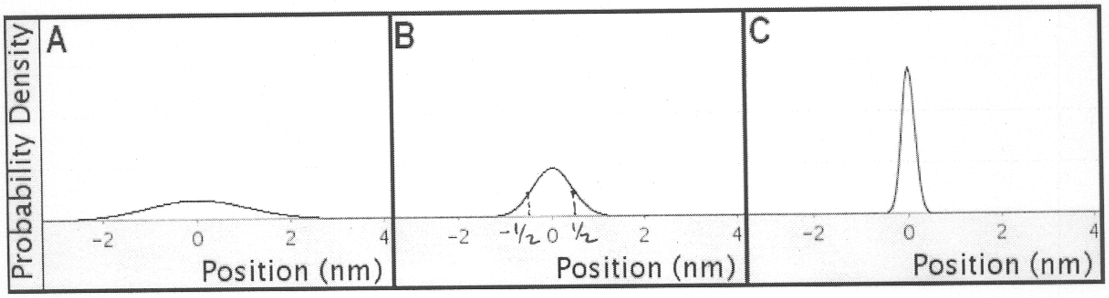
2. We've been using a bunch of new terms and symbols, including:

1. ψ
2. |ψ|2,
3. ψ*ψ,
4. "the wave function", and
5. "the probability density".

Explain the relationship between these terms and symbols. Which could you infer from the intensity of electrons hitting a screen?

The wave function is represented by the greek letter ψ. It represents the wave-like properties of a particle. The probability density, $|\psi|^2 = \psi^* \psi$, represents the probability of finding a particle per unit distance. You can infer the probability density from the intensity of electrons hitting a screen.

4. Below are 3 plots of $|\psi(x)|^2$, the probability density, for 3 free electrons:



i. Which has the greatest uncertainty in position? A

ii. If you define uncertainty in position for a distribution like this as the distance between the points at which the probability density has dropped to ½ its max value, then what is the approximate uncertainty in position for graph b (in nm)?

1 nm

iii. A plane wave has: (check all that apply)

✓ large uncertainty in position
small uncertainty in position
large uncertainty in momentum
✓ small uncertainty in momentum

iv. Relative to a plane wave, a wave packet has: (check all that apply)

larger uncertainty in position
✓ smaller uncertainty in position
✓ larger uncertainty in momentum
smaller uncertainty in momentum

Explain your reasoning:

A plane wave ~~has the~~ extends from $-\infty$ to ∞ with the same amplitude, so the particle could be anywhere with the same probability (large uncertainty in position). A plane wave has a well-defined k (wave number), so the uncertainty in momentum $p = \hbar k$ is zero.

A wave packet is formed by adding several plane waves with different wave numbers, so the uncertainty in momentum is larger than for a single plane wave. Adding different plane waves results in a wave function that is more localized, so the uncertainty in position is smaller.

Normalization: $\psi(x) = 2iA \sin(kx)$, $0<x<L$; $\psi(x)=0$, $x<0$, $x>L$

$$|\psi(x)|^2 = \psi^*(x)\,\psi(x) = -2iA\sin(kx)\cdot 2iA\sin(kx) = 4A^2\sin^2(kx)$$

$$\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = \int_0^L 4A^2\sin^2(kx)\,dx = 4A^2\int_0^L \frac{1}{2}(1-\cos(2kx))\,dx =$$

$$= 2A^2\left(x\Big|_0^L - \frac{\sin(2kx)}{2k}\Big|_0^L\right) = 2A^2\left(L - \frac{\sin(2kL)}{2k}\right) =$$

$$= 2A^2\left(L - \frac{\sin\left(2L\cdot\frac{n\pi}{L}\right)}{2k}\right) = 2A^2\left(L - \cancelto{0}{\frac{\sin(2n\pi)}{2k}}\right) = 2A^2L = 1$$

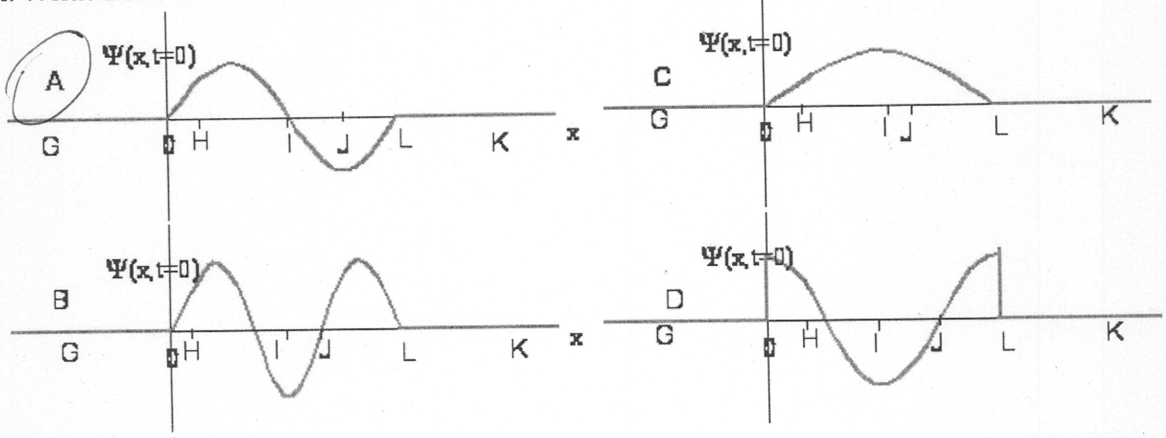
$$\Rightarrow \quad A = \frac{1}{\sqrt{2L}}$$

3. An electron wave function between 0 and L is described by the following function:

$\psi(x,t) = (2/L)^{1/2}\sin(2\pi x/L)e^{-i\omega t}$, $0 < x < L$

$\psi(x,t) = 0$ for $x<0$ and $x>L$

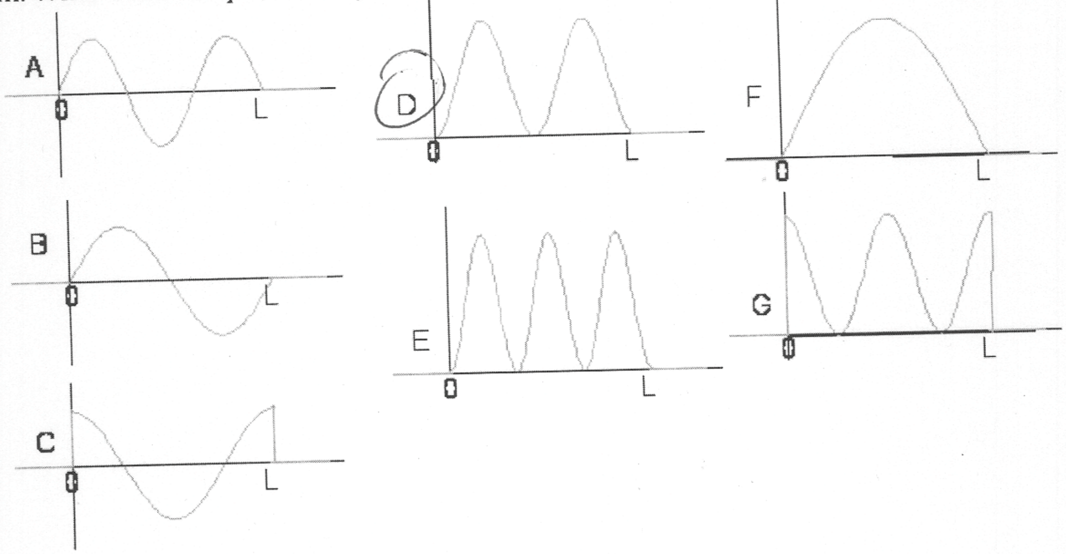
i. What does this wave look like at t=0?



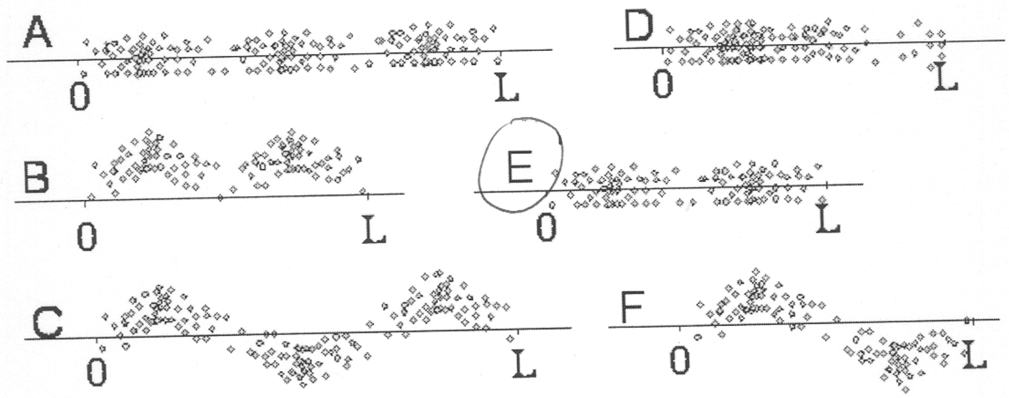
ii. At t=0, how do the probabilities of finding the electron very close (within a very small distance dx) to x=G, H, I, J, and K compare? (G=Probability of finding the electron near point G):

a. G=H=I=J=K
b. H>J=G=K>I
c. H>I=G=K>J
d. J>H>I=G=K
e. H>I>J>G=K
f. I>J>H>G=K
g. I>H>J>G=K

iii. What does the probability density, P(x), look like for this wave function?



iv. If you had a bunch of electrons with this wave function and detected where the electrons were, what pattern would you expect to see:



v. What is the total probability of detecting this electron between L/4 and 3L/4?

$\frac{1}{2}$, by symmetry  the shaded part is $\frac{1}{2}$ of the total area under the curve

vii. Which of the following interpretations of this wave function are valid:

True False The electron's position is higher at G than at J
True False The electrons move up and down as they travel between 0 and L
True False The probability density as a function of position between 0 and L (from question iii) does not change as time passes.
True False The probability of finding the electron at L/2 is 0
True False The probability of finding the electron anywhere between 0 and L/2 is 0.5.