4.12

$$\sigma = \frac{1}{3} e^2 v_F^2 \tau\, g(\varepsilon_F) \qquad \frac{1}{2} m_e v_F^2 = \varepsilon_F \Rightarrow v_F^2 = \frac{2\varepsilon_F}{m_e}$$

$$g(\varepsilon_F) = 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2} \varepsilon_F^{1/2}$$

$$\varepsilon_F = \frac{h^2}{8m_e}\left(\frac{3n}{\pi}\right)^{2/3}$$

$$\sigma = \frac{1}{3} e^2 v_F^2 \tau\, 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2} \varepsilon_F^{1/2} = \frac{1}{3} e^2 \tau \cdot \frac{2\varepsilon_F}{m_e} \cdot 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2} \varepsilon_F^{1/2} =$$

$$= \frac{2}{3}\frac{e^2\tau}{m_e} 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2} \varepsilon_F^{3/2} = \frac{2}{3}\frac{e^2\tau}{m_e} 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2}\left[\frac{h^2}{8m_e}\left(\frac{3n}{\pi}\right)^{2/3}\right]^{3/2} =$$

$$= \frac{2}{3}\frac{e^2\tau}{m_e} 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2}\left(\frac{h^2}{8m_e}\right)^{3/2}\frac{3n}{\pi} = \frac{e^2 n\tau}{m_e}\frac{8\times 2\times\sqrt{2}}{8^{3/2}} = \frac{e^2 n\tau}{m_e}\frac{8\times 2\sqrt{2}}{2^3\cdot 2^{3/2}}$$

$$= \frac{e^2 n\tau}{m_e}\frac{8\times 2\times\sqrt{2}}{8\times 2\sqrt{2}} = \boxed{\frac{e^2 n\tau}{m_e} = \sigma}$$

($\left(\frac{m_e}{h^2}\right)^{3/2}\left(\frac{h^2}{8m_e}\right)^{3/2} = 1$)

Example 4.10

For silver, $E_{F0} = 5.5\,eV$, $\Phi = 4.5\,eV$

a) $$g(E_F) = 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2} E_F^{1/2} = 8\pi\sqrt{2}\left(\frac{9.11\times10^{-31}\,kg}{(6.626\times10^{-34}\,J\,s)^2}\right)^{3/2}\left(5.5\,eV\,\frac{1.6\times10^{-19}\,J}{1\,eV}\right)^{1/2} =$$

$$= 9.97\times10^{46}/m^3\,J \times \frac{1.6\times10^{-19}\,J}{eV} = 1.59\times10^{28}/m^3\,eV$$

b) $$E_F \simeq E_{F0} = \frac{1}{2}mv_F^2 \Rightarrow$$

$$v_F = \sqrt{\frac{2E_{F0}}{m}} = \sqrt{\frac{2(5.5\,eV)(1.6\times10^{-19}\,J/eV)}{9.11\times10^{-31}\,kg}} = 1.39\times10^6\,m/s$$

c) $\sigma = 62.5\times10^6/\Omega m$ at $T = 300\,K$

$$\sigma = \frac{1}{3}e^2 v_F^2 \tau\, g(E_F)$$

$$\tau = \frac{3\sigma}{e^2 v_F^2\, g(E_F)} = \frac{3(62.5\times10^6/\Omega m)}{(1.602\times10^{-19}\,C)^2(1.39\times10^6\,m/s)^2(9.97\times10^{46}/m^3\,J)} =$$

$$= 3.8\times10^{-14}\,s$$

d) $$\ell = v_F\tau = (1.39\times10^6\,m/s)(3.8\times10^{-14}\,s) = 53\times10^{-9}\,m = 53\,nm$$