

Rotation – Set 5

Two meter sticks are stood in the corner where the floor meets the wall. One has a large mass attached to the end furthest from the corner. Which one hits the ground first?

We'll solve the problem by calculating the *Angular Acceleration* of each object and then comparing them.

a) Calculate the angular acceleration of the meter stick falling over under the influence of gravity.

FBD

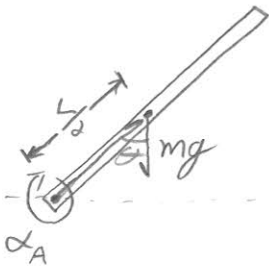
NSL

$$\sum \vec{\tau} = I\alpha$$

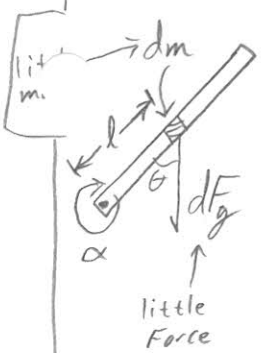
$$mg \frac{L}{2} \sin\theta = I\alpha_A$$

$$\alpha_A = \frac{3}{2} \frac{g}{L} \sin\theta$$

$$mg \frac{L}{2} \sin\theta = \frac{1}{3} ML^2 \alpha_A$$



* Proof that gravity acts at the center of mass



In general: $\vec{\tau} = \vec{r} \times \vec{F}$

In this case: $d\vec{\tau} = \vec{l} \times d\vec{F}_g = l dF_g \sin\theta \hat{i}$

and $dF_g = dm g$ so: $dT = l g \sin\theta dm$

Then, The net Torque is:

$$T = \int dT = \int l g \sin\theta dm = \dots =$$

$$\Rightarrow T = g \sin\theta \int l dm, \quad \text{But } l_{cm} = \frac{1}{M} \int l dm$$

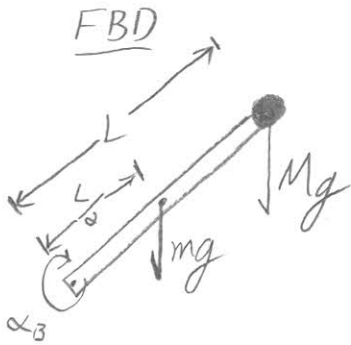
$$\text{so: } \int l dm = \underline{M l_{cm}}$$

and: $T = M g l_{cm} \sin\theta$

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- b) Calculate the angular acceleration of a meter stick with a large mass attached to its end falling over under the influence of gravity.



$$\text{NSL}$$

$$\Sigma T = I \alpha$$

$$\textcircled{1} \quad mg \frac{L}{2} \sin \theta + MgL \sin \theta = I \alpha_B$$

$$I_T = I_{\text{stick}} + I_{\text{mass}}$$

$$\textcircled{2} \quad I_T = \frac{1}{3} mL^2 + ML^2 = \left(\frac{1}{3}m + M\right)L^2$$

$$\textcircled{1} \rightarrow \textcircled{2}: \left(\frac{1}{3}m + M\right)g \sin \theta = \left(\frac{1}{3}m + M\right)L \alpha_B$$

$$\Rightarrow \alpha_B = \frac{\left(\frac{1}{3}m + M\right)g \sin \theta}{\left(\frac{1}{3}m + M\right)L}$$

- c) Take the ratio $\frac{\alpha_a}{\alpha_b}$. Is it greater than or less than one? Which stick hits the ground first?

$$\frac{\alpha_A}{\alpha_B} = \frac{\frac{3}{2}g \sin \theta}{\frac{\left(\frac{1}{3}m + M\right)g \sin \theta}{\left(\frac{1}{3}m + M\right)L}} \Rightarrow \frac{\alpha_A}{\alpha_B} = \frac{3}{2} \cdot \frac{\left(\frac{1}{3}m + M\right)}{\left(\frac{1}{3}m + M\right)}$$

$$\Rightarrow \frac{\alpha_A}{\alpha_B} = \frac{m + 3M}{m + 2M}$$

let's assume:

$$\frac{m + 3M}{m + 2M} > 1 \Rightarrow m + 3M > m + 2M \Rightarrow 3M > 2M \Rightarrow 3 > 2 \text{ True!}$$

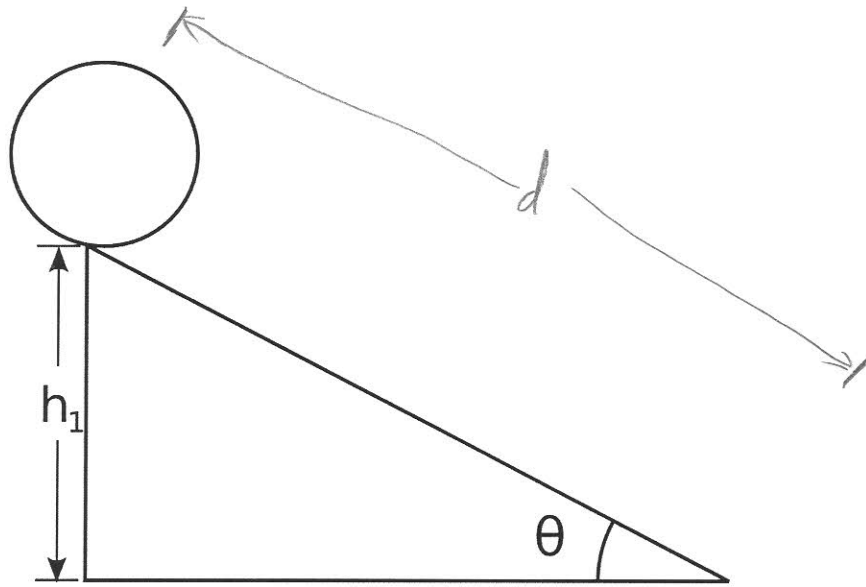
$$\text{So! } \frac{\alpha_A}{\alpha_B} > 1 \Rightarrow \alpha_A > \alpha_B \quad \text{Stick A hits First!}$$

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Use **Torque and Kinematics** to solve this problem.

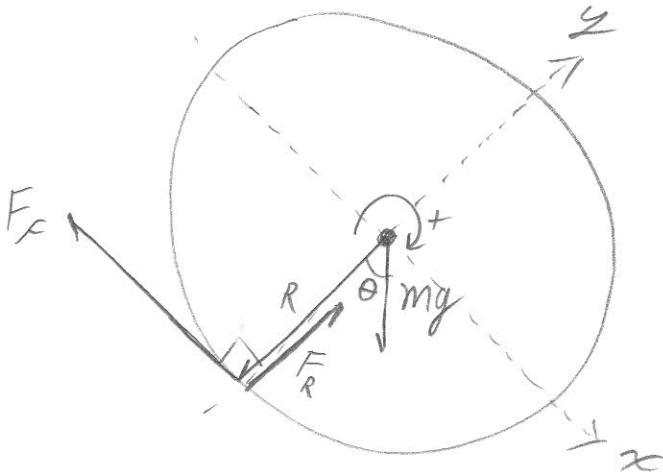
A rolling object with a radius R , mass m , and moment of inertia I , starts from rest at the top of an incline plane of height h that makes an angle θ with the horizontal.

- a) Find an expression for the linear and angular acceleration of the object in terms of I .
- b) Using kinematics, find an expression for the linear and angular ~~acceleration~~ *velocity* of the object in terms of I ?
- c) Assume that the object is a disk with $I = \frac{1}{2}mR^2$ and plug I into your velocity expressions. Verify that your answers are the as when you solved this problem using energy.



d)

Step 1 - FBD



In this problem, we need to consider both Rotation and translation. So we have positive rotation as well as the x - y coordinates labeled

Rotation set 5, P1 continued

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Step 2 - NSL

One object but both translation and rotation

Rotation

$$\sum \vec{\tau} = \vec{I} \alpha$$

$$\Rightarrow \sum R F \sin \theta = I \alpha$$

$$\Rightarrow \underbrace{R F_x \sin(90)}_{\sin 90 = 1} + \underbrace{R F_x \sin(180)}_{\sin(180) = 0} + \underbrace{0 \cdot mg \sin \theta}_{R = 0} = I \alpha$$

$$\Rightarrow R F_x = I \alpha \Rightarrow \boxed{F_x = \frac{I}{R} \alpha}$$

Translation

$$x: \boxed{mg \sin \theta - F_x = ma} \quad (1)$$

$$y: F_R - mg \cos \theta = 0 \leftarrow \text{not useful}$$

$$\text{Plug } (1) \rightarrow (2): mg \sin \theta - \frac{I}{R} \alpha = ma$$

$$\text{Let } a = R \alpha: mg \sin \theta - \frac{I}{R^2} a = ma \Rightarrow mg \sin \theta = (m + \frac{I}{R^2}) a$$

$$\boxed{a = \frac{m}{m + \frac{I}{R^2}} g \sin \theta} \quad (1)$$

$$\boxed{\alpha = \frac{m}{m + \frac{I}{R^2}} \frac{g}{R} \sin \theta} \quad (2)$$

Rotation Set 5, P1 continued

b) Kinematics

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$d = \frac{1}{2} a \frac{v^2}{a^2}$$

$$t = \frac{v}{a}$$

$$d = \frac{1}{2} \frac{v^2}{a} \Rightarrow \boxed{v = (2da)^{1/2}} \quad (3)$$

Plug in a from eq. (1)

$$v = \left[2d \frac{m}{m + I/R^2} g \sin \theta \right]^{1/2}$$

$$\boxed{v = \left[\frac{m}{m + I/R^2} 2gd \sin \theta \right]^{1/2} = \left[\frac{m}{m + I/R^2} 2gh \right]^{1/2}}$$

$$\omega = \frac{v}{R} \Rightarrow \boxed{\omega = \left[\frac{m}{m + I/R^2} \frac{2gh}{R^2} \right]^{1/2}}$$

IF $I = \frac{1}{2} m R^2$, $m + I/R^2 \Rightarrow m + \frac{\frac{1}{2} m R^2}{R^2} \Rightarrow \frac{3}{2} m$

so: $\frac{m}{m + I/R^2} \Rightarrow \frac{m}{\frac{3}{2} m} \Rightarrow \frac{2}{3}$

and: $\boxed{v = \left[\frac{4}{3} gh \right]^{1/2}}$ and $\boxed{\omega = \left[\frac{4}{3} \frac{gh}{R^2} \right]^{1/2}}$

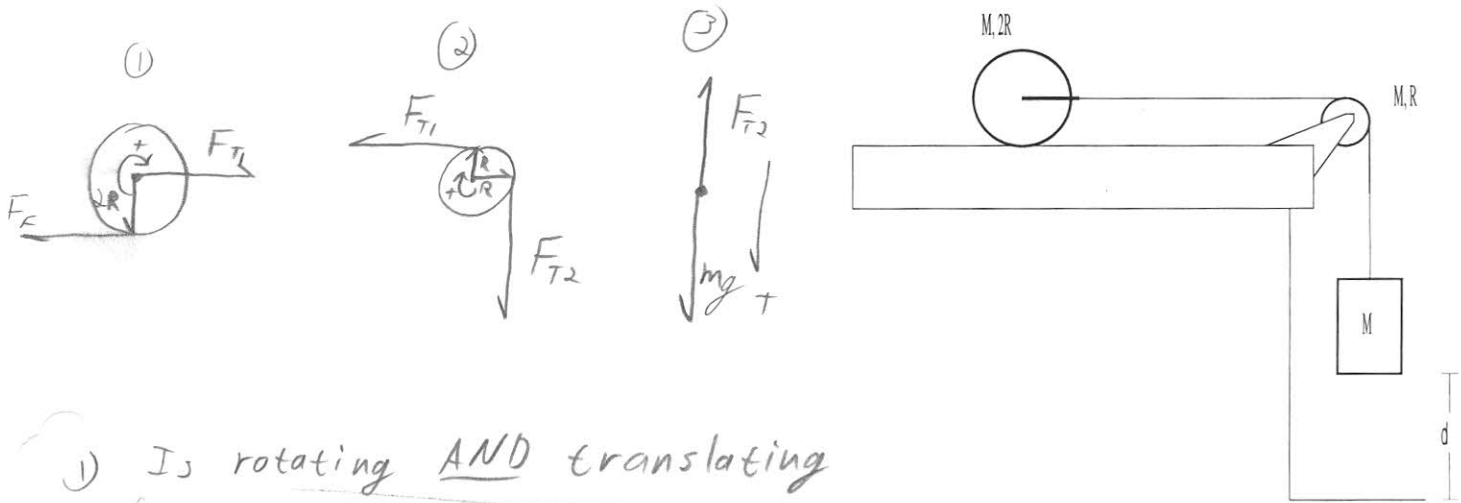
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Use Torque and Newton's Second Law solve this problem.

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

$$I_{cylinder} = \frac{1}{2} MR^2$$

What is the acceleration of the system?



1) Is rotating AND translating

$$-2R F_{T1} + 2R F_f = \frac{1}{2} M (4R^2) \alpha \rightarrow \text{Torque}$$

$$F_f = M R \alpha$$

$$F_{T1} - F_f = m a \rightarrow \text{Force}$$

Put these two together

$$F_{T1} - M R \alpha = m a$$

$$\Rightarrow F_{T1} - \frac{1}{2} m a = m a$$

$$F_{T1} = \frac{3}{2} m a \quad \textcircled{1}$$

continued



Rotation Set 5, P3 continued

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② This pulley is only rotating

$$R F_{T2} - R F_{T1} = \frac{1}{2} m R^2 \alpha \quad \text{and}$$

$$\boxed{F_{T2} - F_{T1} = \frac{1}{2} m a} \quad \text{②}$$

③ The mass is translating

$$m g - F_{T2} = m a$$

$$\Rightarrow \boxed{F_{T2} = m(g - a)} \quad \text{③}$$

Plug ① and ③ into ②

$$m(g - a) - \frac{3}{2} m a = \frac{1}{2} m a$$

$$g - a - \frac{3}{2} a = \frac{1}{2} a$$

$$g = \left(1 + \frac{3}{2} + \frac{1}{2}\right) a$$

$$\Rightarrow \boxed{a = \frac{1}{3} g}$$

