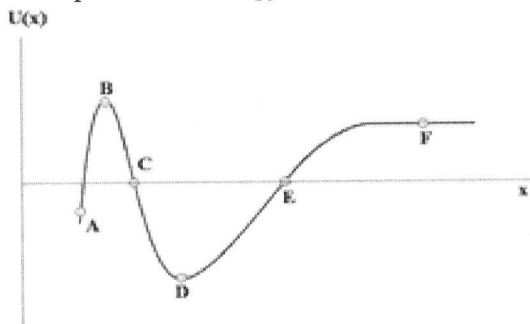# Energy Problems – Set 4

The figure below shows the plot of a potential-energy function for a particle moving along the *x*-axis.

a) At each point indicated, state whether the corresponding force F_x acting on the particle is positive, negative, or zero.

   A: −   D: 0
   B: 0 E: −
   C: +   F: 0

b) At which point does the force have the greatest magnitude? Explain.

   Point A. The slope of the potential curve is greatest there.

c) Identify all points corresponding to stable, unstable, and neutral equilibrium.

   Unstable: B
   Stable : D
   Neutral : F

d) Assuming the particle starts at point A with a large positive velocity, identify the points where the particle's speed is a maximum, minimum, and constant. Explain. (Remember, in order for there to be a potential energy, the force must be conservative.)

   Speed is minimum at point B. Maximum potential, minimum K
   Speed is max at point D. Min U, max K
   Speed is constant at point F. Potential is not changing.

After getting your BS in Physics, you find yourself working as a lab assistant in the stronghold of a mad scientist in a hollowed out volcano in the middle of a remote tropical island.

He is designing a new Mega-Death Ray, but he needs your help with some calculations. The transmogrifier field of the death ray could either have a potential energy function $U_1 = Ax^4$ or $U_2 = Ax^3 - Bx$. A and B are constants and x is the distance as measured from the reaction chamber.
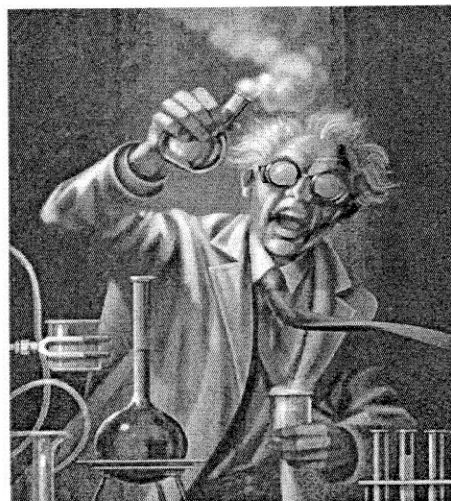
$$\Delta U = -\int \vec{F} \cdot d\vec{s} \iff \vec{F} = -\frac{dU}{d\vec{s}}$$

a) Find the force, F_1, associated with U_1.

$$\vec{F_1} = -\frac{dU_1}{dx}\hat{x} = -\frac{d}{dx}\left(Ax^4\right)\hat{x}$$

$$\boxed{\vec{F_1} = -\tfrac{1}{3}Ax^3\,\hat{x}}$$

b) Find the force F_2 associated with U_2.

$$\vec{F_2} = \frac{dU_2}{dx}\hat{x} = -\frac{d}{dx}\left(Ax^3 - Bx\right)\hat{x}$$

$$\boxed{\vec{F_2} = \left(-2Ax^2 + B\right)\hat{x}}$$

c) If there are any points where the force goes to zero, the entire Island will explode. Are either of the two force fields safe, or are you in mortal danger?
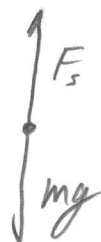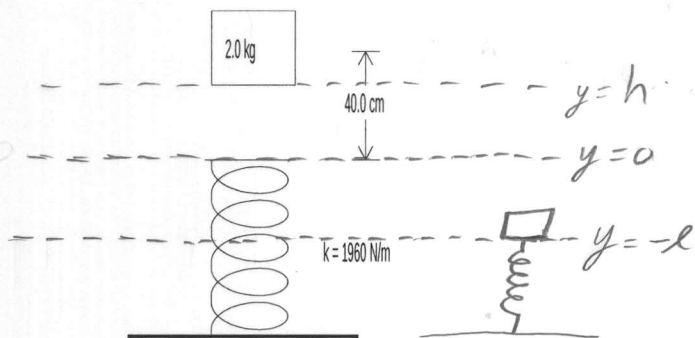
$$|\vec{F_1}| = \tfrac{1}{3}Ax^3 = 0 \implies x = 0 \longrightarrow \boxed{\text{Reaction Chamber Explodes!}}$$

$$|\vec{F_2}| = B - 2Ax^2 = 0 \implies x = \left(\tfrac{1}{2}\tfrac{B}{A}\right)^{1/2} \longrightarrow \boxed{\text{yikes!}}$$

unless either A or B is negative...

Use work-energy techniques to solve the following problem. A 2.0 kg block is dropped from a height of 40 cm onto a spring of spring constant k = 1960 N/m. Find the maximum distance the spring is compressed.

as spring is being compressed

Both forces have a pot. func.

If we set $y=0$ where the spring is uncompressed, life will be easier, since the spring potential and the grav. potential are intertwined.

$$U_I = mgh \qquad\qquad U_F = \tfrac{1}{2}k\ell^2 - mg\ell$$

$$K_I = 0 \qquad\qquad K_F = 0$$

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$mgh + 0 \qquad + 0 \qquad = \tfrac{1}{2}k\ell^2 - mg\ell \qquad \text{hmm... Quadratic.}$$

$$\tfrac{1}{2}k\ell^2 - mg\ell - mgh = 0$$

$$\ell = \frac{1}{k}\left[mg \pm \left((mg)^2 + 2kmgh\right)^{1/2}\right]$$
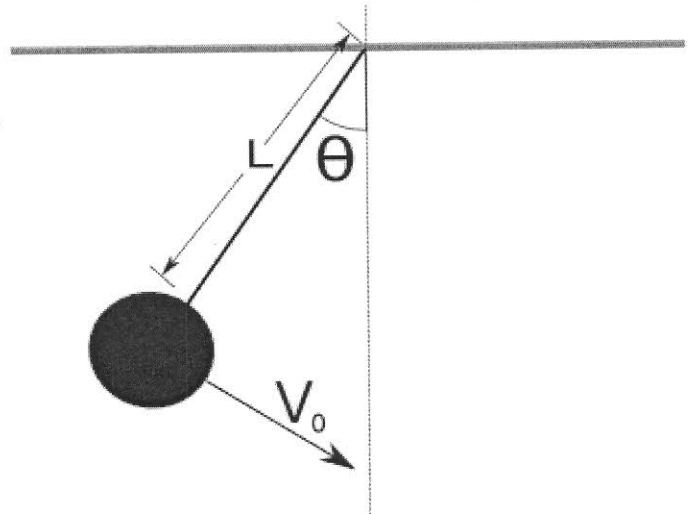
Continued ↓

Negative root gives negative value for l.

But the way we have the potential defined, l must be positive. (l would then be _above_ the $y=0$ line)

$$l = \frac{1}{1960}\left((2.0)(9.8) + \left[\overline{(2 \cdot 9.8)^2 + (2)(1960)(2)(9.8)(0.4_m)}\right]^{1/2}\right)$$

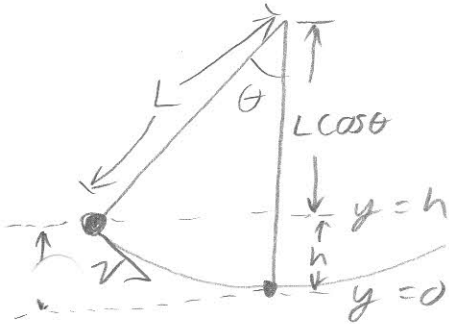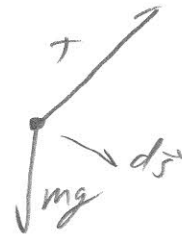$$\boxed{l = 0.1\ m}$$

The picture shows a pendulum with a weight of mass m attached to a light (massless) string of length L. The mass has a speed v_0 when the cord makes an angle θ with the vertical.

a) Derive an expression for the speed of the mass when it is in its lowest position.

b) What is the minimum value of v0 for the chord to make an angle of 90 on the pendulum's upswing?

$$h = L - L\cos\theta$$
$$= L(1 - \cos\theta)$$

$$W_T = 0, \ \vec{T} \perp d\vec{s}$$

$$U_I = mgh \qquad U_F = 0$$

$$K_I = \tfrac{1}{2}mv_0^2 \qquad K_F = \tfrac{1}{2}mv^2$$

$$mgh + \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2$$

$$\Rightarrow v = \left(v_0^2 + 2gh\right)^{1/2} = \boxed{\left(v_0^2 + 2gL(1-\cos\theta)\right)^{1/2}}$$

EP4.4 continued

b) When $\theta = 90$, $h = L(1 - \cos(90)) = L$

$U_I = mg(L - L\cos\theta)$ $U_F = mgL$
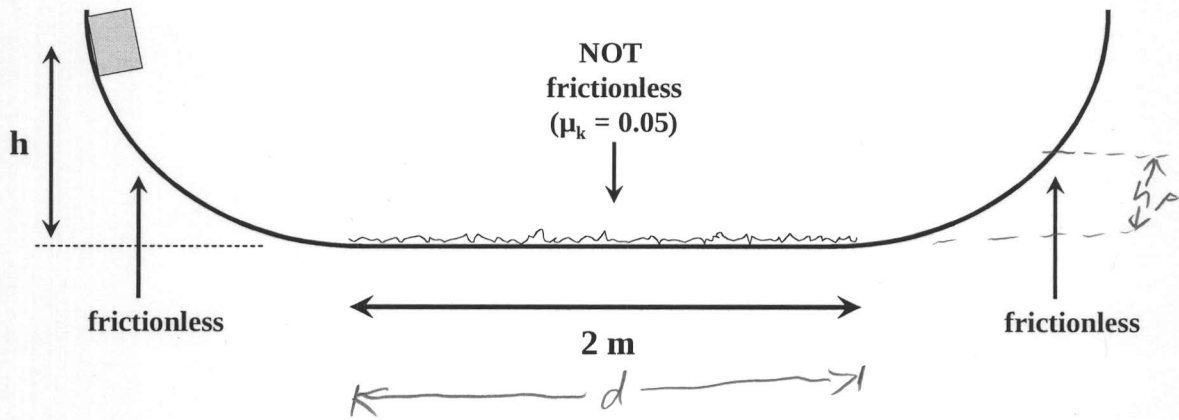
$K_I = \frac{1}{2}mv_0^2$ $K_F = 0$

$mg(L - L\cos\theta) + \frac{1}{2}mv_0^2 = mgL$

$\frac{1}{2}v_0^2 = gL + gL\cos\theta - gL$

$\boxed{v_0^2 = 2gL\cos\theta}$

An 8.75-kg block starts at rest, at height h = 1.0 m, and slides down a frictionless ramp onto a horizontal plane where μ_k = 0.05. If the block has enough energy after passing the plane, it will rise onto another frictionless ramp, and so forth.

(a) The block is released, makes its first trip to the right hand side, returns to the left hand side, and then returns once more to the right. On this second excursion to the right side, how high up the ramp does the block go?

This is a conservation of Energy problem with friction.

The simplest solution is to consider the two endpoints.

It starts a height h on the right and ends

a height h_F on the right after crossing the

friction 3 times.
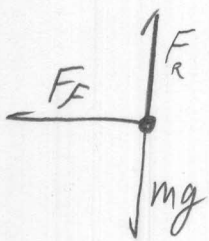
F_R does no work, $\vec{F}_R \perp d\vec{s}$

F_F does work on the flat bottom, non-conservative

mg does work, conservative.

solve for h_F

* $U_I = mgh$ * $U_F = mgh_F$

* $K_I = K_F = 0$

$W_c = \int_0^d \vec{F}_c \cdot d\vec{s} = -\mu_k mgd$ for one trip

EP4, 6 – Continued

Conserve energy accounting for 3 trips across
the friction patch.

$$U_I \quad + \quad K_I \quad + \quad W_F \quad = \quad U_F + K_F$$

$$\cancel{mg}h \quad + \quad 0 \quad + \quad -3\mu_R \cancel{mg}d = \cancel{mg}h_F + 0$$

$$\Rightarrow \boxed{h_F = h - 3\mu_R d}$$

$$h_F = 1.0 - (3)(0.05)(2)$$

$$\boxed{h_F = 0.7 \, m}$$

Energy Problems – Set 4

Use work energy to solve the following problem.

A stone of mass m is thrown vertically upward into the air from ground level with an initial speed of v_o. If a constant drag force equal to 20% of the stone's weight acts on the stone throughout its flight, what is the speed of the stone in terms of v_o when it returns to the ground?

Let's handle the trip up and the trip down separately.

Trip up:
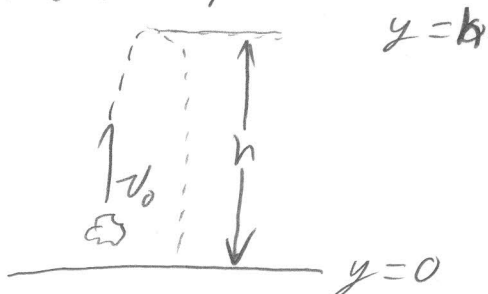
$U_I = 0$ $\qquad\qquad U_F = mgh$

$K_I = \tfrac{1}{2}mv_o^2 \qquad\qquad K_F = 0$

$$W_{NCF} = \int_0^h \vec{F_d} \cdot d\vec{s}, \quad \vec{F_d} = -0.2\,mg\,\hat{j}, \quad d\vec{s} = dy\,\hat{j}$$

$$= \int_0^h (-0.2\,mg\,\hat{j})\cdot(dy\,\hat{j}) = -\tfrac{1}{5}mgh$$

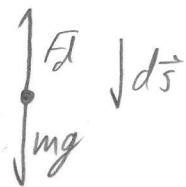$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$0 + \tfrac{1}{2}mv_o^2 - \tfrac{1}{5}mgh = mgh + 0$$

$$\Rightarrow \boxed{\tfrac{1}{2}mv_o^2 = \tfrac{6}{5}mgh} \quad ①$$

EP 4.5

Trip down

$$U_I = mgh \qquad K_I = 0$$

$$U_F = 0 \qquad K_F = \tfrac{1}{2}mv^2$$

$$W_{FCF} = \underline{-0.2mgh}$$

$$mgh - \tfrac{1}{5}mgh = \tfrac{1}{2}mv^2$$

$$\tfrac{4}{5}gh = \tfrac{1}{2}v^2$$

$$\underline{v^2 = \tfrac{8}{5}gh}$$

Plug in h from eq ① : $h = \dfrac{5}{12}\dfrac{v_o^2}{g}$

$$v^2 = \dfrac{\cancel{8}^{2}}{\cancel{5}} \cdot \dfrac{5}{\cancel{12}_{3}}\dfrac{v_o^2}{\cancel{g}}$$

$$v^2 = \tfrac{2}{3}v_o^2$$

$$\boxed{v = \sqrt{\tfrac{2}{3}}\, v_o}$$