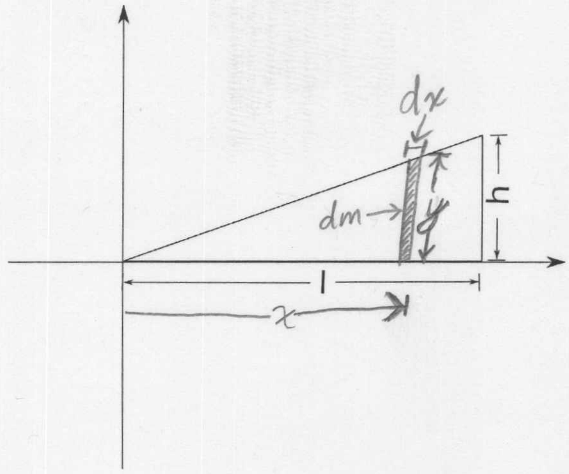
Momentum, Impulse, and Collisions

Calculate the center of mass of a triangular chunk of aluminum of mass M, length l, and height h.



$x_{cm} = \frac{1}{M}\int x\,dm$, x is the x-distance from the origin
dm is the mass of the strip located at x
M is the total mass of the triangle

In general, mass is surface density times area.

The surface density of any object is: $\sigma = \frac{M}{A}$

For our triangle: $A = \frac{1}{2}hl$

so : $\sigma = \frac{2M}{hl}$

The area, dA, of our strip at x is: $dA = y\,dx$

so: $dm = \sigma\,dA = \frac{2M}{hl}\,y\,dx$

continued ↓

We want to integrate in x, so we have to rewrite y in terms of x. Using the law of similar triangles, we have:

$$\frac{h}{\ell} = \frac{y}{x} \Rightarrow y = x\frac{h}{\ell}$$

So, Finally, $dm = \frac{2M}{\cancel{h}\ell}\, x \frac{\cancel{h}}{\ell}\, dx$

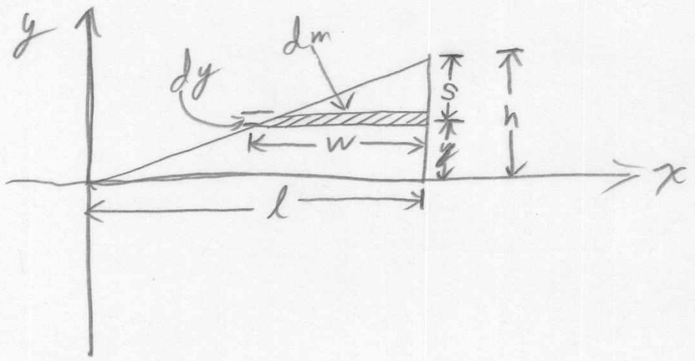
$$dm = \frac{2M}{\ell^2}\, x\, dx$$

Then: $X_{cm} = \frac{1}{\cancel{M}} \int_0^{\ell} x \frac{2\cancel{M}}{\ell^2} x\, dx$

$$= \frac{2}{\ell^2}\int_0^{\ell} x^2\, dx = \frac{2}{\ell^2}\left(\tfrac{1}{3}x^3\right)\Big|_0^{\ell} = \frac{2}{3}\frac{\ell^{\cancel{3}}}{\cancel{\ell^2}}$$

$$\boxed{X_{cm} = \frac{2}{3}\ell}$$

The y-axis is similar



$$y_{cm} = \frac{1}{M}\int y\, dm$$

In this case, consider the strip located at a vertical distance y from the origin

$$dm = \sigma \underbrace{w\, dy}_{\text{Area}}$$

↑ surface density

Now, similar triangles to get w in terms of y: $\frac{h}{\ell} = \frac{s}{w} \Rightarrow \frac{h}{\ell} = \frac{h-y}{w} \Rightarrow w = \frac{\ell(h-y)}{h}$

So: $dm = \frac{2M}{h\cancel{\ell}}\frac{\cancel{\ell}}{h}(h-y)\, dy$ continued ↓

50:

$$y_{cm} = \frac{1}{\cancel{M}} \int_0^h y \frac{2\cancel{M}}{h^2}(h-y)\,dy$$

$$= \frac{2}{h^2}\int_0^h (yh - y^2)\,dy = \frac{2}{h^2}\left(\tfrac{1}{2}hy^2 - \tfrac{1}{3}y^3\right|_0^h$$

$$= \frac{2}{\cancel{h^2}}\left(\frac{1}{2}h^{\cancel{3}} - \frac{1}{3}h^{\cancel{3}}\right)$$

$$= h\left(1 - \tfrac{2}{3}\right)$$

$$\boxed{y_{cm} = \tfrac{1}{3}h}$$

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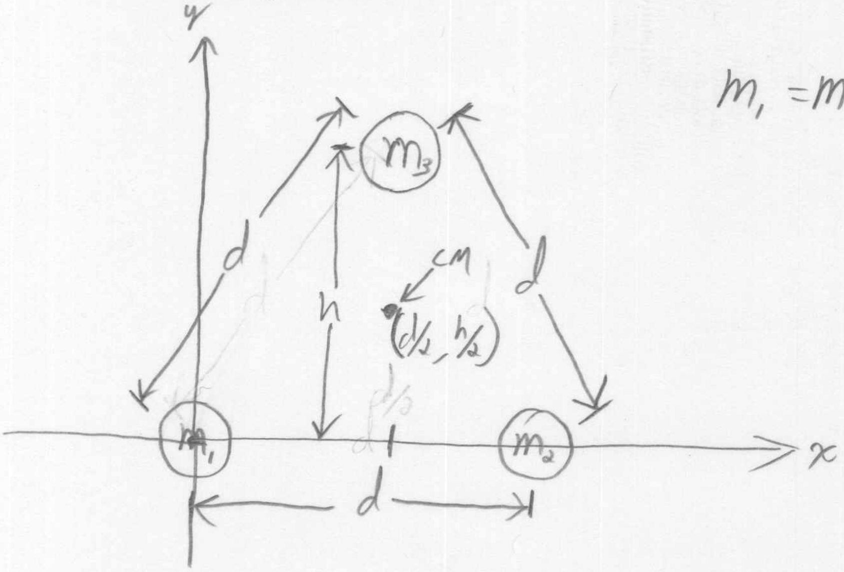
Three particles are positioned at the vertices of an equilateral triangle. m_1 and m_2 lie on the x-axis and are the same mass. The center of mass of the system is exactly in the center of the triangle. What is m_3?



$m_1 = m_2 = m$

$$x_{cm} = \frac{dm + \frac{d}{2}m_3}{2m + m_3}, \quad x_{cm} = \frac{d}{2} \Rightarrow \frac{\cancel{d}}{2} = \frac{\cancel{d}m + \frac{\cancel{d}}{2}m_3}{2m + m_3}$$

$$\cancel{m} + \cancel{\tfrac{1}{2}m_3} = \cancel{m} + \cancel{\tfrac{1}{2}m_3} \Rightarrow \underline{0 = 0}$$

$y_{cm} =$

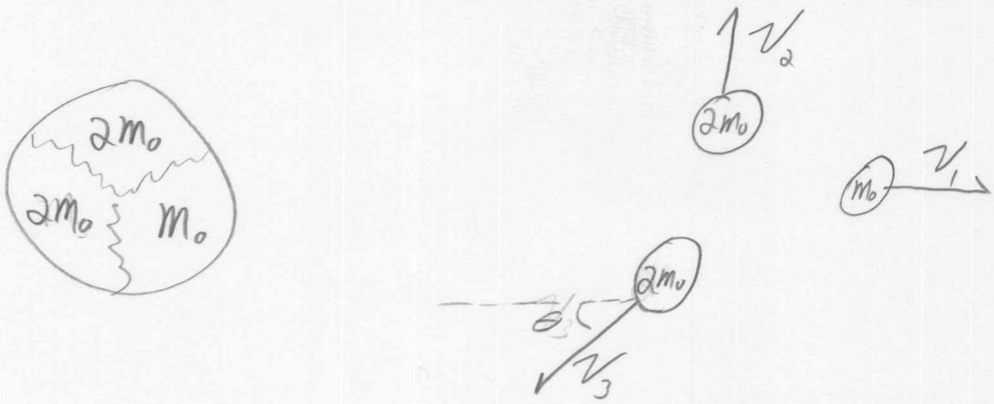
no help from x-axis. m_3 could be anything.

$$y_{cm} = \frac{\cancel{h}}{2} = \frac{\cancel{h}m_3}{2m + m_3} \Rightarrow m + \tfrac{1}{2}m_3 = m_3$$

$$\boxed{m_3 = 2m}$$

Momentum, Impulse, and Collisions

An object with a mass of $5m_o$ explodes at rest breaking into three pieces. One of the pieces with a mass of m_o travels in the x direction at 30.0 m/s. Another piece also with a mass of $2m_o$ travels in the y direction at 20.0 m/s. What is the magnitude and direction of the velocity of the last piece? What is the kinetic energy released in the explosion?



Conserve momentum - (or conserve v_{cm})

2D:

$$x: 0 = \cancel{m_o} v_1 + 2\cancel{m_o} v_{3x} \Rightarrow \boxed{v_{3x} = -\tfrac{1}{2} v_1}$$

$$y: 0 = \cancel{2m_o} v_2 + \cancel{2m_o} v_{3y} \Rightarrow \boxed{v_{3y} = -v_2}$$

$$|v_3| = \left(\left(\tfrac{1}{2} v_1\right)^2 + v_2^2\right)^{1/2}$$

$$= \left(\frac{90}{4} + 40\right)^{1/2} = 7.9 \text{ m/s}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{15}{20}\right) = 37^\circ}$$