Sample Test 3
Phys 111 Spring 2010

Name: ______________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS – YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

1) Starting with the definition of work, derive the **Work Energy Theorem**.

$$W = \int \vec{F}\cdot d\vec{s}, \qquad \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$
$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W = \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow W = \int (F_x\,dx + F_y\,dy + F_z\,dz) \Rightarrow \boxed{W = \int F_x\,dx + \int F_y\,dy + \int F_z\,dz}$$

$$\boxed{W_{net} = \Sigma W} = \Sigma\left(\int F_x\,dx + \int F_y\,dy + \int F_z\,dz\right)$$

$$\Rightarrow W_{net} = \Sigma\int F_x\,dx + \Sigma\int F_y\,dy + \Sigma\int F_z\,dz$$

$$\Rightarrow W_{net} = \int \Sigma F_x\,dx + \int \Sigma F_y\,dy + \int \Sigma F_z\,dz$$

Now: $\Sigma F_x = ma_x$ (NSL) so we can subst.

consider just the x-axis.

$$\int_{x_0}^{x} \Sigma F_x\,dx = \int_{x_0}^{x} ma_x\,dx = \int_{x_0}^{x} m\frac{dv_x}{dt}\,dx \overset{\text{change variable}}{=} \int_{v_{0x}}^{v_x} m\frac{dx}{dt}\,dv = \int_{v_{0x}}^{v_x} mv\,dv$$

$$= \tfrac{1}{2}mv_x^2 - \tfrac{1}{2}mv_{0x}^2 = \boxed{\Delta K_x}$$

So: $\boxed{W_{net} = \Delta K}$ QED.

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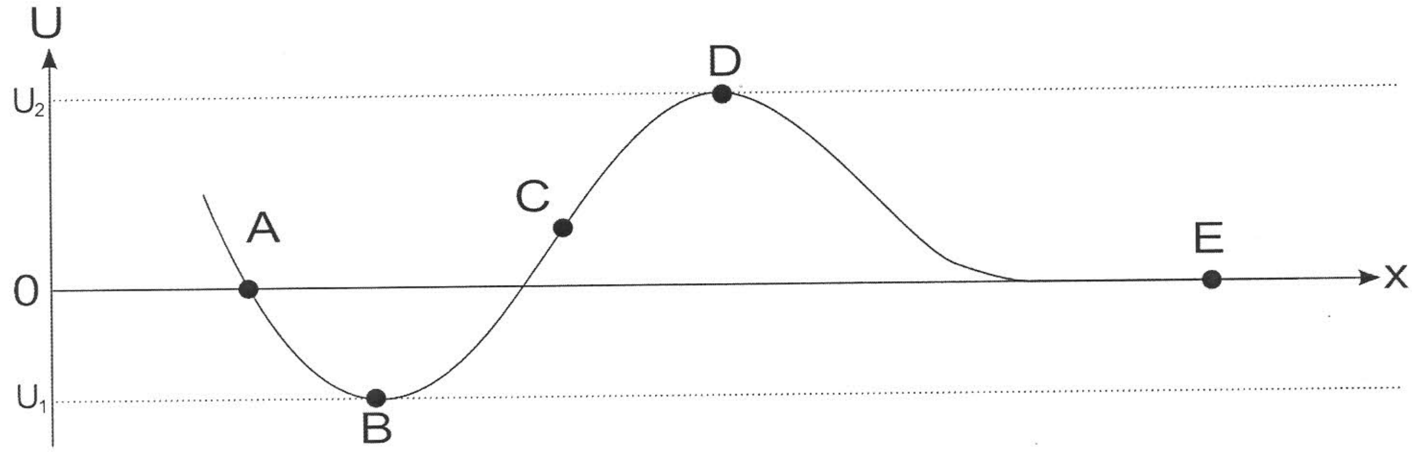
2) Conceptual Questions, 4 points each.



1) Refer to the potential energy curve above. Which point(s) represent **stable** equilibrium?

a) B, D, E
b) A,B,C,D,E
c) B
d) E

2) What is the minimum velocity required by a particle at point A to reach point D?

a) $v=\sqrt{\frac{2U_1}{m}}$

b) $v=\sqrt{\frac{2U_2}{m}}$

c) $v=\sqrt{\frac{2(U_1+U_2)}{m}}$

d) $v=\sqrt{\frac{2(U_1-U_2)}{m}}$

$U_I + K_I = U_F$

$0 + \frac{1}{2}mv^2 = U_2$

$v = \left[\frac{2U_2}{m}\right]^{1/2}$

3) True or False: A force that is always perpendicular to the velocity of a particle does no work on the particle.

a) True
b) False

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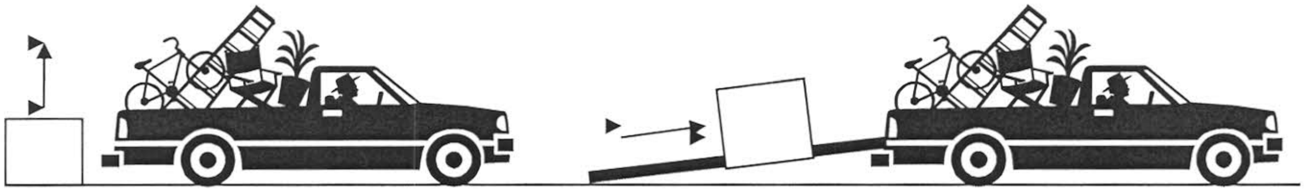
4) You want to load a box into the back of a truck. One way is to lift it straight up through a height h, doing a work W_1. Alternatively, you can slide the box up a loading ramp a distance L, doing a work W_2. Assuming the box slides on the ramp without friction, which of the following is correct?

a) $W_1 < W_2$
b) $W_1 = W_2$
c) $W_1 > W_2$



W_1 W_2

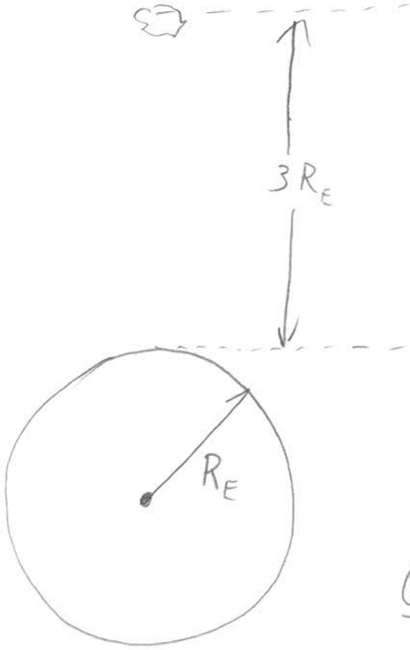
5) The Martians have declared war on us and have begun dropping rocks on us from space. If they release a rock from rest from a height of $3R_{Earth}$ above the Earth's surface, what will its velocity be when it hits us on the ground?

1) $v=\sqrt{\frac{4}{3}\frac{GM_{Earth}}{R_{Earth}}}$

2) $\boxed{v=\sqrt{\frac{3}{2}\frac{GM_{Earth}}{R_{Earth}}}}$

3) $v=\sqrt{\frac{3}{4}\frac{GM_{Earth}}{R_{Earth}}}$

4) $v=\sqrt{\frac{GM_{Earth}}{R_{Earth}}}$



$U_I = \frac{-GM_Em}{4R_E}$ $K_I = 0$

$U_F = \frac{-GM_Em}{R_E}$ $K_F = \frac{1}{2}mv^2$

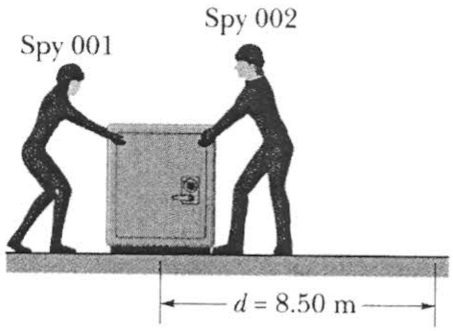
$$-\frac{GM_Em}{4R_E} = -\frac{GM_Em}{R_E} + \frac{1}{2}mv^2$$

$$\frac{GM_E}{R_E}\left[1 - \frac{1}{4}\right] = \frac{1}{2}v^2$$

$$\Rightarrow v = \left[\frac{3}{2}\frac{GM_E}{R_E}\right]^{1/2}$$

Energy Problems – Set 2

Two spies slide an initially stationary 225 kg safe 8.50 m along a straight line towards their truck. Spy 001 pushes 12.0 N at an angle of 30 degrees to horizontal. Spy 002 pulls at an angle of 40 degrees from horizontal. The floor is frictionless (they're stealing the safe from an ice rink).

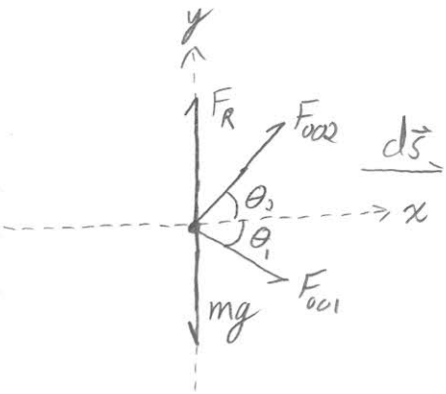


a) What is the total work done on the safe during the 8.5 m displacement.

b) During the displacement, what is the work done on the safe by it's own weight and the normal force from the floor?

c) What is the speed of the safe at the end of the displacement.

d) After the 8.50 m displacement, the spies stop pushing and let the safe slide across the ice and onto the carpet at the edge of the rink. If the coefficient of friction between the safe and the carpet is = 0.6, use the Work Energy Theorem to find how far does the safe slides.



$$\left.\begin{array}{l} W_{F_R} = 0,\ \vec{F}_R \perp d\vec{s} \\ W_g = 0,\ \vec{g} \perp d\vec{s} \end{array}\right] \rightarrow \text{(b)}$$

$$W_{F001} = \int_0^d \left(F_{001}\cos\theta\hat{i} - \cancel{F_{001}\sin\theta\hat{j}}\right)(dx\hat{i})$$

$$W_{F001} = F_{001} d \cos\theta_1$$

$$W_{F002} = \int_0^d \left(F_{002}\cos\theta_2\hat{i} + \cancel{F_{002}\sin\theta_2\hat{j}}\right)(dx\hat{i})$$

$$W_{F002} = F_{002} d \cos\theta_2$$

$$\boxed{W_{net} = (F_{001}\cos\theta_1 + F_{002}\cos\theta_2)d} = ((12)\cos(30) + (10)\cos(40))8.5$$

$$= 153\ J$$

continued ↓

EP2, P4 - continued.

c) WET

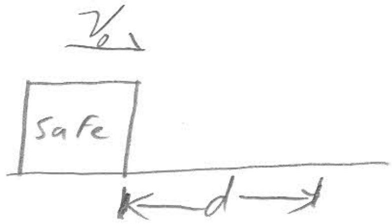
$W_{net} = \Delta K$

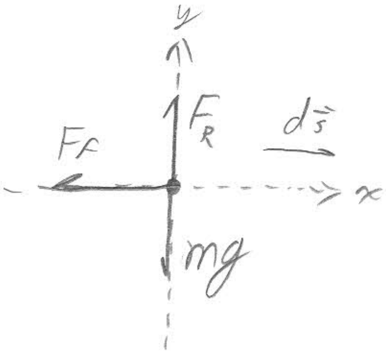
$W_{net} = \frac{1}{2} m v^2 - \cancel{\frac{1}{2} m v_0^2}^{\,0}$

$\Rightarrow \boxed{v = \left(\frac{2 W_{net}}{m}\right)^{1/2}}$

$v = \left(\frac{(2)(153)}{(225)}\right)^{1/2} = 1.2 \text{ m/s}$

d)





Get F_f from NSL

$F_f = \mu_k F_R$

$F_R - mg = 0$

$\Rightarrow \boxed{F_f = \mu_k mg}$

$W_{F_R} = 0, \; F_R \perp d\vec{s}$

$W_g = 0, \; \vec{F}_g \perp d\vec{s}$

$W_{F_f} = \int_0^d (-\mu_k mg \hat{x}) \cdot (dx \hat{x}) = \underline{-\mu_k mgd}$

WET

$W_{net} = \Delta K \Rightarrow -\mu_k mgd = \cancel{\frac{1}{2} m v^2}^{\,0} - \frac{1}{2} m v_0^2$

continued ↓

EP2, P4 - continued

$$+\mu_k \cancel{m} g d = +\tfrac{1}{2} \cancel{m} v_0^2$$

$$\boxed{d = \frac{v_0^2}{2\mu_k g}} = \frac{(1.2)^2}{(2)(0.7)(9.8)} = \boxed{0.1\ m}$$

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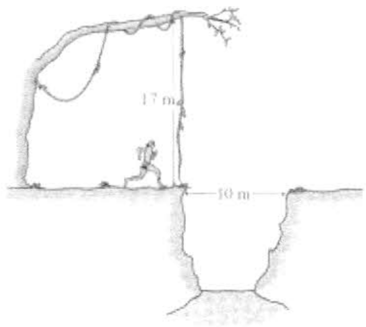
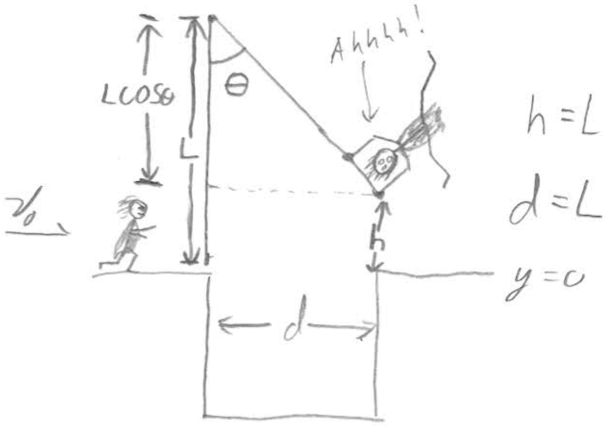
Use work-energy techniques to solve the following problem.



Tarzan is late for a date with Jane and is running as fast as he can to meet her. On the way, he has to get over a 10m wide pit of dangerous croc-a-gators. A 17m vine is hanging vertically from a tree at one side of the pit. Tarzan is going to run up, grab the vine, swing across, and drop vertically to the ground on the other side.

What must his minimum speed be to make it across?



$h = L - L\cos\theta$

$d = L\sin\theta$

$W_{F_T} = 0, \ \vec{F}_T \perp d\vec{s}$

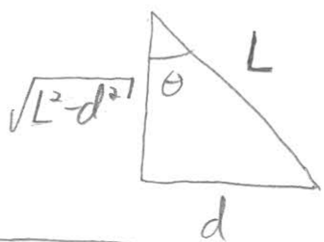
$U_I = 0 \qquad U_F = mgL(1-\cos\theta)$

$K_I = \frac{1}{2}mV_0^2 \qquad K_F = 0$ ← Stops to drop Vertically

$W_{NCF} = 0$

$\frac{1}{2}\cancel{m}V_0^2 = \cancel{m}gL(1-\cos\theta)$

$V_0 = \left(2gL(1-\cos\theta)\right)^{1/2}$



$\cos\theta = \frac{\sqrt{L^2-d^2}}{L}$

$$\boxed{V_0 = \left(2gL\left(1-\frac{\sqrt{L^2-d^2}}{L}\right)\right)^{1/2} = \left(2g\left(L-\sqrt{L^2-d^2}\right)\right)^{1/2}}$$

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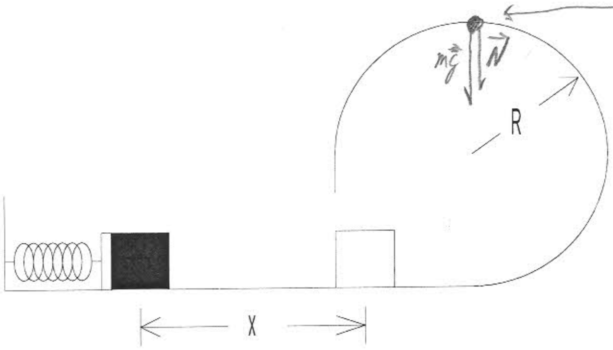
Use Conservation of Energy to solve the following problem.

A block of mass m is pressed against a spring with a spring constant of k a distance x from its starting position and then released. What is the minimum distance x such that the block will travel around the loop and end up back at the starting point? All surfaces are frictionless, the loop has a radius of R, and the size of the block is small compared to the radius of the loop.



at top

$F_{net} = ma$

uniform circular motion

$-mg - \cancel{N}^{\to 0} = -\cancel{m}\frac{v^2}{R}$, N goes to zero Just when block falls.

$\Rightarrow \boxed{v = \sqrt{gR}}$

$U_I = \frac{1}{2}kx^2$ $\quad K_I = 0$

$U_F = mg2R$ $\quad K_F = \frac{1}{2}mv^2$

$\frac{1}{2}kx^2 = 2mgR + \frac{1}{2}mgR$

$x^2 = \frac{1}{k}\left[4mgR + mgR\right]$

$\Rightarrow \boxed{x = \left[5\frac{mgR}{k}\right]^{1/2}}$

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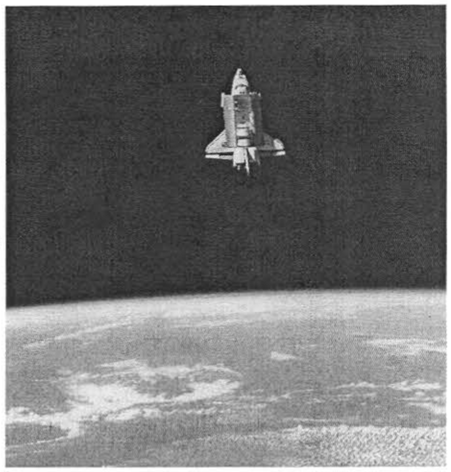


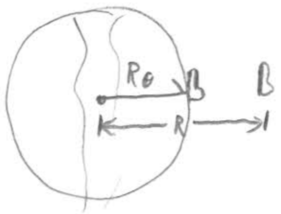
a) Derive a general expression for the amount of energy required, E, to put the space shuttle into an orbit of radius R. Ignore the Earth's rotation. (Let $K_i = 0$)

b) Now let's use the equation from part a to compare two orbital energies.

Let $R_1 = (1.05)R_{Earth}$ (the normal orbit of the space shuttle) and let $R_2 = \infty$. Show that, in terms of energy, low Earth orbit is halfway across the Universe.

In other words, show that: $\frac{E_1}{E_2} = 0.52$

a)



$$U_I = -\frac{GM_\oplus m_s}{R_\oplus} \qquad U_F = -\frac{GM_\oplus m_s}{R}$$

$$K_I = \emptyset \qquad K_F = \frac{1}{2} m_s v^2$$

What's v? have to use $F = ma$ to get it

$$F = ma$$

$$\frac{GM_\oplus \cancel{m_s}}{R^{\cancel{2}}} = \cancel{m_s}\frac{v^2}{\cancel{R}} \Rightarrow v = \sqrt{\frac{GM_\oplus}{R}}$$

$$U_I + K_I + W_{NCF} = U_F + K_F \leftarrow \tfrac{1}{2} m v^2$$

$$-\frac{GM_\oplus m_s}{R_\oplus} + 0 + W_{NCF} = -\frac{GM_\oplus m_s}{R} + \frac{1}{2}\frac{GM_\oplus m_s}{R}$$

$$W_{NCF} = +\frac{GM_\oplus m_s}{R_\oplus} - \frac{1}{2}\frac{GM_\oplus m_s}{R}$$

Work by rocket engines

continued ↓

ST 3, Shuttle Orbit - continued.

$$W_{NCF} = \boxed{\underbrace{E}_{\text{Energy to Orbit}} = GM_{\oplus}m\left(\frac{1}{R_{\oplus}} - \frac{1}{2R}\right)}$$

b) Compare two Energies

$$E_1 = GM_{\oplus}m_s\left(\frac{1}{R_{\oplus}} - \frac{1}{2(1.05)R_{\oplus}}\right)$$

$$= \frac{GM_{\oplus}m_s}{R_{\oplus}}\left(1 - \frac{1}{2.1}\right) = 0.52\,\frac{GM_{\oplus}m_s}{R_{\oplus}}$$

$$E_2 = GM_{\oplus}m_s\left(\frac{1}{R_{\oplus}} - \frac{1}{\infty}\right) = \frac{GM_{\oplus}m_s}{R_{\oplus}}$$

$$\frac{E_1}{E_2} = \frac{0.52\,\frac{\cancel{GM_{\oplus}m_s}}{\cancel{R_{\oplus}}}}{\frac{\cancel{GM_{\oplus}m_s}}{\cancel{R_{\oplus}}}} = \boxed{0.52}$$