

An oscillating block-spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s.

- What is the spring constant?
- What is the mass of the block?
- What is the frequency of oscillation?

$$E_T = U + K \Rightarrow E_T = \frac{1}{2}kx^2 + \frac{1}{2}mV^2$$

- a) Use the amplitude. When the spring is compressed,
 $K=0$ and $x=A$

$$\text{So: } E_T = \frac{1}{2}kA^2 \Rightarrow \boxed{k = \frac{2E_T}{A^2}} \Rightarrow \boxed{k = \frac{2(1.0\text{ J})}{(0.1\text{ m})^2} = 200 \text{ N/m}}$$

- b) Use the max speed.

Max speed is at equilibrium when $U=0$

$$E_T = \frac{1}{2}mV_{\text{max}}^2 \Rightarrow \boxed{m = \frac{2E_T}{V_{\text{max}}^2}} \Rightarrow \boxed{m = \frac{2(1.0)}{(1.2\text{ m/s})^2} = 1.4 \text{ kg}}$$

- c) We can use V_{max} and A to find ω .

$$v(t) = -\omega A \cos(\omega t + \phi)$$

$$\Rightarrow V_{\text{max}} = \omega A \Rightarrow \boxed{\omega = \frac{V_{\text{max}}}{A}} \Rightarrow \boxed{\omega = \frac{1.2\text{ m/s}}{0.1\text{ m}} = 12 \text{ rad/s}}$$

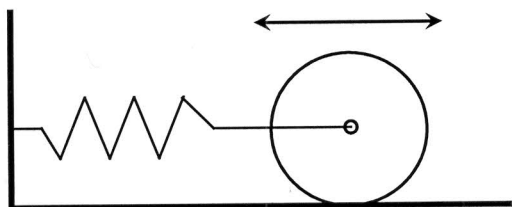
Oscillation - Set 2

1

Use **ENERGY** techniques to answer the following question.

A solid cylinder of mass $M=2$ kg and radius $R=1.0$ m is attached to a horizontal spring with spring constant $k=100$ N/m. The cylinder can roll without slipping along the horizontal plane. When the system is displaced from the equilibrium position, it executes simple harmonic motion.

- a) Derive an expression for the period of the oscillations in terms of M , k , and I .
b) If the cylinder has a translational velocity of $v_0=5.0$ m/s as it passes through equilibrium, find the phase constant, the amplitude, and the maximum acceleration of the system.



$$I = \frac{1}{2} m R^2$$
$$\omega = \frac{v}{R}$$
$$\alpha = \frac{a}{R}$$

$$a) E_T = \frac{1}{2} k x^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

$$\frac{dE_T}{dt} = \frac{1}{2} k \cdot 2x \frac{dx}{dt} + \frac{1}{2} I \cdot 2\omega \frac{d\omega}{dt} + \frac{1}{2} m \cdot 2v \frac{dv}{dt} = 0$$

$$\Rightarrow kxv + I\omega\alpha + mva = 0$$

$$\Rightarrow kxv + \frac{1}{2} m R^2 \frac{v}{R} \frac{a}{R} + mv a = 0$$

$$\Rightarrow kx + \frac{1}{2} ma + ma = 0$$

$$\Rightarrow \frac{3}{2} ma = -kx$$

$$\Rightarrow a = -\frac{2}{3} \frac{k}{m} x \Rightarrow \frac{d^2x}{dt^2} = -\left[\frac{2}{3} \frac{k}{m}\right] x$$

$$\omega = \left[\frac{2}{3} \frac{k}{m}\right]^{1/2}$$

Oscillation set 3, P1 continued

b) $v_0 = 5.0 \text{ m/s}$. Let's let $t=0$ then.

$$\boxed{v(0) = 5.0 \text{ m/s}}$$

At equilibrium, $x = 0$ so:

$$\boxed{x(0) = 0}$$

General Solution

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -\omega A \sin(\omega t + \phi)$$

$$x(0) = A \cos(\phi) \Rightarrow 0 = A \cos(\phi) \Rightarrow \boxed{\phi = 0, \pi}$$

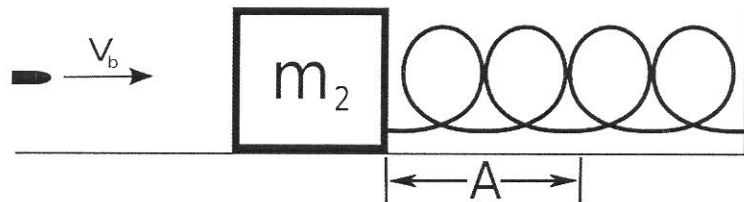
$$v(0) = -\omega A \sin \phi \Rightarrow 5.0 = -\omega A \sin \phi, \text{ let } \underline{\phi = \pi}$$

$$\Rightarrow A = \frac{5.0}{\omega} = \sqrt{\frac{3 \text{ N}}{2 \text{ kg}}} 5.0$$

$$\boxed{A = \left(\frac{3}{2} \frac{\text{N}}{100}\right)^{1/2} \cdot 5.0 = 0.87 \text{ m}}$$

A block of mass $m_2 = 10 \text{ kg}$ attached to a spring with spring constant $k = 5 \text{ N/m}$ is oscillating with an amplitude of $A = 1.5 \text{ m}$ horizontally on a frictionless surface. When the spring has reached its maximum extension to the left, it collides with a bullet with a mass $m_1 = 5 \text{ g}$ moving $V_b = 200 \text{ m/s}$ towards the right.

- a) What is the period of the oscillator after the collision?
- b) What is the amplitude of the oscillator after the collision?



a) Find the oscillator freq. with energy:

$$E_T = \frac{1}{2} k x^2 + \frac{1}{2} m_T v^2$$

$$\frac{dE_T}{dt} = \frac{1}{2} k \cancel{x} \frac{dx}{dt} + \frac{1}{2} m_T \cancel{v} \frac{dv}{dt}$$

$$\Rightarrow 0 = k x v + m_T v a$$

$$\Rightarrow 0 = k x + m_T a \Rightarrow a = - \left[\frac{k}{m_T} \right] x$$

ω^2

$m_T = m_B + m_2$

$$\omega = \left[\frac{k}{m_B + m_2} \right]^{1/2}$$

Continued
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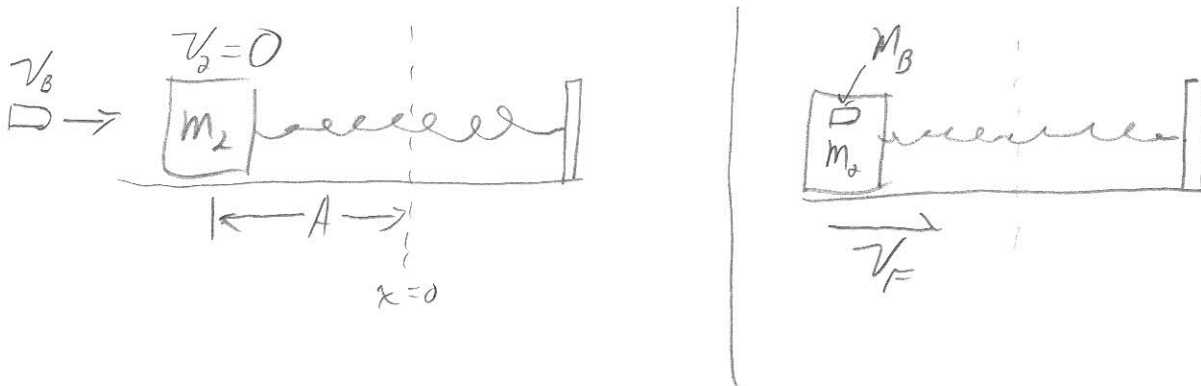
b) First, we have to determine the initial conditions.

Let's let the moment of collision be $t=0$.

at $t=0$, $x(0) = A$, the Amplitude of the old oscillator (pre-collision)

at $t=0$, $v(0) = v_F$, the post collision velocity of the Block/Bullet system.

To get v_F , conserve momentum



$$m_B v_B = (m_B + m_2) v_F$$

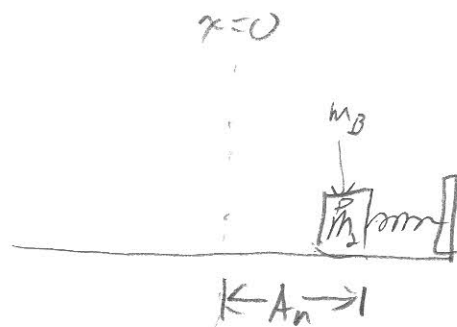
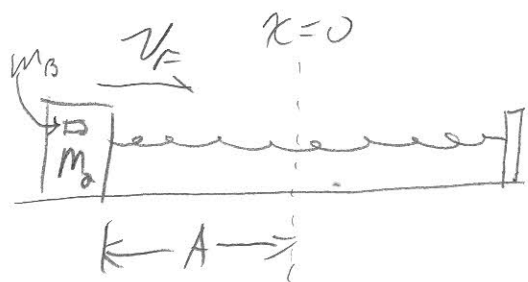
$$\Rightarrow v_F = \frac{m_B}{m_B + m_2} v_B$$

Now, the new oscillator will have a new amplitude A_n , and we can conserve energy to get A

continued



Oscillation Set 3 P3 continued



$$U_I = \frac{1}{2} k A^2$$

$$K_I = \frac{1}{2} (m_2 + m_B) v_F^2$$

$$U_F = \frac{1}{2} k A_n^2$$

$$K_F = 0$$

$$\cancel{\frac{1}{2}} k A^2 + \cancel{\frac{1}{2}} (m_2 + m_B) v_F^2 = \cancel{\frac{1}{2}} k A_n^2$$

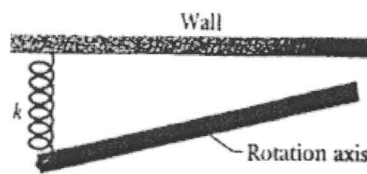
$$\Rightarrow A_n = \left[A^2 + \frac{1}{k} (m_2 + m_B) v_F^2 \right]^{\frac{1}{2}}$$

$$\Rightarrow A_n = \left[A^2 + \frac{1}{k} \cancel{(m_2 + m_B)} \frac{m_B^2}{(m_2 + m_B)^{\cancel{x}}} v_B^2 \right]^{\frac{1}{2}}$$

$$\boxed{A_n = \left[A^2 + \frac{1}{k} \frac{m_B^2}{(m_2 + m_B)} v_B^2 \right]^{\frac{1}{2}}}$$

Oscillation – Set 3

A long uniform rod of length L and mass m is free to rotate in a horizontal plane about a vertical axis through its center (the picture shows a top view). A spring with force constant k is connected horizontally between one end of the rod and a fixed wall. When the rod is in equilibrium, it is parallel to the wall.



What is the period of the *small* oscillations that result when the rod is rotated slightly and then released?

$I_{cm} = \frac{1}{12}ML^2$ for the rod.

- a) Use **Newton's Second Law** to find the oscillator frequency.
- b) Use **Energy Techniques** to find the oscillator frequency.



$$\sum T = I\alpha \Rightarrow \alpha = \frac{\sum T}{I}$$

$$\Rightarrow \alpha = \frac{\frac{1}{2}F_s}{\frac{1}{12}ML^2} \Rightarrow \alpha = \frac{-\frac{1}{2}kx}{\frac{1}{12}ML^2}, \quad x = \theta \frac{L}{2}$$

$$\Rightarrow \alpha = \frac{-\theta \frac{1}{2}k \frac{L}{2}}{\frac{1}{12}ML^2} \Rightarrow \boxed{\alpha = -\frac{3k}{M}\theta}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\left[\frac{3k}{M}\right]\theta \Rightarrow \boxed{\omega = \left[\frac{3k}{M}\right]^{1/2}}$$

b) $E_T = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2$

$$\frac{dE_T}{dt} = \frac{1}{2}k \frac{dx}{dt} + \frac{1}{2}I \frac{d\omega}{dt} = 0 \Rightarrow kxv + I\omega\alpha = 0$$

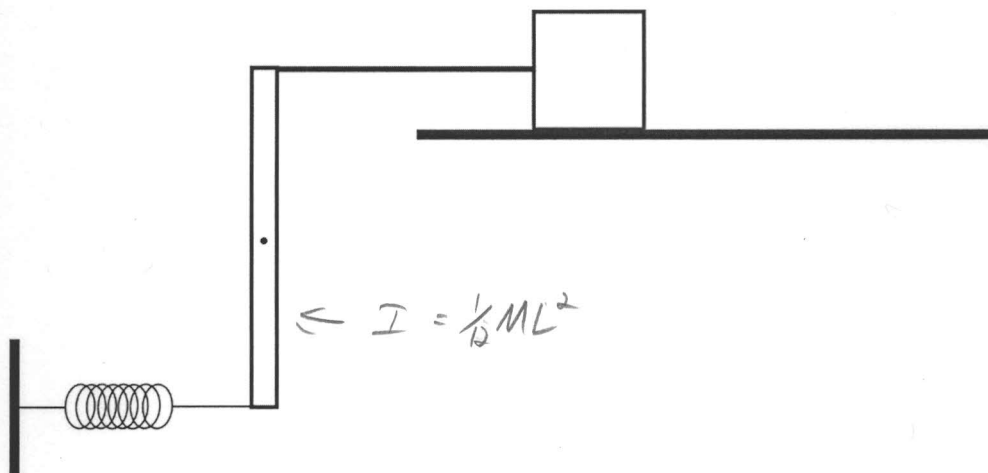
$$x = \frac{L}{2}\theta, \quad v = \frac{L}{2}\omega$$

$$k \frac{L}{2}\theta \frac{L}{2}\omega + \frac{1}{12}ML^2\omega\alpha = 0 \Rightarrow \frac{1}{3}ML^2\alpha = -\frac{1}{4}kL^2\theta \Rightarrow \boxed{\alpha = -\frac{3k}{M}\theta}$$

$$\boxed{\omega = \left[\frac{3k}{M}\right]^{1/2}}$$

Use **ENERGY** techniques to answer the following question.

A block of mass M resting on a frictionless surface is attached to a stiff rod of negligible mass. The other end of the rod is attached to the top of a thin bar of length l mass M that is allowed to rotate about its center. The bottom of a bar is attached to a light spring of spring constant k . The spring is relaxed when the bar is vertical. Find the frequency of small oscillations.



$$E_T = \frac{1}{2} k x^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

$$\frac{dE_T}{dt} = \frac{1}{2} k 2x \frac{dx}{dt} + \frac{1}{2} I 2\omega \frac{d\omega}{dt} + \frac{1}{2} m 2v \frac{dv}{dt} = 0$$

$$\Rightarrow kxv + I\omega\alpha + mv a = 0, \quad \omega = \frac{v}{l/2}, \quad \alpha = \frac{a}{l/2}$$

$$\Rightarrow kx\cancel{v} + \frac{1}{12} M l^2 \frac{\cancel{v}}{l/2} \cdot \frac{a}{l/2} + m\cancel{v} a = 0$$

$$\Rightarrow kx + \frac{1}{3} M a + m a = 0$$

$$\Rightarrow \frac{4}{3} m a = -kx \Rightarrow a = -\frac{3}{4} \frac{k}{m} x \Rightarrow \left(\frac{d^2 x}{dt^2} = - \left[\frac{3}{4} \frac{k}{m} \right] x \right)$$

$$\left(\omega = \left[\frac{3}{4} \frac{k}{m} \right]^{1/2} \right)$$

Oscillation – Set 3

5

Two particles are in simple harmonic motion in a straight line. They have the same amplitude and a period of 1.5 s but differ in phase by $\pi/6$ radians.

- How far apart are they from one another (in terms of A) when the lagging particle is at its maximum position?
- Are they moving in the same direction or opposite directions?
- How far apart are they 0.5 seconds later?
- Are they moving in the same or opposite directions then?

$$A_1 = A_2 = A, \quad T_1 = T_2 = T, \quad \phi_1 = 0, \quad \phi_2 = \pi/6,$$

$$\begin{aligned} \text{a) } x_1(t) &= A_1 \cos(\omega_1 t + \phi_1) \\ x_2(t) &= A_2 \cos(\omega_2 t + \phi_2) \end{aligned}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

What's t_0 ?

$$x_1(t_0) = A \cos(\omega t_0) \quad \text{At max, } \underline{x = A}$$

$$A = A \cos(\omega t_0) \Rightarrow \cos(\omega t) = 1$$

$$\Rightarrow \omega t = 0 \Rightarrow \underline{\underline{t = 0}}$$

$$x_2(0) = A \cos(\phi_2) \Rightarrow x_2(0) = A \cos(\pi/6) = \frac{\sqrt{3}}{2} A$$

$$\boxed{x_2 - x_1 = \left(\frac{\sqrt{3}}{2} - 1\right) A}$$

$$b) v_1(0) = -\omega A \sin(0) = 0$$

hmm... neither...

trick question :-

c) at $t=0.5$

$$x_1(0.5) = A \cos(-\omega \cdot 0.5), \quad T = \frac{3}{2} \Rightarrow \frac{2\pi}{3/2} = \frac{4}{3}\pi = \omega$$

$$x_1(0.5) = A \cos\left(\frac{4}{3}\pi \cdot \frac{1}{2}\right) \quad \omega = \frac{4}{3}\pi$$

$$x_1(0.5) = A \cos\left(\frac{2}{3}\pi\right)$$

$$\underline{x_1(0.5) = -\frac{1}{2}A}$$

$$x_2(0.5) = A \cos\left(\frac{4}{3}\pi \cdot \frac{1}{2} + \frac{\pi}{6}\right)$$

$$= A \cos\left(\frac{5}{6}\pi\right)$$

$$= -\frac{\sqrt{3}}{2}A$$

$$\boxed{x_2 - x_1 = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)A}$$

$$d) v_1(0.5) = -\omega A \sin\left(\frac{2}{3}\pi\right) = -\omega A \frac{\sqrt{3}}{2}$$

$$v_2(0.5) = -\omega A \sin\left(\frac{5}{6}\pi\right) = -\omega A \frac{1}{2}$$

same sign, same direction