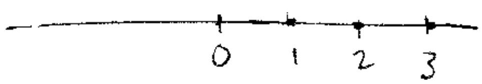
a) in 1D: $E_n = \frac{\pi^2\hbar^2}{2mL^2} n^2$



the number of states is given by the length of the line (positive side only)

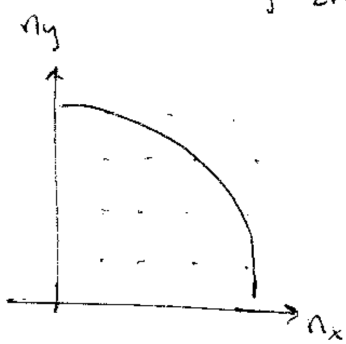
$$S(E) = \underbrace{2}_{\text{spin}} \times n' = 2\,\frac{\sqrt{2mE}}{\pi\hbar}\,L$$

per unit length,

$$S_L(E) = \frac{S(E)}{L} = \frac{2\sqrt{2mE}}{\pi\hbar}$$

$$g(E) = \frac{dS_L(E)}{dE} = \frac{2\sqrt{2m}}{\pi\hbar}\,\frac{1}{2}\,E^{-1/2} = \frac{1}{\pi\hbar}\left(\frac{2m}{E}\right)^{1/2}$$

b) in 2D: $E_{n_x,n_y} = \frac{\pi^2\hbar^2}{2mL^2}(n_x^2+n_y^2)$, $E_{n'} = \frac{\pi^2\hbar^2}{2mL^2}n'^2$



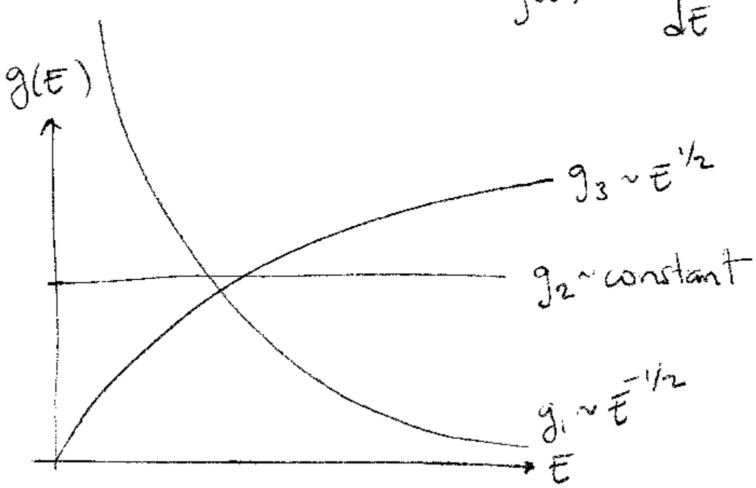
$$S(E) = \underbrace{2}_{\text{spin}} \times \underbrace{\frac{1}{4}}_{\text{positive } n_x,\,n_y} \times \pi n'^2 = \frac{1}{\cancel{2}}\cancel{\pi}\,\frac{\cancel{2}mEL^2}{\pi^{\cancel{2}}\hbar^2} = \frac{mEL^2}{\pi\hbar^2}$$

per unit area,

$$S_A(E) = \frac{S(E)}{L^2} = \frac{mE}{\pi\hbar^2}$$

$$g(E) = \frac{dS_A(E)}{dE} = \frac{m}{\pi\hbar^2}$$

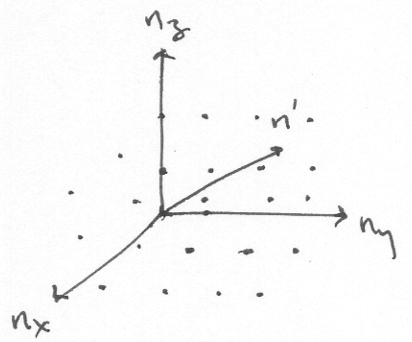
c)



Density of states in 3D:

$$E = \frac{\pi^2 \hbar^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2) \quad \text{where} \quad \begin{matrix} n_x = 1, 2, \ldots \\ n_y = 1, 2, \ldots \\ n_z = 1, 2, \ldots \end{matrix}$$

The possible values of (n_x, n_y, n_z) correspond to points in the positive octant of a cubic lattice:



$$n' = \sqrt{n_x^2 + n_y^2 + n_z^2}$$

$$E' = \frac{\pi^2 \hbar^2}{2mL^2} n'^2 \longrightarrow n' = \frac{L\sqrt{2mE}}{\pi\hbar}$$

The number of states with $E < E'$:

$$S(n') = \underbrace{2}_{\text{spin}} \times \underbrace{\frac{1}{8}}_{\text{positive octant only}} \times \underbrace{\frac{4}{3}\pi n'^3}_{\text{volume of sphere with radius } n'} = \frac{1}{3}\pi n'^3 = \frac{1}{3}\pi\left(\frac{L\sqrt{2mE}}{\pi\hbar}\right)^3 = \frac{\pi L^3 (8mE')^{3/2}}{3h^3} = S(E'$$

per unit volume $(V = L^3)$:

$$S_v(E') = \frac{\pi (8mE')^{3/2}}{3h^3}$$

Then the density of states (number of states per unit volume per unit energy) is:

$$g(E') = \frac{dS_v}{dE'} = \frac{\pi (8m)^{3/2}}{\cancel{3}h^3} \cdot \frac{\cancel{3}}{2} E'^{1/2} = \boxed{8\pi\sqrt{2}\left(\frac{m}{h^2}\right)^{3/2} E'^{1/2} = g(E')}$$