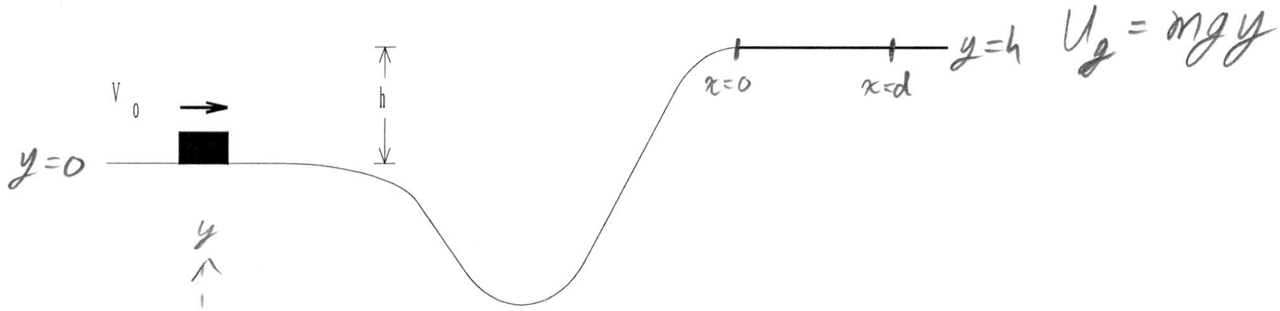
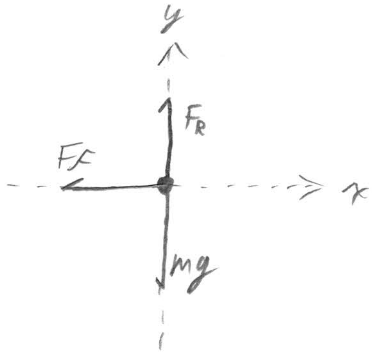
Energy Problems – Set 3

A block slides along the track shown below. The track is frictionless until the block reaches the level portion at the top of the hill, where the coefficient of friction is μ_k.





$$F_f = -\mu_k F_R \hat{x} = -\mu_k m g \hat{x}$$

$$U_I = 0 \qquad U_F = mgh$$

$$K_I = \tfrac{1}{2} m v_0^2 \qquad K_F = 0$$

$$W_F = \int_0^d \vec{F}_f \cdot d\vec{s} = \int_0^d (-\mu_k m g \hat{x}) \cdot (dx\,\hat{x}) = -\mu_k m g d$$

$$U_I + K_I + W_F = U_F + K_F$$

$$0 + \tfrac{1}{2} m v_0^2 - \mu_k m g d = mgh$$

$$\boxed{v_0 = \left[2g(\mu_k d + h)\right]^{1/2}}$$

Energy Problems – Set 3

A brick is lifted to a certain height and then dropped to the ground. Next, a second identical brick is lifted twice as high as the first and also dropped to the ground. When the second brick strikes the ground, it has

A. half as much kinetic energy as the first.
B. as much kinetic energy as the first.
C. twice as much kinetic energy as the first.
D. four times as much kinetic energy as the first.

Explain:

Energy goes as mgh. Double the height, Double the energy.

Using ratios: $E_T = U + K$

$E_1 = mgh_1$

$E_2 = mgh_2 \qquad h_2 = 2h_1$

$$\frac{E_2}{E_1} = \frac{mgh_2}{mgh_1} = \frac{\cancel{mg}2\cancel{h_1}}{\cancel{mgh_1}} = 2$$

A bottle dropped from a balcony strikes the sidewalk with a particular speed. To double the speed of impact, you would have to drop the bottle from a balcony

A. twice as high.
B. three times as high.
C. four times as high.
D. five times as high.
E. six times as high.

Explain: Let's use ratios.

In general: $W_{net} = \Delta K \Rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \Rightarrow h = \frac{v^2}{2g}$

So: $h_1 = \frac{v_1^2}{2g}$ and $h_2 = \frac{(2v_1)^2}{2g} = \frac{\cancel{4}^{2} v_1^2}{2g}$

and $\frac{h_2}{h_1} = \frac{2\cancel{v_1^2}}{\cancel{g}} \cdot \frac{2\cancel{g}}{\cancel{v_1^2}} = 4$

Energy Problems – Set 3

A car is going 10 mph. The driver hits the brakes. The car travels 3 feet after the brakes are applied. A while later, the same car is going 20 mph. The driver hits the brakes. About how far does the car go after the brakes are applied?

ratios: $W_{net} = \Delta K \Rightarrow -\mu_k mgd = 0 - \frac{1}{2}mv^2$

A. 3 ft.
B. 6 ft.
C. 9 ft.
D. 12 ft.
E. 15 ft.

$\Rightarrow d = \frac{v^2}{2\mu_k g}$ $\quad \frac{d_2}{d_1} = \frac{v_2^2}{2\mu_k g} \cdot \frac{2\mu_k g}{v_1^2} \Rightarrow d_2 = \frac{v_2^2}{v_1^2} d_1$

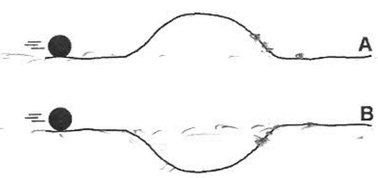
Explain: so: $d_2 = \frac{20^2}{10^2} d_1 = 4 \cdot d_1 = 12$

twice the velocity = 4 times the energy

Friction is linear (brakes)

$3 \cdot 4 = 12$

Two identical balls are rolled across tracks A and B. The height of the bump in track A is equal to the depth of the dip in track B. Both balls begin rolling at the same time and with the same speed. Ignore friction. Circle ALL of the following statements that are true.



A. The final speed of ball A is greater than that of ball B.
B. The final speed of ball B is greater than that of ball A.
C. The two balls have the same final speed.
D. The ball on track A finishes the race first.
E. The ball on track B finishes the race first.
F. The two balls take the same amount of time to reach the ends of their tracks.

Explain:

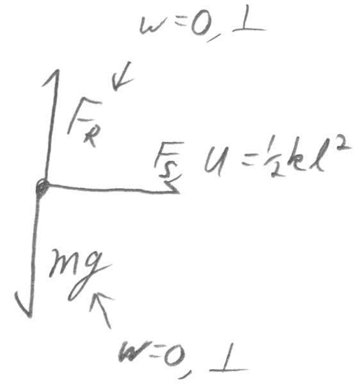
The change in potential is the same for both, (0) so the final velocity is the same.

At all points, $v_B \geq v_A$ so B must finish first.

A block of mass m is pushed against a spring of spring constant k and the spring is compressed a distance l. The block is released and slides across a frictionless surface for a short distance before encountering a surface with a coefficient of friction μ_k.

a. Use conservation of energy to find an expression for the velocity of the block after it leaves the spring.

b. Use conservation of energy to find an expression for how far it slides on the surface with friction before coming to a stop.



a) $U_I = \frac{1}{2}kl^2$ $U_F = 0$

$K_I = 0$ $K_F = \frac{1}{2}mv^2$

$$\cancel{\tfrac{1}{2}}kl^2 = \cancel{\tfrac{1}{2}}mv^2 \Rightarrow \boxed{v = \sqrt{\frac{k}{m}}\,l}$$

b) $U_I = 0$ $U_F = 0$

$K_I = \frac{1}{2}mv^2$ $K_F = 0$

$$W_F = \int_0^d \vec{F}_F \cdot d\vec{r} = \int_0^d (-\mu_k mg\hat{x})\cdot(dx\hat{x}) = -\mu_k mgd$$

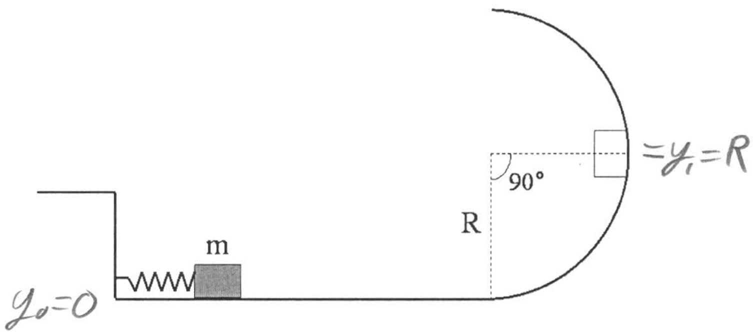
$$\frac{1}{2}mv^2 - \mu_k \cancel{m}gd = 0 \Rightarrow \frac{1}{2}\frac{k}{m}l^2 = \mu_k gd$$

$$\boxed{d = \frac{kl^2}{2\mu_k mgd}}$$

Energy Problems – Set 3

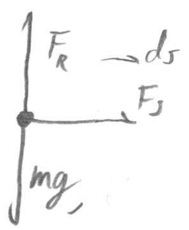
A mass m rests on a frictionless horizontal track while compressing a horizontal spring of spring constant k. The mass is released and it along a frictionless horizontal track before sliding up a frictionless circular surface of radius R.

a. Find an expression for the compression d such that the mass just comes to rest at a radius position of θ=90° as shown in the picture?



b. Now include friction in the problem. If the block stops at a radius position of θ=35°, how much work was done by the frictional force acting on the block?

a)



$W_{F_R}=0,\ \vec{F}_R \perp d\vec{s}$

$U_g = mgy$

$U_s = \frac{1}{2}kd^2$

$U_I = \frac{1}{2}kd_0^2 + \cancel{mgy_0}^{\,0}$ $\quad U_F = \cancel{\frac{1}{2}kd_1^2}^{\,0} + mgy_1$

$K_I = 0$ $\quad K_F = 0$

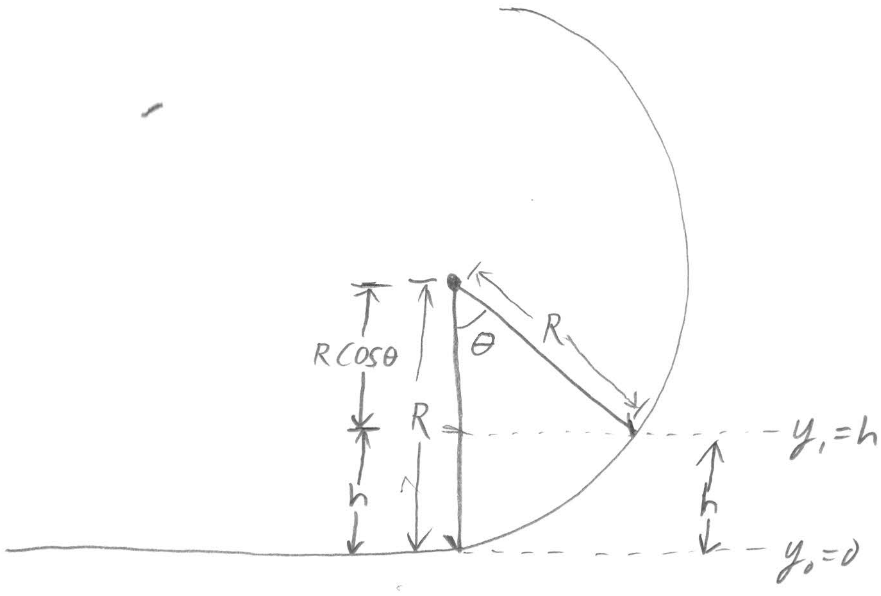
$U_I + K_I + W_{NCF} = U_F + K_F$

$\frac{1}{2}kd^2 + 0 + 0 = mgR + 0$

$$\boxed{d = \left[\frac{2mgR}{k}\right]^{1/2}}$$

EP3, 5 - continued

b)



$h = R - R\cos\theta$

$h = R(1 - \cos\theta)$

With friction, we don't reach as high. We only reach $h = R(1-\cos\theta)$.

So, let's write the energy balance.

$U_I = \frac{1}{2}kd_0^2$ $\quad$ $U_F = mgh$

$K_I = 0$ $\quad$ $K_F = 0$

Work done by friction

$$U_I + K_I + W_F = U_F + K_F$$

$$\frac{1}{2}kd^2 + 0 + W_F = mgh + 0$$

$$\Rightarrow W_F = mgh - \frac{1}{2}kd^2$$

$$= mgh - \cancel{\frac{1}{2}}\cancel{k}\,\frac{\cancel{2}mgR}{\cancel{k}} = mgR(1-\cos\theta) - mgR$$

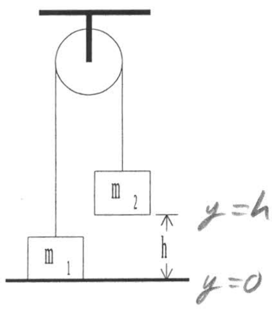
$$\boxed{W_F = -mgR\cos\theta}$$

Energy Problems – Set 3

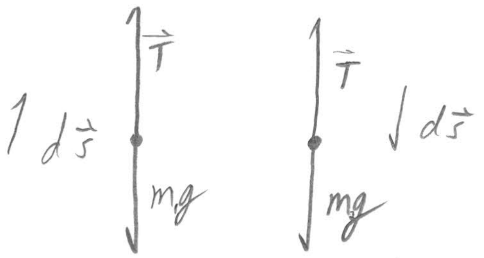
Use work-energy techniques to solve the following problem.

Two masses are connected by a light string passing over a light frictionless pulley. The mass m_2 is released from rest at a height of 4.0 m above the ground.

Determine the speed of m_1 just as m_2 hits the ground and the maximum height m_1 rises above the ground.



$m_1 = 3.0$ kg
$m_2 = 5.0$ kg



T and T do work
No pot. Func.

mg does work, conservative

consider Work done on the system by tension.

on mass 1: $W_T = \int_0^h (\vec{T}\cdot d\vec{s}) = \int_0^h (T\hat{j})\cdot(dy\,\hat{j}) = \underline{Th}$

on mass 2: $W_T = \int_0^h (\vec{T}\cdot d\vec{s}) = \int_0^h (T\hat{j})\cdot(-dy\,\hat{j}) = -\underline{Th}$

Net work on the <u>system</u> by T is <u>zero</u>

Now consider the <u>entire</u> system and conserve energy

a) $U_I = m_2 g h$ $\qquad U_F = m_1 g h$

$K_I = 0$ $\qquad K_F = \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2$

Same velocity since they are linked by the rope.

continued ↓

EP3,6 - continued

$$m_2 gh = m_1 gh + \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2$$

$$\tfrac{1}{2}(m_1+m_2)v^2 = (m_2-m_1)gh$$

$$\boxed{v = \left[\frac{(m_2-m_1)}{m_1+m_2} 2gh\right]^{1/2}} = \boxed{4.4\ m/s}$$

b) Find max height of m_1. The floor has interfered. Some energy ($\tfrac{1}{2}m_2 v^2$) is lost.

consider just m_1

$U_I = m_1 gh \qquad U_F = m_1 g h_{max}$

$K_I = \tfrac{1}{2} m_1 v^2 \qquad K_F = 0$

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$m_1 gh + \tfrac{1}{2} m_1 v^2 = m_1 g h_{max} + 0$$

$$\Rightarrow h_{max} = h + \frac{v^2}{2g} = h + \frac{1}{\cancel{2g}} \frac{m_2-m_1}{m_1+m_2} \cancel{2g} h = h\left[1 + \frac{m_2-m_1}{m_1+m_2}\right]$$

$$= h\left[\frac{\cancel{m_1} + m_2 + m_2 - \cancel{m_1}}{m_1+m_2}\right] = \boxed{\frac{2m_2}{m_1+m_2} h} = 5$$

yay!