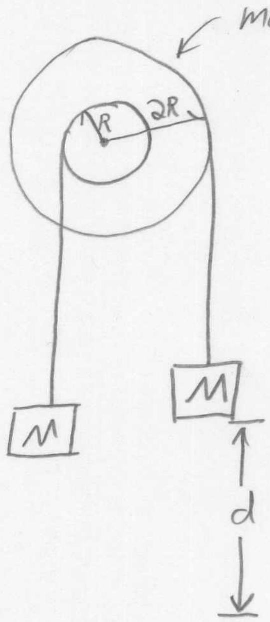
Problems From Lecture - Modified Atwood.

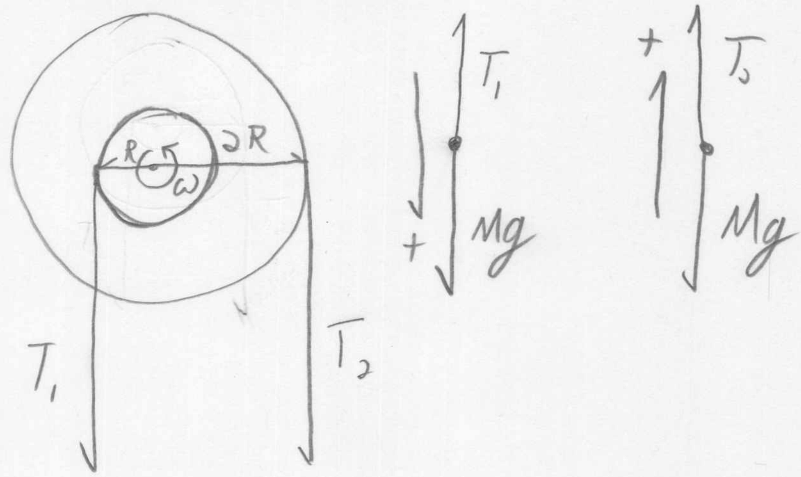


what's v after the mass drops a distance d?

Let:

$I = 2MR^2$

FBD



ω is positive counter-clockwise, so T_1 is negative and T_2 is positive in the linear FBDs. In the angular FBD, T_1R is positive and $2T_2R$ is negative.

Torque

① $T_1R - 2T_2R = I\alpha$

④ $\alpha = \frac{a_2}{2R}$

Linear

② $Mg - T_1 = Ma_1$ ③ $T_2 - Mg = Ma_2$

$a_1 = R\alpha$, $a_2 = 2R\alpha$

$\Rightarrow a_2 = 2a_1$ ⑤

④ and ⑤ are from the general law $a = R\alpha$

continued ↓

Solve (2) and (3) for T_1 and T_2 and plug into (1)

$$T_1 = Mg - Ma_1 , \quad T_2 = Mg + Ma_2 , \quad I = 2MR^2$$

$$(Mg - Ma_1)R - 2(Mg + Ma_2)R = 2MR^2\alpha$$

$$(6) \quad g - a_1 - 2g - 2a_2 = 2R\alpha$$

plug in (4) and (5) into (6)

$$g - \tfrac{1}{2}a_2 - 2g - 2a_2 = 2R\frac{a_2}{2R}$$

$$-a_2 + \tfrac{1}{2}a_2 - 2a_2 = 2g - g$$

$$-\tfrac{7}{2}a_2 = g \Rightarrow \boxed{a_2 = -\tfrac{2}{7}g}$$

Now find v after d from kinematics

$$d = \tfrac{1}{2}at^2 \qquad v = at$$

$$d = \tfrac{1}{2}a\frac{v^2}{a^2} \Rightarrow v^2 = 2da$$

$$\boxed{v^2 = \tfrac{4}{7}gd}$$