d}_{12})\,\hat{d}_{12} + \vec{v}_{2i} - (\vec{v}_{2i}\cdot\hat{d}_{12})\,\hat{d}_{12} =$$

$$= \left[(\vec{v}_{1i} - \vec{v}_{2i})\cdot\hat{d}_{12}\right]\hat{d}_{12} + \vec{v}_{2i}$$