The Zeronians live on a planet with a mass of 5.0×10^{23} kg, a radius of 3.0×10^{6} m. Their planet is rapidly running out of atmosphere (so there's no wind resistance) and, because they enjoy skydiving AND breathing, they have built a space-craft to leave.

a) Their space-craft, which weighs 10 kg (the zeronians are tiny), launched with an initial velocity of 3000 m/s. What will be its velocity at when it is 4.0×10^{6} m from the center of the planet?

b) Prove that the maximum altitude of a space craft launched with an initial velocity V_0 from the surface of a planet is:

$$H_{max} = \frac{2G R_p M_p}{2GM_p - R_p V_0^2}$$

where R_p is the radius of the planet, M_p is the mass of the planet, and G is the gravitational constant.

c) If the ship launches with a low V_0, it will go up to H_{max}, stop and come back down. As V_0 increases, H_{max} increases. Calculate the required initial velocity for the particle to go up and NOT come back down. (HINT: What is H_{max} if the particle never comes back?)

A satellite in a perfectly circular orbit is in *Uniform Circular Motion*.

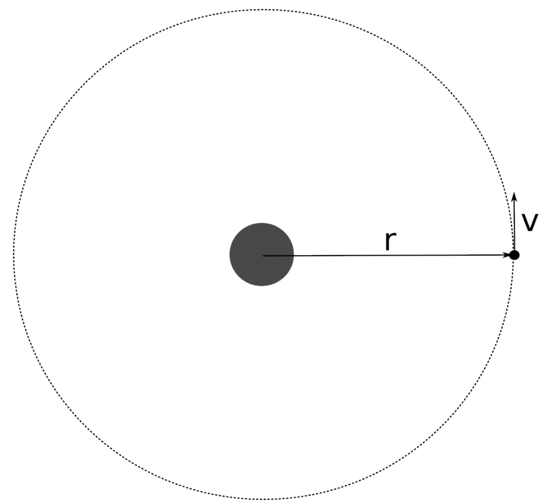
a) Using Newton's Second Law and the problem solving techniques from last unit, find an expression for the orbital velocity of a satellite a distance r from the center of the Earth.

Satellites in geosynchronous orbit always remain above the same geographic spot on the Earth's surface, making them extremely handy for communications.

b) Calculate the radius of the geosynchronous orbit.

c) Calculate the orbital velocity of the geosynchronous orbit.

The radius of the Earth is: $R_{Earth} = 6.37x10^6$ m.
The mass of the Earth is : $M_{Earth} = 5.97x10^{24}$ kg
The gravitaional constant: $G = 6.67x10^{-11}$ $m^3/(kg\ s^2)$

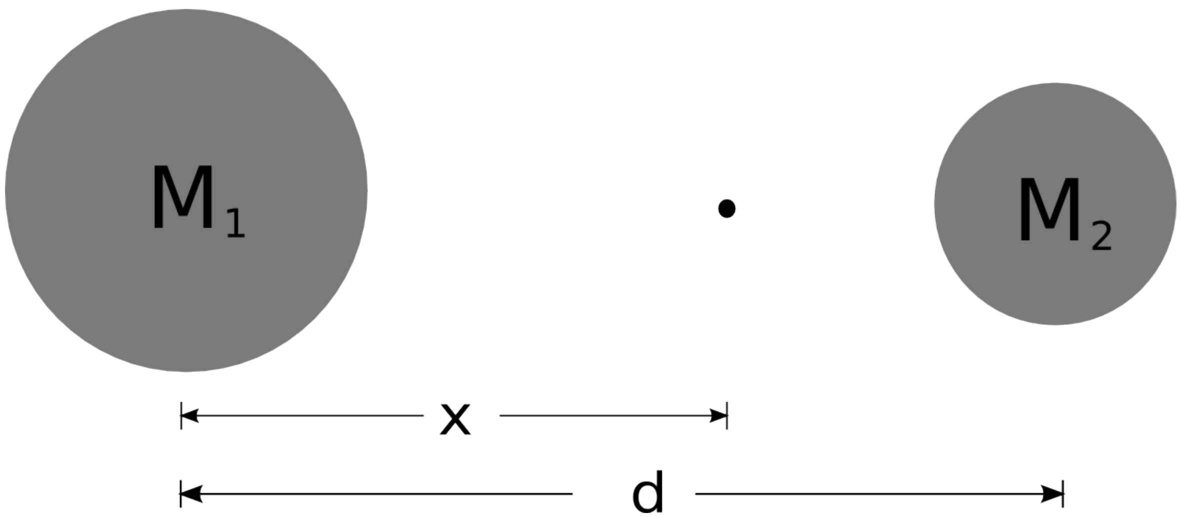


A planet of mass M_1 and its moon of mass M_2 are separated by a distance d. Find the equilibrium point x between the two planets by:

a) Using Newton's Universal Law of Gravitation to find where the force goes to zero.

b) Sketch the potential function. Is the equilibrium point stable or unstable?

c) By writing the total gravitational potential at a distance x from M_1 and finding any minima and maxima.

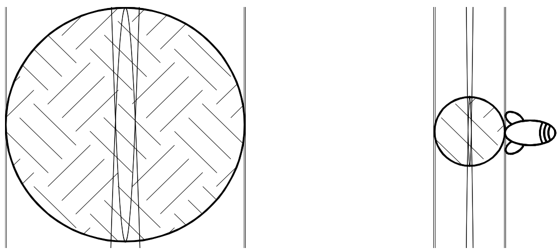


Energy Problems – Set 4

Suppose the Earth were suddenly to stop revolving around the Sun. The gravitational force would then pull it directly into the Sun. What would be the Earth's speed as it crashed (i.e., just when the Earth's *surface* hits the Sun's *surface*)? Assume the Earth starts at rest and the Sun doesn't move toward the earth.

$M_{sun} = 1.99 \times 10^{30}$ kg
$R_{sun} = 695{,}500$ km
$R_{Earth} = 6378.1$ km
$M_{Earth} = 5.9742 \times 10^{24}$ kg

Determine the escape speed of a rocket from the far side of Ganymede, the largest of Jupiter's moons. The radius of Ganymede is 2.640×10^{6} m, and its mass is 1.495×10^{23} kg. The mass of Jupiter is 1.900×10^{27} kg, and the distance between the center of mass of Ganymede and the center of mass of Jupiter is 1.071×10^{9} m. Be sure to include the gravitational effect of Jupiter, but you can ignore the motion of Jupiter and Ganymede as they revolve about the systems center of mass.



3. Two identical stars with mass M orbit around their center of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides of the circle.

(a) Find the gravitational force of one star on the other.

(b) Find the orbital speed of each star and the period of the orbit.

(c) How much energy must be added to the system to separate the two stars to infinity, where they are at rest? Write your answer in terms of M and R. (Note: the addition of energy, however it physically happens, is tantamount to non-conservative work.)