Two meter sticks are stood in the corner where the floor meets the wall. One has a large mass attached to the end furthest from the corner. Which one hits the ground first?

We'll solve the problem by calculating the *Angular Acceleration* of each object and then comparing them.

a) Calculate the angular acceleration of the meter stick falling over under the influence of gravity.

FBD

NSL

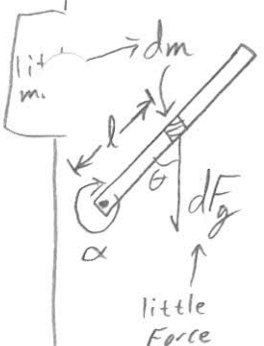
$$\sum \vec{T} = I\alpha$$

$$mg\frac{L}{2}\sin\theta = I\alpha_A$$

$$\cancel{m}g\frac{\cancel{L}}{2}\sin\theta = \frac{1}{3}\cancel{m}L^{\cancel{2}}\alpha_A$$

$$\boxed{\alpha_A = \frac{3}{2}\frac{g}{L}\sin\theta}$$

* Proof that gravity acts at the center of mass



In general: $\vec{T} = \vec{r}\times\vec{F}$

In this case: $d\vec{T} = \vec{l}\times d\vec{F}_g = l\,dF_g\sin\theta\,\hat{i}$

and $dF_g = dm\,g$ so: $dT = lg\sin\theta\,dm$

Then, The net Torque is:

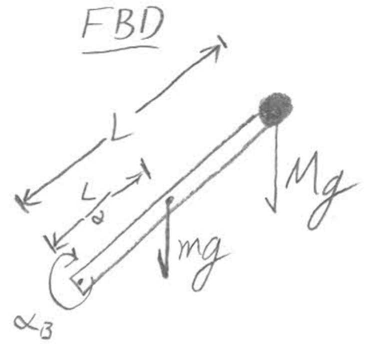
$$T = \int dT = \int lg\sin\theta\,dm$$

$\Rightarrow T = g\sin\theta\int l\,dm$, But $l_{cm} = \frac{1}{M}\int l\,dm$

so: $\int l\,dm = M l_{cm}$

and: $\boxed{T = Mgl_{cm}\sin\theta}$

b) Calculate the angular acceleration of a meter stick with a large mass attached to its end falling over under the influence of gravity.



NSL

$\Sigma T = I\alpha$

① $\boxed{mg\frac{L}{2}\sin\theta + MgL\sin\theta = I\alpha_B}$

$I_T = I_{stick} + I_{mass}$

② $\boxed{I_T = \frac{1}{3}mL^2 + ML^2 = \left(\frac{1}{3}m + M\right)L^2}$

②→①: $\left(\frac{1}{2}m + M\right)g\cancel{L}\sin\theta = \left(\frac{1}{3}m + M\right)L^{\cancel{2}}\alpha_B$

$\Rightarrow \boxed{\alpha_B = \frac{\left(\frac{1}{2}m + M\right)}{\left(\frac{1}{3}m + M\right)}\frac{g}{L}\sin\theta}$

c) Take the ratio $\frac{\alpha_a}{\alpha_b}$. Is it greater than or less than one? Which stick hits the ground first?

$\frac{\alpha_A}{\alpha_B} = \frac{\frac{3}{2}\frac{\cancel{g}}{\cancel{L}}\cancel{\sin\theta}}{\frac{\left(\frac{1}{2}m + M\right)}{\left(\frac{1}{3}m + M\right)}\frac{\cancel{g}}{\cancel{L}}\cancel{\sin\theta}} \Rightarrow \frac{\alpha_A}{\alpha_B} = \frac{3}{2}\cdot\frac{\left(\frac{1}{3}m + M\right)}{\left(\frac{1}{2}m + M\right)}$

$\Rightarrow \boxed{\frac{\alpha_A}{\alpha_B} = \frac{m + 3M}{m + 2M}}$

let's assume:

$\frac{m + 3M}{m + 2M} > 1 \Rightarrow m + 3M > m + 2M \Rightarrow 3\cancel{M} > 2\cancel{M} \Rightarrow 3 > 2$ True!

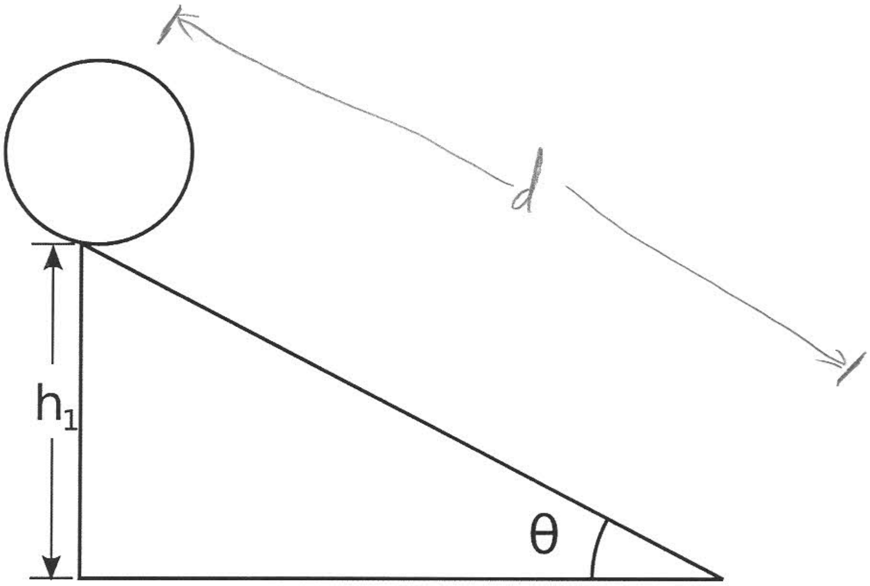
So! $\frac{\alpha_A}{\alpha_B} > 1 \Rightarrow \boxed{\alpha_A > \alpha_B}$ $\boxed{\text{Stick A hits First}}$

Rotation – Set 5

Use **Torque and Kinematics** to solve this problem.

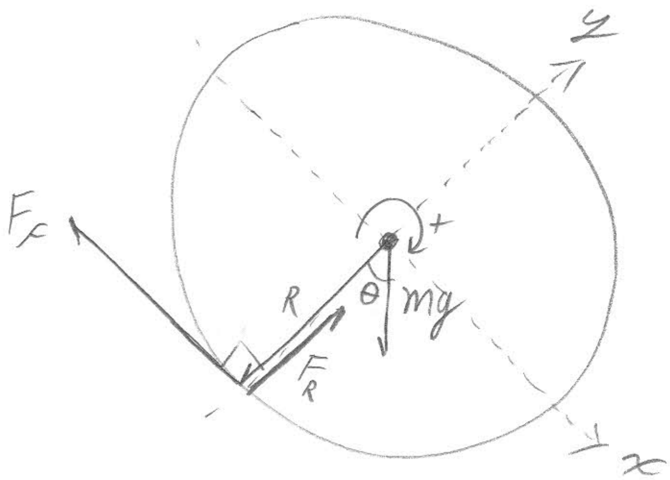
A rolling object with a radius *R*, mass *m*, and moment of inertia I, starts from rest at the top of an incline plane of height *h* that makes an angle θ with the horizontal.

a) Find an expression for the linear and angular acceleration of the object in terms of I.
b) Using kinematics, find an expression for the linear and angular ~~acceleration~~ Velocity of the object in terms of I?
c) Assume that the object is a disk with $I=\frac{1}{2}mR^2$ and plug I into your velocity expressions. Verify that your answers are the as when you solved this problem using energy.



a)

Step 1 - FBD



In this problem, we need to consider both Rotation and translation. So we have positive rotation as well as the x-y coordinates labeled

continued ↓

Rotation set 5, P1 continued

Step (2) - NSL

One object but both translation and rotation

Rotation

$\sum \vec{T} = \vec{I}\alpha$

$\Rightarrow \sum RFSIN\theta = I\alpha$

$\Rightarrow \underbrace{RF_{f}SIN(90)}_{SIN90=1} + \underbrace{\cancel{RF_{R}SIN(180)}^{=0}}_{SIN(180)=0} + \underbrace{\cancel{0 \cdot mg\,SIN\theta}^{=0}}_{R=0} = I\alpha$

$\Rightarrow RF_f = I\alpha \Rightarrow \boxed{F_f = \frac{I}{R}\alpha}$

Translation

$x: \boxed{mg\,SIN\theta - F_f = ma}$ (2)

$y: F_R - mg\cos\theta = 0 \leftarrow$ not useful

Plug (1) → (2): $mg\,SIN\theta - \frac{I}{R}\alpha = ma$

Let $a = R\alpha$: $mg\,SIN\theta - \frac{I}{R^2}a = ma \Rightarrow mg\,SIN\theta = (m + I/R^2)a$

$$\boxed{a = \frac{m}{m + I/R^2} g\,SIN\theta}$$ (1)

$$\boxed{\alpha = \frac{m}{m + I/R^2} \frac{g}{R} SIN\theta}$$ (2)

Rotation set 5, P1 continued

b) kinematics

$$x = \cancel{x_0}^{0} + \cancel{v_0 t}^{0} + \tfrac{1}{2}at^2 \qquad v = \cancel{v_0}^{0} + at$$

$$t = \frac{v}{a}$$

$$d = \tfrac{1}{2}\cancel{a}\frac{v^2}{a^{\cancel{2}}}$$

$$d = \tfrac{1}{2}\frac{v^2}{a} \Rightarrow \boxed{v = (2da)^{1/2}} \quad 3$$

Plug in a from eq. 1

$$v = \left[2d\,\frac{m}{m + I/R^2}\,g\sin\theta\right]^{1/2}$$

$$\boxed{v = \left[\frac{m}{m + I/R^2}\,2gd\sin\theta\right]^{1/2} = \left[\frac{m}{m + I/R^2}\,2gh\right]^{1/2}}$$

$$\omega = \frac{v}{R} \Rightarrow \boxed{\omega = \left[\frac{m}{m + I/R^2}\,\frac{2gh}{R^2}\right]^{1/2}}$$

$$\text{IF } I = \tfrac{1}{2}mR^2,\quad m + I/R^2 \Rightarrow m + \frac{\tfrac{1}{2}m\cancel{R^2}}{\cancel{R^2}} \Rightarrow \tfrac{3}{2}m$$

$$\text{so: } \frac{m}{m + I/R^2} \Rightarrow \frac{\cancel{m}}{\tfrac{3}{2}\cancel{m}} \Rightarrow \underline{2/3}$$

$$\text{and: } \boxed{v = \left[\tfrac{4}{3}gh\right]^{1/2}} \text{ and } \boxed{\omega = \left[\tfrac{4}{3}\frac{gh}{R^2}\right]^{1/2}}$$

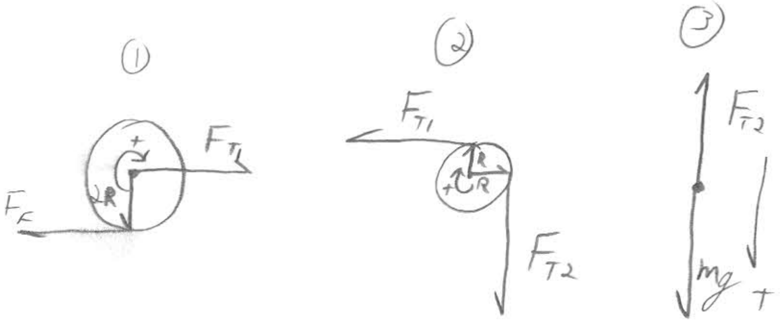
Rotation – Set 5

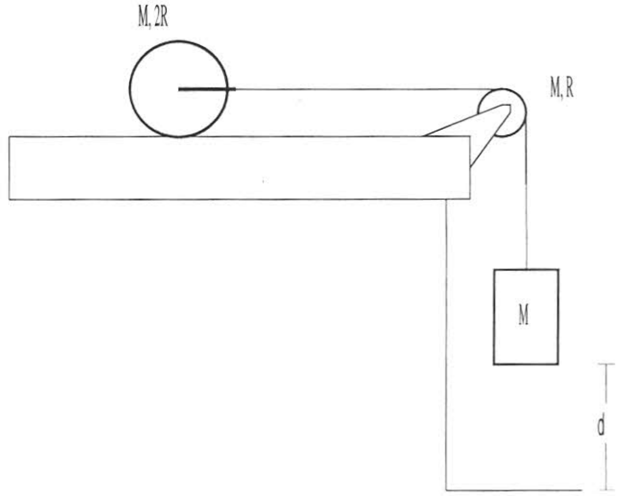
Use **Torque and Newton's Second Law** solve this problem.

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

$$I_{cylinder} = \frac{1}{2}MR^2$$

What is the acceleration of the system?





1) Is rotating AND translating

$$-2(0)\cancel{F_{T1}}^{0} + \cancel{2}RF_f = \frac{1}{\cancel{2}}m4R^{\cancel{2}}\alpha \rightarrow \text{Torque}$$

$$F_f = mR\alpha$$

$$F_{T1} - F_f = ma \rightarrow \text{Force}$$

Put these two together

$$F_{T1} - mR\alpha = ma$$

$$\Rightarrow F_{T1} - \frac{1}{2}ma = ma$$

$$\boxed{F_{T1} = \frac{3}{2}ma} \quad (1)$$

continued ↓

Rotation Set 5, P3 continued

2 This pulley is only rotating

$$\cancel{R}F_{T2} - \cancel{R}F_{T1} = \tfrac{1}{2}mR^{\cancel{2}}\alpha \qquad a = r\alpha$$

$$\boxed{F_{T2} - F_{T1} = \tfrac{1}{2}ma} \quad ②$$

3 The mass is translating

$$mg - F_{T2} = ma$$

$$\Rightarrow \boxed{F_{T2} = m(g-a)} \quad ③$$

Plug ① and ③ into ②

$$\cancel{m}(g-a) - \tfrac{3}{2}\cancel{m}a = \tfrac{1}{2}\cancel{m}a$$

$$g - a - \tfrac{3}{2}a = \tfrac{1}{2}a$$

$$g = (1 + \tfrac{3}{2} + \tfrac{1}{2})a$$

$$\Rightarrow \boxed{a = \tfrac{1}{3}g}$$

Rotation – Set 5

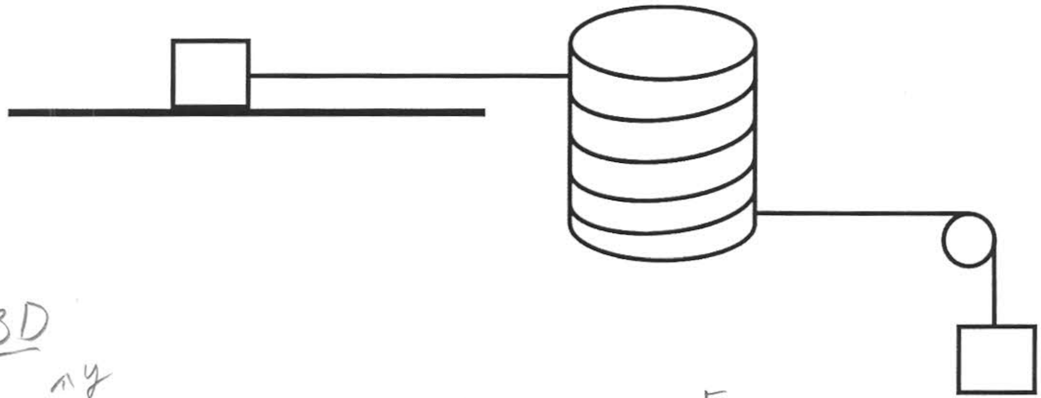
4

Use **Torque and Newton's Second Law** solve this problem.

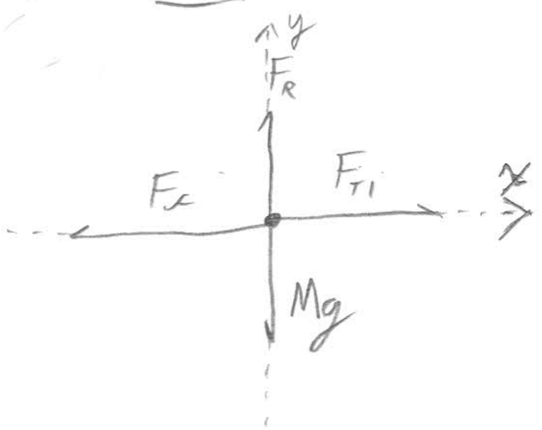
A block of mass M rests on a rough table with $\mu_k = 0.3$. A massless string is attached to the block, wrapped around a solid cylinder having a mass M and a radius R, runs over a massless frictionless pulley, and is attached to a second block of mass M that is hanging freely.

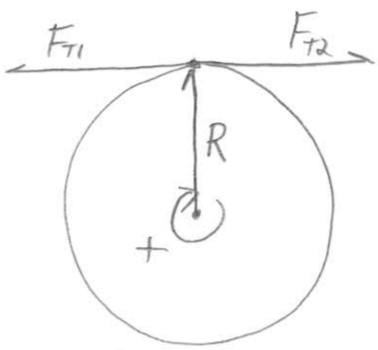
Find the acceleration of this system.

$$I_{cylinder} = \frac{1}{2}MR^2$$



FBD





NSL $\sum \vec{F} = ma$

x: $F_{T1} - F_f = Ma$

y: $F_R - Mg = 0$

$F_R = Mg$

$\Rightarrow \boxed{F_{T1} - \mu_k Mg = Ma}$ ①

$\sum \vec{\tau} = I\vec{\alpha}$

$\sum RF\sin\theta = I\alpha$

$RF_{T2} - RF_{T1} = \frac{1}{2}MR^2\alpha$

$\boxed{F_{T2} - F_{T1} = \frac{1}{2}MR\alpha}$ ②

$\sum \vec{F} = m\vec{a}$

$\boxed{-F_{T2} + Mg = Ma}$ ③

continued ↓

Rotation Set 5, P4 continued

Eliminate F_{T1} and F_{T2}:

From ①: $F_{T1} = \mu_K Mg + Ma$

From ③: $F_{T2} = Mg - Ma$

into ②: $\cancel{M}g - \cancel{M}a - \mu_K \cancel{M}g - \cancel{M}a = \frac{1}{2}\cancel{M}R\alpha$

$g(1-\mu_K) = 2a + \frac{1}{2}R\alpha \Rightarrow g(1-\mu_K) = 2R\alpha + \frac{1}{2}R\alpha$

$\Rightarrow g(1-\mu_K) = \frac{5}{2}R\alpha$

$\Rightarrow \boxed{\alpha = \frac{2}{5(1-\mu_K)}\frac{g}{R}}$

or $\boxed{a = \frac{2}{5(1-\mu_K)}g}$