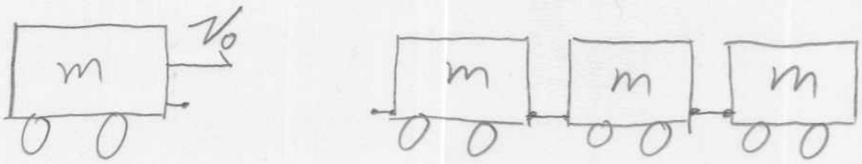
A 4000 kg railroad car inelastically collides with three other 4000 kg cars sitting at rest on a rough track. The four cars travel together down the rough track for 1.5 m before they stop. Assuming $\mu_k = 0.10$, what is the velocity of the first car at impact?

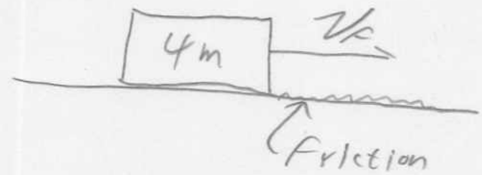


Find velocity after collision

$$\cancel{m}V_0 = 4\cancel{m}V_F \quad : \text{ conserve momentum}$$

$$V_F = \frac{V_0}{4}$$

Find distance to stop



$$K_i = \tfrac{4}{2} m V_F^2 \qquad U_i = 0$$

$$K_f = 0 \qquad U_f = 0$$

$$E_f = N\mu_k d = 4mg\mu_k d$$

$$\tfrac{\cancel{4}}{2}\cancel{m}\frac{V_0^2}{4^{\cancel{2}}} = 4\cancel{m}g\mu_k d$$

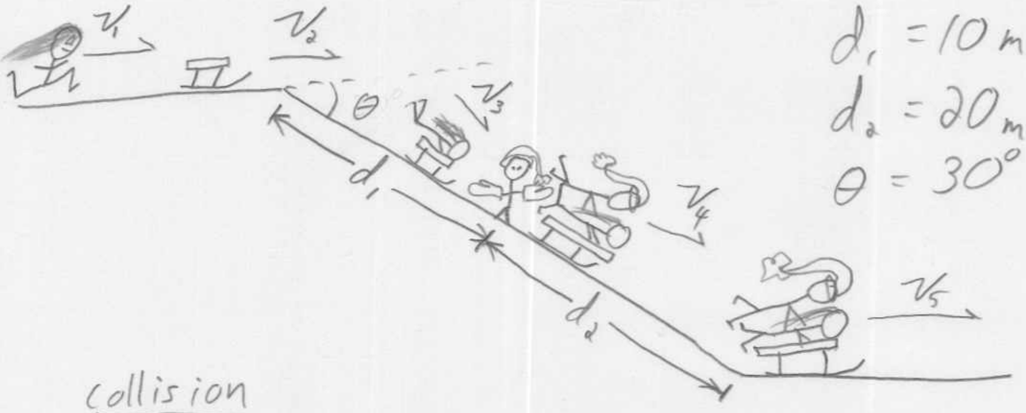
$$\boxed{V_0 = \sqrt{32 g \mu_k d}} = \boxed{6.9\,\text{m/s}}$$

Momentum, Impulse, and Collisions

Gayle runs at a speed of 4.0 m/s and dives onto a sled that is initially at rest on top of a frictionless snow covered hill sloping down at 30°. After she has traveled 10 m down the slope, her brother Billy hops on the sled; and together they travel on down the slope another 20 m. (Billy was at rest initially) What is their speed at the bottom of the hill?

$m_{Gayle} = 50.0$ kg, $m_{Billy} = 30.0$ kg, $m_{sled} = 5.0$ kg

$v_1 = 4.0$ m/s
$d_1 = 10$ m
$d_2 = 20$ m
$\theta = 30°$



collision

① $m_g v_1 = (m_g + m_s) v_2$: conserve momentum

$$v_2 = \frac{m_g}{(m_g + m_s)} v_1 = \frac{50}{50+5}(4) = \boxed{3.63 \text{ m/s}}$$

② work-energy

$K_i = \frac{1}{2}(m_g + m_s) v_2^2$ $\qquad$ $K_f = \frac{1}{2}(m_g + m_s) v_3^2$

$U_i = (m_g + m_s) g h = (m_g + m_s) g d_1 \sin\theta$. $U_f = 0$

$$\frac{1}{2}\cancel{(m_g + m_s)} v_2^2 + \cancel{(m_g + m_s)} g d_1 \sin\theta = \frac{1}{2}\cancel{(m_g + m_s)} v_3^2$$

$$v_3 = (v_2^2 + 2 g d_1 \sin\theta)^{1/2} = (3.63^2 + 2(9.8)(10)\sin(30))^{1/2}$$

$$\boxed{= 10.5 \text{ m/s}}$$

3 collision #2

$$(m_s + m_g) v_3 = (m_s + m_g + m_B) v_4 \quad \text{; conserve momentum}$$

$$v_4 = \frac{m_s + m_g}{m_s + m_g + m_B} v_3 = \frac{50+5}{50+5+30} \cdot 10.5$$

$$= \underline{6.79 \ m/s}$$

4 Work - Energy

$$K_i = \tfrac{1}{2}(m_s + m_g + m_B) v_4^2 \qquad K_f = \tfrac{1}{2}(m_s + m_g + m_B) v_5^2$$

$$U_i = (m_s + m_g + m_B) g d_2 \sin\theta \qquad U_f = 0$$

$$\tfrac{1}{2}\cancel{(m_s + m_g + m_B)} v_4^2 + \cancel{(m_s + m_g + m_B)} g d_2 \sin\theta = \tfrac{1}{2}\cancel{(m_s + m_g + m_B)} v_5^2$$

$$v_5 = \left[v_4^2 + 2 g d_2 \sin\theta\right]^{1/2}$$

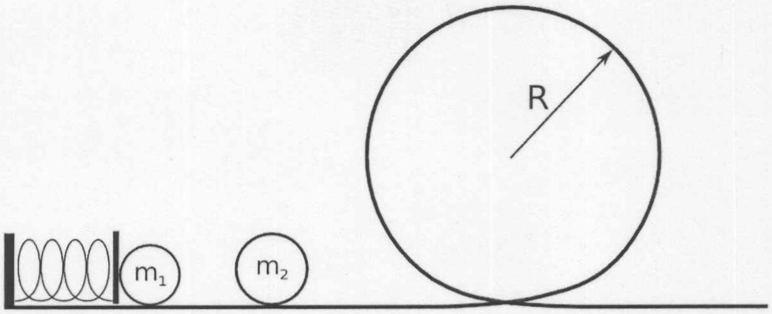
$$= \left[(6.79)^2 + (2)(9.8)(20)\sin(30)\right]^{1/2}$$

$$\boxed{= 15.6 \ m/s}$$

Momentum, Impulse, and Collisions

In the system below, a ball of mass m_1 is placed against a spring with spring constant k that has been compressed a distance d. It is released from rest and collides with a second ball of mass m_2 which then goes around the loop the loop of radius R.

What is the minimum spring compression d such that m_2 makes it around the loop?



Stage 1 - Get m_1 off of the spring

Work - Energy

$U_I = \frac{1}{2}kd^2 \qquad U_F = 0$

$K_I = 0 \qquad K_F = \frac{1}{2}mV_1^2$

$$\cancel{\tfrac{1}{2}}kd^2 = \cancel{\tfrac{1}{2}}mV_1^2 \Rightarrow \boxed{V_1 = \sqrt{\frac{kd^2}{m}}}$$

Stage 2 - Collision, Elastic

$m_1V_1 = m_1V_{1F} + m_2V_2$ conserve momentum

$\cancel{\tfrac{1}{2}}m_1V_1^2 = \cancel{\tfrac{1}{2}}m_1V_{1F}^2 + \cancel{\tfrac{1}{2}}m_2V_2^2$ conserve kinetic energy

Gather terms and divide

$$\frac{\cancel{m_1}(V_1^2 - V_{1F}^2)}{\cancel{m_1}(V_1 - V_{1F})} = \frac{\cancel{m_2}V_2^2}{\cancel{m_2}\cancel{V_2}}$$

Continued ↓

Remember: $(a+b)(a-b) = a^2 - b^2$

$$\frac{(v_1 + v_{1F})\cancel{(v_1 - v_{1F})}}{\cancel{(v_1 - v_{1F})}} = v_2 \Rightarrow \underline{v_{1F} = v_2 - v_1}$$

Plug back into momentum eq.

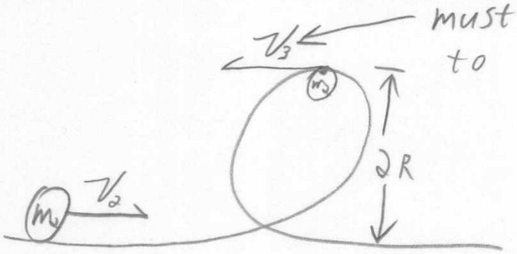
$$m_1 v_1 = m_1(v_2 - v_1) + m_2 v_2$$

$$m_1 v_1 = m_1 v_2 - m_1 v_1 + m_2 v_2$$

$$2 m_1 v_1 = (m_1 + m_2) v_2$$

$$\boxed{v_2 = \frac{2 m_1}{(m_1 + m_2)} v_1}$$

<u>Stage 3</u> - Get around the loop



<u>NSL</u>

$$\cancel{-N}^{\,0} + mg = +m\frac{v_3^2}{R}$$

goes to zero when it <u>just</u> loses contact

$$\underline{v_3^2 = gR}$$

$U_I = 0$ $\quad U_F = 2mgR$

$K_I = \frac{1}{2} m v_2^2$ $\quad K_F = \frac{1}{2} m v_3^2 = \frac{1}{2} mgR$

$$\frac{1}{2}\cancel{m} v_2^2 = 2\cancel{m}gR + \frac{1}{2}\cancel{m}gR$$

$$\boxed{\cancel{\tfrac{1}{2}} v_2^2 = \frac{5}{\cancel{2}} gR}$$

continued

↓