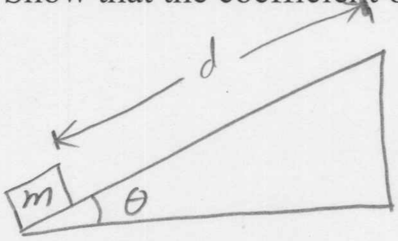
Test 2
Phys 111, Fall 2009, Section 1

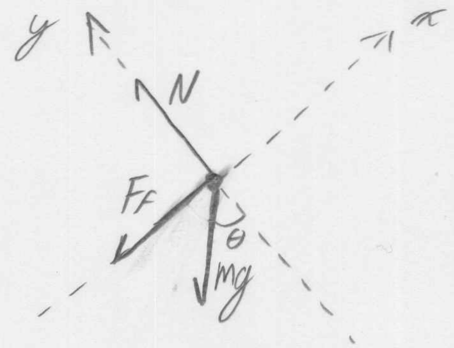
A block is projected up an incline plane making an angle θ with the horizontal. It returns to its initial position with half of its initial speed. Show that the coefficient of kinetic friction is: $\mu_k = 3/5\tan(\theta)$.



So: the block has an initial velocity V_0, it goes up the ramp a distance d, stops, and comes back down. Its final velocity is:

$$V_f = \tfrac{1}{2} V_0$$

On the way up – Friction opposes motion



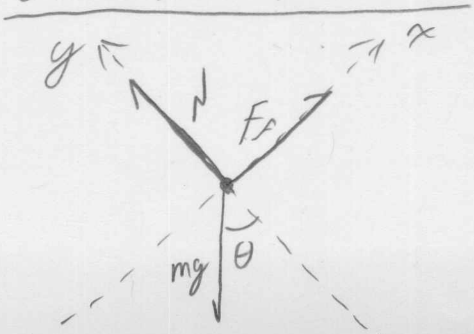
$x: -\mu_k N - mg\sin\theta = ma_{up}$

$y: N - mg\cos\theta = 0$

$\Rightarrow \mu_k \cancel{m}g\cos\theta + \cancel{m}g\sin\theta = -\cancel{m}a_{up}$

$\Rightarrow \boxed{a_{up} = -g(\sin\theta + \mu_k\cos\theta)}$

On the way down – Friction opposes motion



$x: \mu_k N - mg\sin\theta = ma_{dwn}$

$y: N - mg\cos\theta = 0$

$\boxed{a_{dwn} = -g(\sin\theta - \mu_k\cos\theta)}$

So the acceleration up is different than the acceleration down. So we'll take the kinematiccs in ~~two~~ two parts, up and then down.

up - a distance d and stops

$x_f = x_0 + v_0 t + \frac{1}{2} a_{up} t^2$ $\qquad$ $v_f = v_0 + a_{up} t$

$d = 0 + v_0 t + \frac{1}{2} a_{up} t^2$ $\qquad$ $0 = v_0 + a_{up} t$

Eliminate t $\qquad$ $t = -\frac{v_0}{a_{up}}$

$$d = -\frac{v_0^2}{a_{up}} + \frac{1}{2} \frac{v_0^2}{a_{up}}$$

$$\boxed{d = -\frac{1}{2} \frac{v_0^2}{a_{up}}}$$

down - goes a distance d from rest

$x_f = x_0 + v_0 t + \frac{1}{2} a_{dwn} t^2$ $\qquad$ $v_f = v_0 + a_{dwn} t$

$0 = d + 0 + \frac{1}{2} a_{dwn} t^2$ $\qquad$ $v_f = a_{dwn} t$

$t = \frac{v_f}{a_{dwn}}$

$$0 = d + \frac{1}{2} \frac{v_f^2}{a_{dwn}}$$

Plug in d

$\Rightarrow 0 = -\cancel{\frac{1}{2}} \frac{v_0^2}{a_{up}} + \cancel{\frac{1}{2}} \frac{v_f^2}{a_{dwn}}$ and use $v_f = \frac{1}{2} v_0$

$$\frac{\cancel{v_0^2}}{a_{up}} = \frac{1}{4} \frac{\cancel{v_0^2}}{a_{dwn}} \Rightarrow 4 a_{dwn} = a_{up}$$

continued ↓

$$+4\cancel{g}(\sin\theta - \mu_k \cos\theta) = \cancel{g}(\sin\theta + \mu_k \cos\theta)$$

$$4\sin\theta - 4\mu_k \cos\theta = \sin\theta + \mu_k \cos\theta$$

$$3\sin\theta = 5\mu_k \cos\theta$$

$$\boxed{\mu_k = \tfrac{3}{5}\tan\theta}$$