

HW #22

1. One way of calculating the population of a city is to find the density of houses in that city (i.e. the number of houses per unit area), multiply that by the probability of finding a given number of humans in a house, and finally, integrate the result over the area of the city. The problem is working out the chances of a house being occupied by a certain number of people, using a mathematical formula. If $g(A)$ is the density of houses, and $f(A)$ is the probability that a house is occupied by a certain number of people,
- Write down the integral that would give you the population of a city.
 - For a hypothetical city, whose area is a circle, f and g are given by:

$$f(r,\theta) = 4e^{-r/20\text{km}}$$

$$g(r,\theta) = 1000e^{-r^2/200\text{km}^2}$$

where distances are in units of kilometers, and house density is given in units of houses / km^2 .

- Sketch f and g as a function of r . What information do they give you?
- If the city limits are at a radius $R = 15$ km from the city centre, what is the total population of the city? (You can use Mathematica to calculate the integral.)
- What percentage of the population lives downtown? Define downtown as within 1 km of the city centre.
- At what distance R from the city centre is the average number of people per household the highest?
- At what distance R from the city centre is the density of houses the highest?
- What is the average household occupation for the entire city?
- Make a 2D (contour or density) plot of g and compare it to the satellite Google map for Houston, TX (if you remove the labels on the map it is easier to see the housing concentration). Is this model realistic? How would you improve it?