$$E_{ave} = \frac{\int E\, g(E)\, f(E)\, dE}{\int g(E)\, f(E)\, dE}$$

at T=0K all states up to E_{F0} are full, and all states above are empty:

$$E_{ave}(T=0K) = \frac{\int_0^{E_{F0}} E \cdot g(E)\, dE}{\int_0^{E_{F0}} g(E)\, dE} = \frac{\cancel{8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2}} \int_0^{E_{F0}} E \cdot E^{1/2}\, dE}{\cancel{8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2}} \int_0^{E_{F0}} E^{1/2}\, dE}$$

$$= \frac{\int_0^{E_{F0}} E^{3/2}\, dE}{\int_0^{E_{F0}} E^{1/2}\, dE} = \frac{\frac{\cancel{2}}{5} E^{5/2} \Big|_0^{E_{F0}}}{\frac{\cancel{2}}{3} E^{3/2} \Big|_0^{E_{F0}}} = \frac{3}{5} \frac{E_{F0}^{5/2}}{E_{F0}^{3/2}} = \frac{3}{5} E_{F0}$$

HW #21

4.7 a) Since in the free-electron model we set the potential energy at zero inside the metal, all the energy is kinetic:

$$E_F = \frac{1}{2} m_e v_F^2$$

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2\,(7\,\text{eV})(1.602\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = 1.57\times10^{6}\,\text{m/s}$$

From the equipartition theorem, we add $\frac{1}{2}kT$ of energy per degree of freedom. The electrons can move in 3 directions and there are no internal degrees of freedom (rotation or vibration), then

$$\frac{1}{2} m v_{thermal}^2 = \frac{3}{2} kT$$

$$v_{thermal} = \sqrt{\frac{3kT}{m_e}} = \sqrt{\frac{3\,(1.381\times10^{-23}\,\text{J/K})(293\,\text{K})}{9.11\times10^{-31}\,\text{kg}}} = 1.15\times10^{5}\,\text{m/s}$$

$$\frac{v_F}{v_{thermal}} = 13.7$$

The thermal velocity is much smaller than v_F because it does not take into account the Pauli exclusion principle. Quantum states with higher energies need to be filled in ~~of~~ order to have only one electron per state.

b) $$\lambda = \frac{h}{p} = \frac{h}{m_e v_F} = \frac{6.626\times10^{-34}\,\text{Js}}{(9.11\times10^{-31}\,\text{kg})(1.57\times10^{6}\,\text{m/s})} = 4.63\times10^{-10}\,\text{m} = 4.63\,\text{Å}$$

$\lambda = 2d\sin\theta$, $d = 2.09\,\text{Å}$

$$\sin\theta = \frac{\lambda}{2d} = \frac{4.63\,\text{Å}}{2\times2.09\,\text{Å}} = 1.11 > 1$$

Since $\sin\theta > 1$ there can be no diffraction, i.e. no value of θ would satisfy the equation above.

HW #22

4.8 Both sodium & gold have one valence electron

$$n_{Na} = (\#\text{valence } e^-)\frac{N_A \cdot \rho}{M_{Na}} = 1\,\frac{(6.023\times10^{23}/\text{mol})(968\ \text{kg/m}^3)}{23\times10^{-3}\ \text{kg/mol}} = 2.53\times10^{28}/\text{m}^3$$

$$n_{Au} = 1\,\frac{(6.023\times10^{23}/\text{mol})(19300\ \text{kg/m}^3)}{197\times10^{-3}\ \text{kg/mol}} = 5.9\times10^{28}/\text{m}^3$$

at $T = 0\text{K}$,

$$E_{F0}(Na) = \frac{h^2}{8m_e}\left(\frac{3n}{\pi}\right)^{2/3} = \frac{(6.626\times10^{-34}\,\text{Js})^2}{8\times9.11\times10^{-31}\,\text{kg}}\left[\frac{3\times2.53\times10^{28}/\text{m}^3}{\pi}\right]^{2/3} =$$

$$= 5.04\times10^{-19}\ \text{J} = 3.15\ \text{eV}$$

$$E_{F0}(Au) = \frac{(6.626\times10^{-34}\,\text{Js})^2}{8\times9.11\times10^{-31}\,\text{kg}}\left[\frac{3\times5.9\times10^{28}/\text{m}^3}{\pi}\right]^{2/3} = 8.863\times10^{-19}\ \text{J} = 5.54\ \text{eV}$$

at $T = 300\text{K}$,

$$E_F(Na) = E_{F0}(Na)\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F0}(Na)}\right)^2\right] = 3.15\text{eV}\left[1 - \frac{\pi^2}{12}\left(\frac{0.02585\ \text{eV}}{3.15\ \text{eV}}\right)\right]^2 = 3.15\text{eV}$$

$$E_F(Au) = E_{F0}(Au)\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F0}(Au)}\right)^2\right] = 5.54\text{eV}\left[1 - \frac{\pi^2}{12}\left(\frac{0.02585\ \text{eV}}{5.54\text{eV}}\right)^2\right] = 5.54\text{eV}$$

Mean speed: $\frac{1}{2}m_e v_e^2 = E_{ave} = \frac{3}{5}E_{F0} \Rightarrow v_e = \left(\frac{6E_{F0}}{5m_e}\right)^{1/2}$

$$v_e(Na) = \left(\frac{6\times3.15\times10^{-19}\ \text{J}}{5\times9.11\times10^{-31}\,\text{kg}}\right)^{1/2} = 8.15\times10^5\ \text{m/s}$$

$$v_e(Au) = \left(\frac{6\times5.54\times1.6\times10^{-19}\ \text{J}}{5\times9.11\times10^{-31}\,\text{kg}}\right)^{1/2} = 1.08\times10^6\ \text{m/s}$$

Speed at E_F : $\frac{1}{2} m_e v_e^2 = E_{F0} \Rightarrow v_e = \left(\frac{2E_{F0}}{m_e} \right)^{1/2}$

$$v_e^{Na} = \left(\frac{2 \times 3.15 \times 1.602 \times 10^{-19} J}{9.11 \times 10^{-31} kg} \right)^{1/2} = 1.05 \times 10^6 m/s$$

$$v_e^{Au} = \left(\frac{2 \times 5.54 \times 1.602 \times 10^{-19} J}{9.11 \times 10^{-31} kg} \right)^{1/2} = 1.4 \times 10^6 m/s$$

Density of states: $g(E) = 8\pi\sqrt{2} \left(\frac{m_e}{h^2} \right)^{3/2} E^{1/2}$

for Na:

$$g_{Na}(E_F) = 8\pi\sqrt{2} \left(\frac{9.11 \times 10^{-31} kg}{(6.626 \times 10^{-34} Js)^2} \right)^{3/2} (3.15 \times 1.602 \times 10^{-19} J)^{1/2} =$$

$$= 7.54 \times 10^{46} / J m^3 = 1.2 \times 10^{22} / eV\, cm^3$$

$$\frac{E_F + \Phi}{2} = \frac{3.15 eV + 2.75 eV}{2} = 2.95\ eV$$

$$g_{center}(Na) = 7.3 \times 10^{46} / m^3 J = 1.17 \times 10^{22} / eV\, cm^3$$

for Au:

$$g_{Au}(E_F) = 10 \times 10^{46} / J m^3 = 1.6 \times 10^{22} / eV\, cm^3$$

$$\frac{E_F + \Phi}{2} = \frac{5.54 eV + 5.1 eV}{2} = 5.32\ eV$$

$$g_{center}(Au) = 9.8 \times 10^{46} / J m^3 = 1.57 \times 10^{22} / eV\, cm^3$$