

Classical Mechanics

SOLUTIONS ASSIGNMENT 4

1. $\vec{r} = \frac{1}{2} \omega \times \vec{r} \times \vec{r}$ (i)
 $\vec{v} = \omega \times \vec{r}$ (ii)
 $\frac{d}{dt}(\frac{1}{2} \omega \times \vec{r} \times \vec{r}) = \omega \times \vec{r} \times \vec{v} + \frac{1}{2} \dot{\omega} \times \vec{r} \times \vec{r} + \frac{1}{2} \omega \times \dot{\vec{r}} \times \vec{r} + \frac{1}{2} \omega \times \vec{r} \times \dot{\vec{r}}$
 $\text{USE: } \omega \times \vec{r} \times \vec{v} = \frac{d}{dt}(\frac{1}{2} \omega \times \vec{r} \times \vec{r})$ (iii)

2. $L = \int dV \rho(\vec{r}) (\frac{1}{2} \dot{\vec{r}}^2 - \frac{1}{2} \omega^2 r^2) \rightarrow L = \frac{1}{2} \int dV \rho(\vec{r}) (\dot{\vec{r}}^2 - \omega^2 r^2)$ (i)
REWRITE: $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2) = \dot{\vec{r}} \cdot \ddot{\vec{r}}$ (ii)
 $\frac{d}{dt}(\frac{1}{2} \omega^2 r^2) = \omega^2 \vec{r} \cdot \dot{\vec{r}} = \omega^2 \frac{d}{dt}(\frac{1}{2} r^2)$
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2 - \frac{1}{2} \omega^2 r^2) = \dot{\vec{r}} \cdot (\ddot{\vec{r}} - \omega^2 \vec{r}) = \dot{\vec{r}} \cdot \vec{a}$ (iii)

YOUR CLASSIC CASE:
 $\vec{r} = r \hat{e}_r \rightarrow \dot{\vec{r}} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta \rightarrow \dot{\vec{r}}^2 = \dot{r}^2 + r^2 \dot{\theta}^2$
 $\vec{a} = \ddot{r} \hat{e}_r - r \dot{\theta}^2 \hat{e}_r + 2\dot{r} \dot{\theta} \hat{e}_\theta + r \ddot{\theta} \hat{e}_\theta - r \dot{\theta} \ddot{\theta} \hat{e}_\theta$

3. **POINT PARTICLES:**
 $\vec{r} = r \hat{e}_r \rightarrow \dot{\vec{r}} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2) = \dot{\vec{r}} \cdot \ddot{\vec{r}} = \dot{r} \ddot{r} - r \dot{\theta}^2 + 2\dot{r} \dot{\theta} + r \ddot{\theta}$
 $\frac{d}{dt}(\frac{1}{2} \omega^2 r^2) = \omega^2 \vec{r} \cdot \dot{\vec{r}} = \omega^2 r \dot{r}$
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2 - \frac{1}{2} \omega^2 r^2) = \dot{r} (\ddot{r} - \omega^2 r) + 2\dot{r} \dot{\theta} + r \ddot{\theta} - \omega^2 r \dot{r}$
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2 - \frac{1}{2} \omega^2 r^2) = \dot{r} (\ddot{r} - \omega^2 r) + 2\dot{r} \dot{\theta} + r \ddot{\theta} - \omega^2 r \dot{r}$ (i)

4. $\vec{r} = r \hat{e}_r \rightarrow \dot{\vec{r}} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2) = \dot{\vec{r}} \cdot \ddot{\vec{r}} = \dot{r} \ddot{r} - r \dot{\theta}^2 + 2\dot{r} \dot{\theta} + r \ddot{\theta}$
 $\frac{d}{dt}(\frac{1}{2} \omega^2 r^2) = \omega^2 \vec{r} \cdot \dot{\vec{r}} = \omega^2 r \dot{r}$
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2 - \frac{1}{2} \omega^2 r^2) = \dot{r} (\ddot{r} - \omega^2 r) + 2\dot{r} \dot{\theta} + r \ddot{\theta} - \omega^2 r \dot{r}$ (ii)

ANSWER:
 $\frac{d}{dt}(\frac{1}{2} \dot{\vec{r}}^2 - \frac{1}{2} \omega^2 r^2) = \dot{r} (\ddot{r} - \omega^2 r) + 2\dot{r} \dot{\theta} + r \ddot{\theta} - \omega^2 r \dot{r}$ (iii)

TIPS:
 - MAKE SURE AT THE END, $\dot{r} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$
 - MAKE SURE AT THE END, $\ddot{r} = \ddot{r} \hat{e}_r - r \dot{\theta}^2 \hat{e}_r + 2\dot{r} \dot{\theta} \hat{e}_\theta + r \ddot{\theta} \hat{e}_\theta - r \dot{\theta} \ddot{\theta} \hat{e}_\theta$

Classical
MECHANICS
THIRD EDITION



off the mark by Mark Paris
www.offthemark.com



SIR ISAAC NEWTON WOULD HAVE DISCOVERED GRAVITY YEARS EARLIER HAD WILLIAM TELL NOT WANDERED BY

$a, p)$

$\frac{1}{2} m (\dot{P}_x^2 + \dot{P}_y^2)$

$H' = \frac{1}{2m} (\dot{P}_r^2 + \dot{P}_\theta^2 r^2) + U'(r)$

Overview

A. Definitions

1. Inertia
2. Force
3. Position
4. Velocity
5. Acceleration
6. Motion

Overview

B. Newton Laws

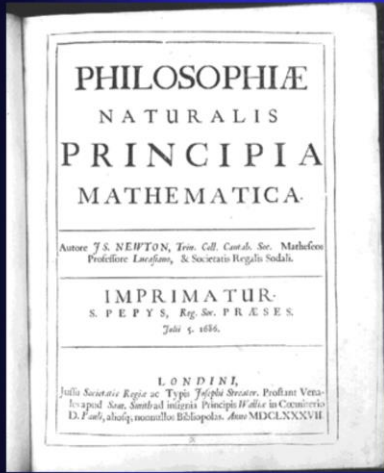
1. What's different?
2. Position, Velocity, Acceleration
3. Three Laws
4. Gravitation
5. Orbits!
6. Kepler's Second.
7. Unification!

Overview

C. Conservation Laws

1. What's that?
2. Conservation of Momentum and rockets
3. Conservation of Angular Momentum and Kepler's Second Law
4. Conservation of Energy and Kepler's Second Law
5. Scientific Models

The Rise of Modern Physics

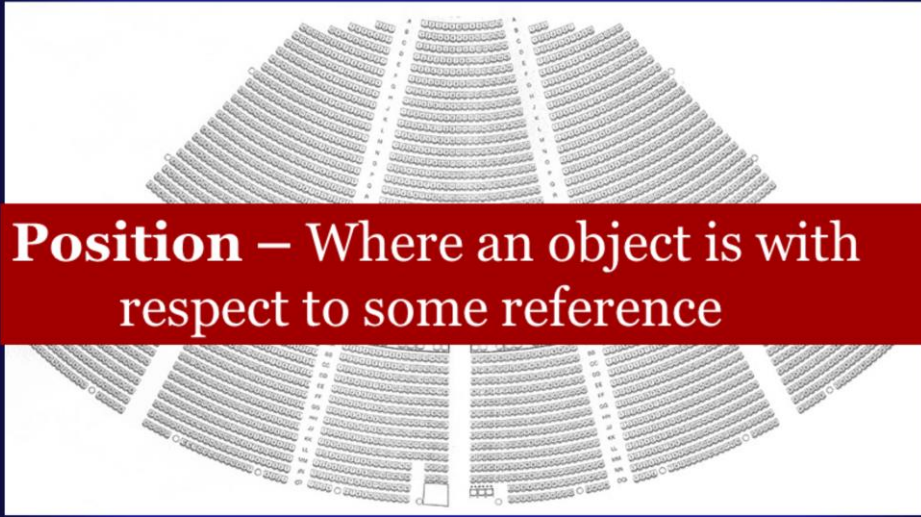


Isaac Newton

- Three laws of motion
- Universal Gravitation
- The Calculus

Simple central principals from which all motion, either on Earth or in the heavens, can be derived.

Position



Position – Where an object is with respect to some reference

Coordinates describe position

Velocity



Velocity – The rate of change of position

Velocity has something to do with changes in SPATIAL position.

The other element is time.

Velocity is the RATE of change in position.

It takes into account direction.

Acceleration



Acceleration – The rate of change of velocity

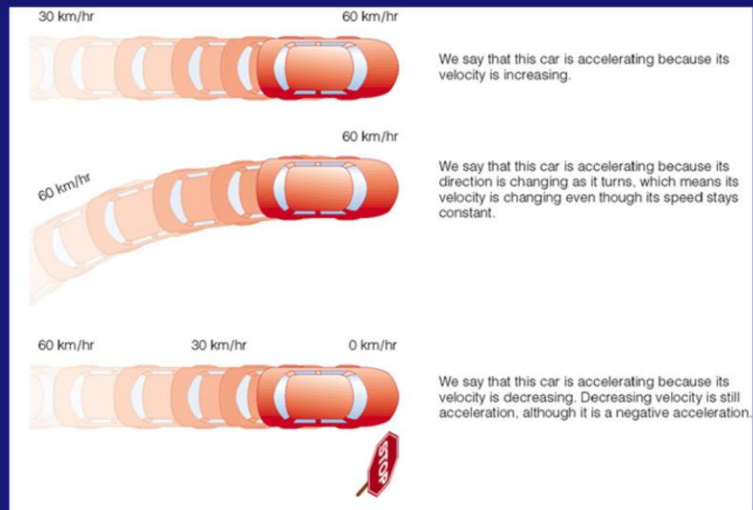
Define velocity... Note that it is a combination of Speed AND Direction.

Define force as causing a change in velocity

A change in direction... IS a change in velocity

Note net force... sum of all forces...

Acceleration



A change in *direction* counts as a change in *velocity*

So... forces cause changes in velocity... or accelerations.

The acceleration is proportional to the force... and inversely proportional to the mass.

ABCD

My car is going 60 mph in a straight line.
Its acceleration is:

A. 60

B. zero

C. less than zero

D. greater than zero but not 60

B

ABCD

My car is going 60 mph in a circle. It's acceleration is:

A. zero

B. not zero

C. depends on the circle

B

Newton's First Law

The Law of Inertia



Inertia – A massive object's natural tendency to **Resist Change**.



DONT BE
AFRAID OF CHANGE.

Left on their own, massive bodies have a **constant velocity**

Aristotle says:

The natural state of objects on the Earth is a state of rest.

An “impelling force” is required for objects to remain in motion.

Galileo Said NO!

The natural state of motion of an object is to keep doing what it's doing.

No “impelling

Newton repeated it.

This tendency to “keep doing what you're doing” is called Inertia.

The ball rolls to the back of the wagon when you pull the wagon because...

The ball's natural tendency is to stay where it is.

Newton's Second Law

Forces cause a change in the **velocity**



Force – A push or pull resulting from an **interaction** between two objects

Galileo didn't say this.

To get an object to change its state of motion, a force must be applied.

When we push the gas pedal in the car, the engine generates a force causing the car to move.

Why does the car stop if I stop pushing with the engine?

ABCD

A rocket in space is travelling with a non-zero constant velocity. This means that:

- A.** Its position **MUST** be changing
- B.** Its position is **NOT** changing
- C.** We don't know whether or not its position is changing

A

ABCD

A rocket in space is experiencing a constant acceleration. This means that:

- A.** Its velocity **MUST** be changing
- B.** Its velocity is **NOT** changing
- C.** We don't know whether or not its velocity is changing

A

ABCD

A rocket in space is experiencing a constant Force. This means that:

- A. Its acceleration **MUST** be changing
- B.** Its acceleration is **NOT** changing
- C. We don't know whether or not its acceleration is changing

B

ABCD

A rocket experiences a constant force. This means that:

- A.** Its inertia is changing
- B.** Its acceleration is changing
- C.** Its position is not changing
- D.** Its experiencing a constant acceleration

D

Newton's Third Law

If you **PUSH** on something, it *pushes back*



Forces come in pairs.

If you apply a force to something, it responds by pushing back.

As I push against a baseball to throw it, I can FEEL it pushing back.

This is how rockets work. We push mass out the back of the rocket and the opposing force pushes back, propelling the rocket forward.

ABCD

When I throw a ball in the air, it keeps going up after I release it because:

- A.** There is an upward pointing force on it
- B.** its Inertia carries it upward without the need for a force.
- C.** The Moon pulls it
- D.** My hand is pushing it.

B

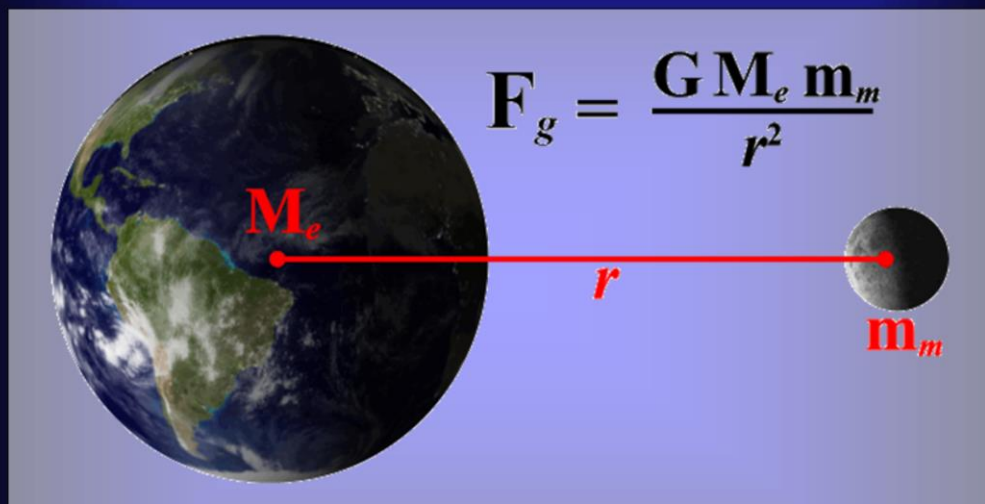
ABCD

When making a sharp left turn in a car, we feel like we are being flung across the car to the right. What's happening?

- A.** There is a force on us pointing to the right.
- B.** We are trying to go straight (inertia) but the car is pulling our bottoms left.
- C.** The Moon is pulling us.
- D.** There is a centrifugal force.

B

Universal Law of Gravitation



Objects with **mass** exert an attractive force on one another.

ABCD

When you are on the top of a building, your weight is ____ when you are on the ground floor.

- A. slightly greater than
- B. the same as
- C. slightly less than

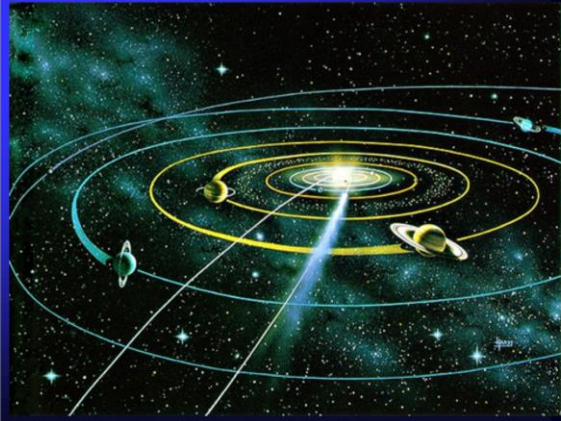
C

What is an Orbit?

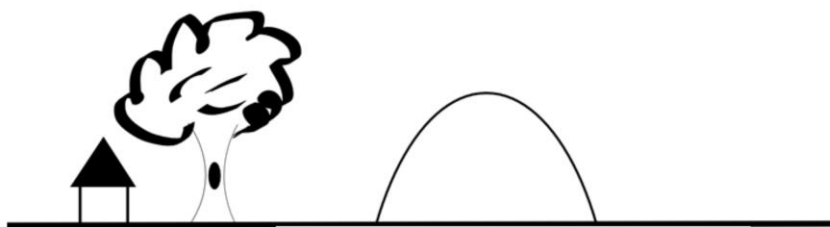


The knack lies in learning how to throw yourself at the ground and miss.

~Arthur Dent



Heaven and Earth United



Newton versus Kepler

Kepler's laws are a special case of
Newton's laws.

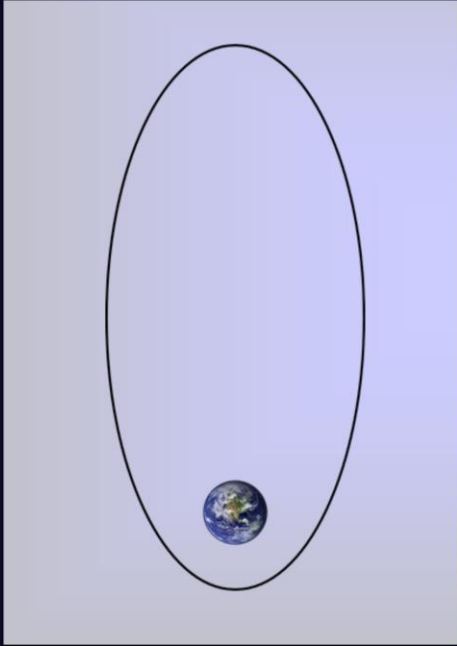


My laws make
very *accurate*
predictions!



My laws are
more general!

Heaven and Earth United



Newton's Laws lead
to Kepler's Laws

Ellipses are a
consequence

Fast at perihelion
Slow at aphelion

Longer period for
larger orbits

Unification!

Physicists seek
Unifying Principles

Newton's model
Unifies the Heavens
and the Earth



Conservation Laws



The Law of
Conservation of Blocks

Momentum

$$\mathbf{p} = \mathbf{mv}$$

Momentum is **conserved**

Our first conservation law.

In a closed system **MOMENTUM IS ALWAYS CONSERVED.**

This includes situations where the gravitational force is involved.

It also includes situations where energy is transferred such as:

Friction robs kinetic energy

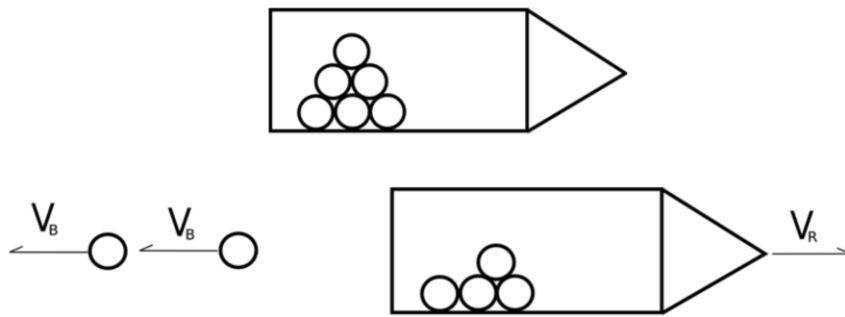
and explosion occurs (potential energy transfers into kinetic energy)

If we add up $(M1 \times V1) + (M2 \times V2) + \dots$ and wait for some time

And add them up again, the answer will be the **SAME.**

As long as no external forces act on the system.

Momentum



Consider the Momentum. Which statement is true?

- A. Momentum A < Momentum B
- B. Momentum A = Momentum B
- C. Momentum A > Momentum B

B

Rockets Conserve Momentum



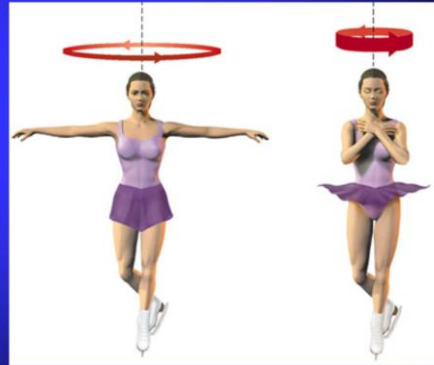
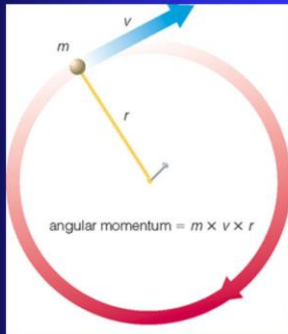
In the rocket, an EXPLOSION occurs that propels exhaust out the back of the rocket and propels the rocket forward.

The system conserves momentum.

If the rocket fuel system has zero momentum initially, the momentum of the rocket plus the momentum of the exhaust will be zero afterward.

Angular Momentum

$$L = mvr$$



Also conserved...

Closed systems also conserve ANGULAR momentum.

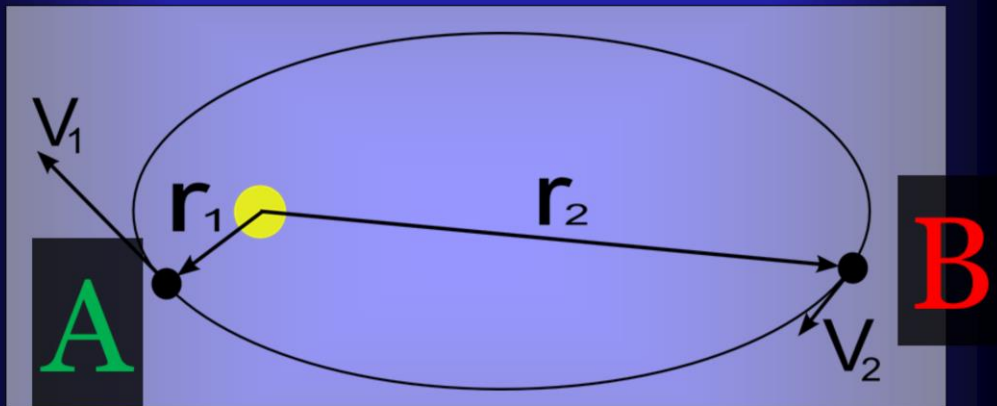
Angular momentum takes into account the distance of the massive object from a pivot point.

The skater spins faster when her arms are tucked in.

Initially, r is large (her arms are far from her body) and v is small.

Later, r is small (her arms are close to her body) and v is big.

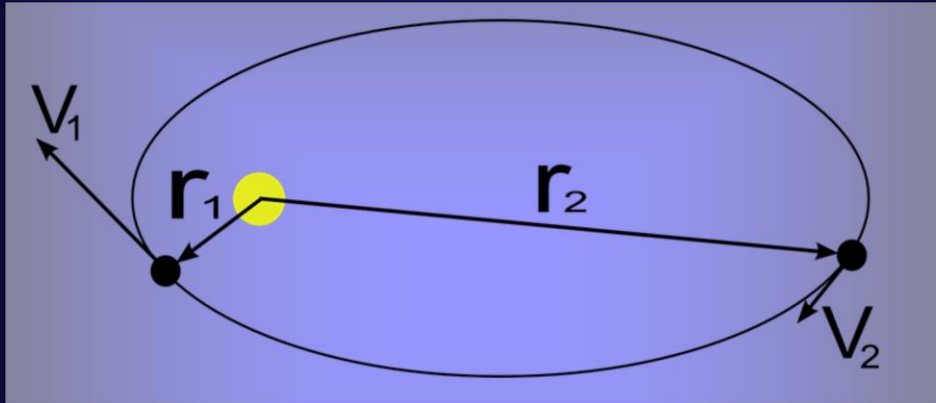
Orbital Angular Momentum



Which point has greater angular momentum?

Neither! It's the same at A and B

Orbital Angular Momentum



How does conservation of angular momentum explain Kepler's Second Law?

Energy

What is Energy?



We buy it in the form of electricity and gas

We gather it when we eat food and expend it when we do things.

But what IS it?

Types of Energy

Kinetic

- Motion



Potential

- Gravitational
- Chemical
- Springs/Rubber Bands



Radiative

- Light



All of the energy forms that you can think of fit into one of these three types.

Potential (stored) Energy



Energy that is stored. Potential energy can be converted into either kinetic or radiative (light) energy.

Sources of Potential Energy:

Chemical-

Gasoline, Hydrogen (both are forms of combustion, or combining with oxygen)

Gravitational-

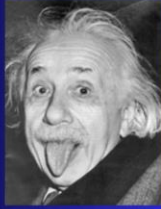
Water flowing down a mountain.

A space rock falling towards the earth

Molecular bonds

Springs and rubber bands

Mass Energy



$$E = mc^2$$

Principle behind

- Nuclear Fission (Nuclear reactors)
- Nuclear Fusion (The Sun)

It is another form of potential energy

Einstein says, Mass is another form of energy.

And... there's a LOT of energy locked up in massive objects.

The total energy is the mass times the speed of light squared.

Really, the universe is made of nothing but energy.

Some is kinetic, some is light, some is locked up in mass, some is stored in fields.

Kinetic Energy

$$K = \frac{1}{2} mv^2$$



Kinetic energy depends on

mass

A truck moving at 10 miles an hour has more energy than a bicycle moving at moving at 10 miles per hour

And the SQUARE of the velocity.

You saw this in the Impact lab.

2 times the velocity is 4 times the energy

4 times the velocity is 16 times the energy

Fast moving asteroids carry ENORMOUS kinetic energy.

Energy is CONSERVED



**Energy can be *transformed*
but not created or
destroyed**

Where does the energy in the gasoline come from?

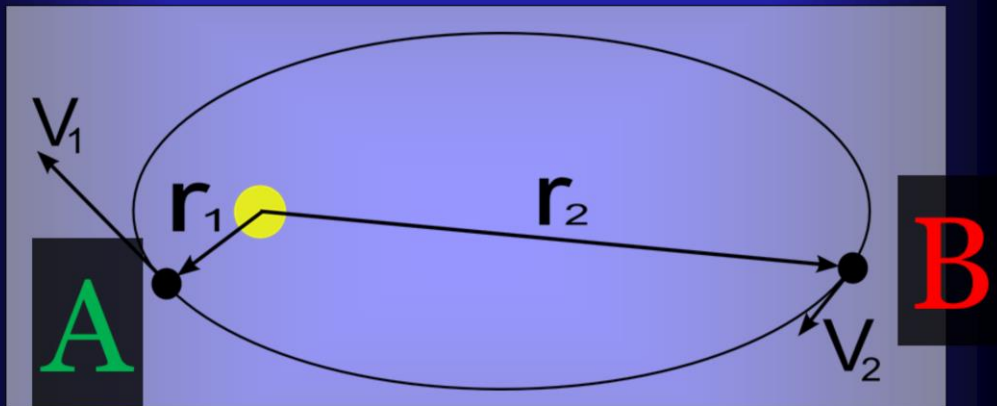
ABCD

You throw a rock into the air. After leaving your hand, it has kinetic energy. At the top of its flight when its velocity is zero, its TOTAL energy is

- A) It has zero energy
- B) Its energy is the same as when it left your hand
- C) It has less energy than when it left your hand
- D) It has more energy than when it left your hand.

B

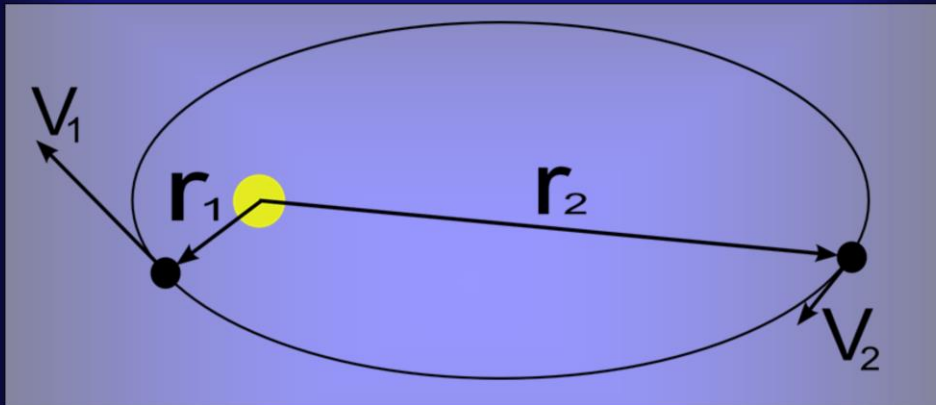
Orbital Energy



Which point has greater energy?

Neither! Total energy is conserved.

Orbital Angular Momentum



How does conservation of angular momentum explain Kepler's Second Law?

Scientific Models

Three Models

All give the **Same**
answer.

Which one is “**right?**”
What does it **Mean?**



The question “Which one is right?” is probably nonsense.
The right model is the one that provides the best predictions.

What it means is probably more a matter of philosophy than science.