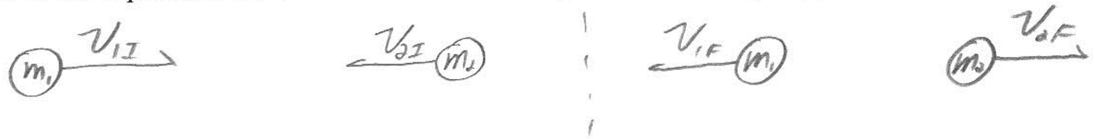
SAMPLE TEST 4
PHYS 111 SPRING 2010

Name: ____________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS – YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

1) Derive the equations for the final velocities of particles undergoing an elastic collision in 1 dimension.



conserve momentum: $m_1 V_{1I} + m_2 V_{2I} = m_1 V_{1F} + m_2 V_{2F}$ (1)

conserve energy: $\frac{1}{2} m_1 V_{1I}^2 + \frac{1}{2} m_2 V_{2I}^2 = \frac{1}{2} m_1 V_{1F}^2 + \frac{1}{2} m_2 V_{2F}^2$ (2)

Get all m_1 on right and all m_2 on left

$m_1(V_{1I} - V_{1F}) = m_2(V_{2F} - V_{2I})$ (3)

$m_1(V_{1I}^2 - V_{1F}^2) = m_2(V_{2F}^2 - V_{2I}^2)$ (4)

Divide $\frac{(4)}{(3)}$: $\frac{\cancel{m_1}(V_{1I}^2 - V_{1F}^2)}{\cancel{m_1}(V_{1I} - V_{1F})} = \frac{\cancel{m_2}(V_{2F}^2 - V_{2I}^2)}{\cancel{m_2}(V_{2F} - V_{2I})}$

Remember:
$(a^2 - b^2) = (a+b)(a-b)$: $\frac{\cancel{(V_{1I} - V_{1F})}(V_{1I} + V_{1F})}{\cancel{(V_{1I} - V_{1F})}} = \frac{\cancel{(V_{2F} - V_{2I})}(V_{2F} + V_{2I})}{\cancel{(V_{2F} - V_{2I})}}$

$$V_{1I} + V_{1F} = V_{2I} + V_{2F} \quad (5)$$

Solve (5) for V_{2F}: $V_{2F} = V_{1I} + V_{1F} - V_{2I}$ (6)

continued
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Sample Test 4, P1 continued

Put ⑥ → ①

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 (v_{1I} + v_{1F} - v_{2I})$$

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{1I} + m_2 v_{1F} - m_2 v_{2I}$$

$$v_{1F}(m_1 + m_2) = (m_1 - m_2) v_{1I} + 2 m_2 v_{2I}$$

$$\boxed{v_{1F} = \frac{m_1 - m_2}{m_1 + m_2} v_{1I} + \frac{2 m_2}{m_1 + m_2} v_{2I}}$$

And swap subscripts for v_{2F}

$$\boxed{v_{2F} = \frac{m_2 - m_1}{m_1 + m_2} v_{2I} + \frac{2 m_1}{m_1 + m_2} v_{1I}}$$

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2)

2.1 When you fire a rifle, the bullet flies out and the rifle recoils. The magnitude of the momentum of the bullet is _____ the momentum of the rifle.

a. greater than
b. the same as
c. less than

$$0 = m_R v_R + m_B v_B \Rightarrow \boxed{m_R v_R = -m_B v_B}$$

magnitudes are equal

2.2 When you fire a rifle, the bullet flies out and the rifle recoils. Assuming that the bullet weighs less than the rifle, the Kinetic Energy of the bullet is _____ the Kinetic Energy of the rifle.

a. greater than
b. the same as
c. less than

$$m_R v_R = -m_B v_B \Rightarrow v_R = -\frac{m_B}{m_R} v_B$$

$$\frac{K_B}{K_R} = \frac{\cancel{\tfrac{1}{2}} m_B v_B^2}{\cancel{\tfrac{1}{2}} m_R v_R^2} = \frac{\cancel{m_B}}{\cancel{m_R}} \cancel{v_B^2} \frac{m_R^2}{m_B^2} \cdot \frac{1}{\cancel{v_B^2}} = \frac{m_R}{m_B} > 1 \Rightarrow \boxed{K_B > K_R}$$

2.3) According to Newton's Second Law for systems of particles, if the sum of the external forces is zero:

a. The position of the center of mass must be constant
b. The velocity of the center of mass must be zero
c. The velocity of the center of mass must be constant.
d. None of the above statements is true.

2.4) Which of the following systems do **NOT** conserve momentum?

a. While slipping on a patch of ice a car collides with another car on the ice. (Ignore friction with the ice.) *System: Both cars.*
b. A single car sliding across the ice. (Ignore friction with the ice.) *System: The car.*
c. A ball drops to earth. (Ignore air friction.) *System: The ball.*
d. A billiard ball collides with another billiard ball on a pool table. (Ignore friction with the table during the collision.) *System: Both balls.*

2.5) Two friends are standing on opposite ends of a canoe. The canoe is initially at rest with respect to the lake. The person on the right throws a very massive ball to the left and the person on the left catches it. After the ball is caught, the canoe is (ignore friction between the canoe and the water)

a. stationary
b. moving to the right
c. moving to the left.

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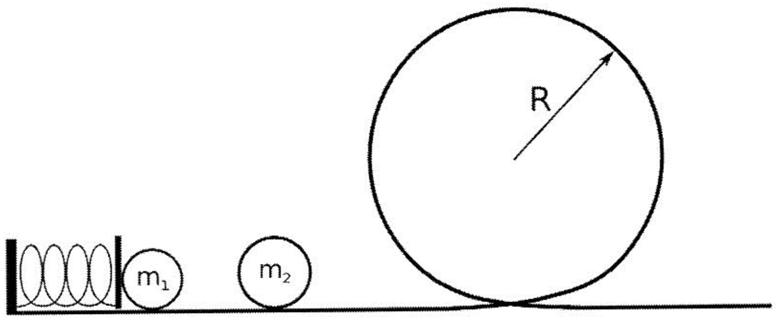
3) In the system below, a ball of mass m_1 is placed against a spring with spring constant k that has been compressed a distance d. It is released from rest and collides with a second ball of mass m_2 which then goes around the loop the loop of radius R.

Find an expression for the minimum spring compression d in terms of m_1, m_2, k, R, and g such that m_2 makes it around the loop.



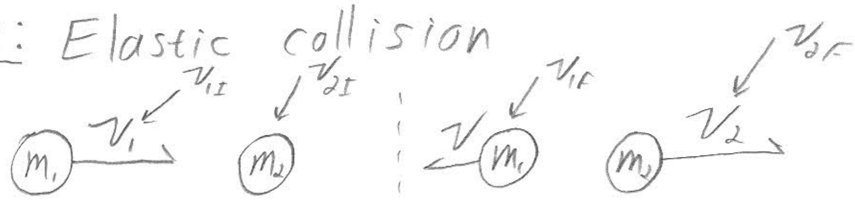
Stage 1: Spring releases

$U_I = \frac{1}{2}kd^2$ $\qquad$ $U_F = 0$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2}m_1 v_1^2$

$$\cancel{\tfrac{1}{2}}kd^2 = \cancel{\tfrac{1}{2}}m_1 v_1^2 \Rightarrow \boxed{d = \sqrt{\frac{m_1}{k}}\,v_1} \quad (1)$$

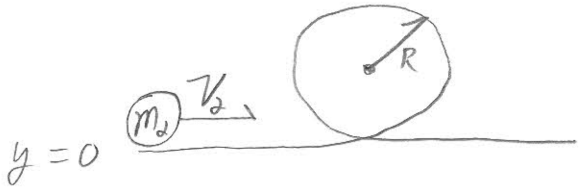
Stage 2: Elastic collision



$$v_{2F} = \cancel{\frac{m_2 - m_1}{m_1 + m_2} v_{2I}}^{\,0} + \frac{2m_1}{m_1+m_2} v_{1I}$$

$$\Rightarrow v_2 = \frac{2m_1}{m_1+m_2} v_1 \Rightarrow \boxed{v_1 = \frac{m_1+m_2}{2m_1} v_2} \quad (2)$$

Stage 3: Loop the Loop

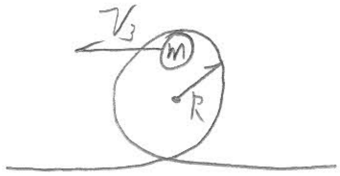


$U_I = 0$ $\qquad$ $U_F = m_2 g(2R)$

$K_I = \frac{1}{2} m_2 v_2^2$ $\qquad$ $K_F = \frac{1}{2} m_2 v_3^2$

$$\frac{1}{2} m_2 v_2^2 = 2 m_2 g R + \frac{1}{2} m_2 v_3^2$$

But, what is v_3 so that m_2 loops?



NSC

$$\sum F = ma$$

uniform circular motion

$$\Rightarrow m_2 g + \cancel{N}^{\,0} = m_2 \frac{v_3^2}{R}$$

minimum speed ... $N \to 0$

$$\Rightarrow \cancel{m_2} g = \cancel{m_2} \frac{v_3^2}{R}$$

$$\Rightarrow \boxed{v_3 = \sqrt{gR}}$$

So: $\frac{1}{2} \cancel{m_2} v_2^2 = 2 \cancel{m_2} g R + \frac{1}{2} \cancel{m_2} g R$

$$\Rightarrow \boxed{v_2 = \sqrt{5gR}}$$ 3

continued ↓

Sample Test 4, P3 continued

Put it all together

From ①: $d = \sqrt{\frac{m_1}{k}}\, v_1$

plug in ②: $d = \sqrt{\frac{m_1}{k}}\, \frac{m_1 + m_2}{2 m_1}\, v_2$

plug in ③: $d = \sqrt{\frac{m_1}{k}}\, \frac{m_1 + m_2}{2 m_1} \sqrt{5gR}$

$$\Longrightarrow \boxed{d = \sqrt{\frac{5gR}{k}} \cdot \frac{m_1 + m_2}{2\sqrt{m_1}}}$$

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4. In many classic westerns, gunfighters fly backwards several meters after being shot, often crashing through windows or saloon doors. Assume that a typical bullet weights 2 g and that a typical cowboy weights 80 kg.

a) If the bullet leaves the gun at 200 m/s, what is the velocity of the cowboy/bullet system after the impact?

b) What velocity does the bullet need for the cowboy to slide 3 meters across the floor after being shot (assuming $U_k = 0.5$)?

a)

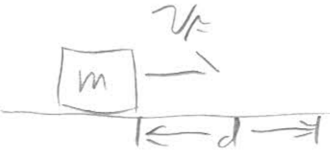
Given
$m_B = 2\times10^{-3}$ kg
$M_c = 80$ kg
$V_{BI} = 200$ m/s
$V_F = ?$

$$P_i = m_B V_{BI} + \cancel{m_c V_{CI}}^{\,0} \qquad P_F = (m_B + m_c) V_F$$

$$\Rightarrow \boxed{V_F = \frac{m_B}{(m_B + m_c)} V_{BI}}$$

$$V_F = \frac{2\times10^{-3}}{80.002}\cdot 200 = \boxed{0.4 \text{ m/s}}$$

b) Slide to a stop in a distance d



$$K_I = \tfrac{1}{2}(m_c + m_B) V_F^2 \qquad W_f = -\mu_k (m_c + m_B) g d$$

$$K_f = U_I = U_F = 0$$

$$\Rightarrow \tfrac{1}{2}\cancel{(m_c + m_B)} V_F^2 = \mu_k \cancel{(m_c + m_B)} g d$$

$$V_F = (2\mu_k g d)^{1/2}$$

continued

Sample Test 4, P4 continued

$$\Rightarrow \frac{m_B}{(m_c + m_B)} v_{BI} = (2\mu_k g d)^{1/2}$$

$$\Rightarrow \boxed{v_{BI} = \frac{m_c + m_B}{m_B} (2\mu_k g d)^{1/2}}$$

$$v_{BI} = \frac{80.002}{2\times 10^{-3}} \left((2)(0.5)(9.8)(3)\right)^{1/2}$$

$$= \boxed{7.4\times 10^{3}\ m/s} = 16{,}000\ \text{miles/hour}$$

For comparison, the M16 muzzle velocity is approximately 1,000 m/s or 2,200 miles/hour

So... Flying backwards is bogus...

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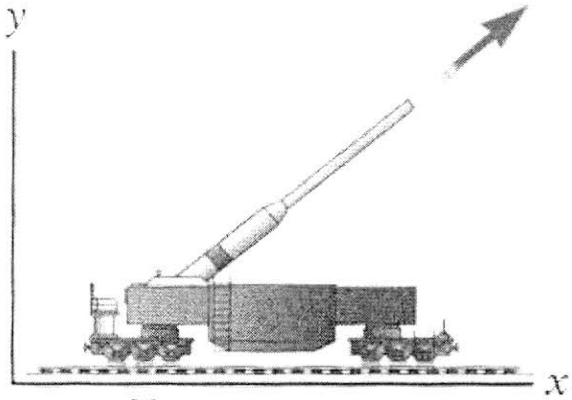
5. 4. A gun is mounted on a railroad car as shown in the picture. There is no friction between the car and the track, the track is perfectly horizontal, and the car is initially at rest. The gun fires a shell of mass 30 kg with a velocity of 300 m/s at an angle of 40° with respect to the horizontal. What is the recoil velocity of the car?



$m_B = 30\,kg$ $\quad m_c = 3000\,kg$

$v_B = 300\,m/s$ $\quad v_c = ?$

$\theta = 40°$

$\vec{P}_I = 0$ $\qquad$ $\vec{P}_F = m_B \vec{v}_B + m_c \vec{v}_c$

This system does <u>not</u> conserve momentum in the y due to the external Normal Force.

The bullet undergoes acceleration in the y that the car does not experience.

We <u>can</u> conserve P_x however.

$$0 = m_B v_{Bx} + m_c v_{cx} = \boxed{v_{cx} = \frac{m_B}{m_c} v_B \cos\theta}$$

$$v_{cx} = \frac{30}{6\times10^4} \cdot (300)\cos(40) = \boxed{1.15\times10^{-1}\,m/s}$$