

SAMPLE TEST 4
 PHYS 111 SPRING 2010

Name: _____

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS — YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS (IF NOT MORE) IMPORTANT THAN) YOUR FINAL ANSWER.

1) Derive the equations for the final velocities of particles undergoing an elastic collision in 1 dimension.



Conserve momentum: $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (1)

Conserve energy: $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ (2)

Get all m_1 on right
 and all m_2 on left

$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$ (3)

$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$ (4)

Divide $\frac{(4)}{(3)}$: $\frac{m_1 (v_{1i}^2 - v_{1f}^2)}{m_1 (v_{1i} - v_{1f})} = \frac{m_2 (v_{2f}^2 - v_{2i}^2)}{m_2 (v_{2f} - v_{2i})}$

Remember: $(a^2 - b^2) = (a+b)(a-b)$: $\frac{(v_{1i} - v_{1f})(v_{1i} + v_{1f})}{(v_{1i} - v_{1f})} = \frac{(v_{2f} - v_{2i})(v_{2f} + v_{2i})}{(v_{2f} - v_{2i})}$

$v_{1i} + v_{1f} = v_{2i} + v_{2f}$ (5)

Solve (5) for v_{2f} : $v_{2f} = v_{1i} + v_{1f} - v_{2i}$ (6)

continued
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Sample Test 4, P1 continued

Put (6) \rightarrow (1)

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 (v_{1I} + v_{1F} - v_{2I})$$

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{1I} + m_2 v_{1F} - m_2 v_{2I}$$

$$v_{1F} (m_1 + m_2) = (m_1 - m_2) v_{1I} + 2 m_2 v_{2I}$$

$$v_{1F} = \frac{m_1 - m_2}{m_1 + m_2} v_{1I} + \frac{2 m_2}{m_1 + m_2} v_{2I}$$

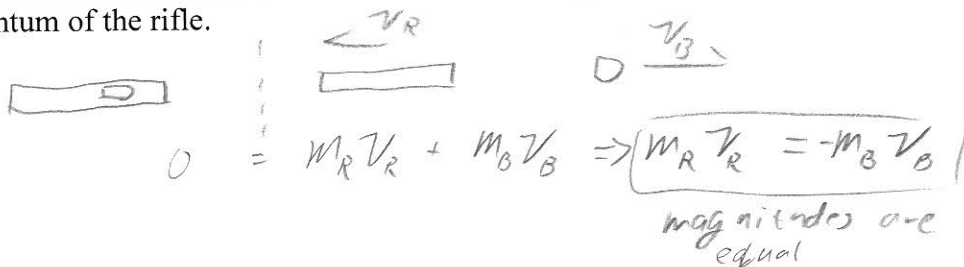
And swap subscripts for v_{2F}

$$v_{2F} = \frac{m_2 - m_1}{m_1 + m_2} v_{2I} + \frac{2 m_1}{m_1 + m_2} v_{1I}$$

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2) 2.1 When you fire a rifle, the bullet flies out and the rifle recoils. The magnitude of the momentum of the bullet is _____ the momentum of the rifle.

- a. greater than
- b. the same as
- c. less than



2.2 When you fire a rifle, the bullet flies out and the rifle recoils. Assuming that the bullet weighs less than the rifle, the Kinetic Energy of the bullet is _____ the Kinetic Energy of the rifle.

- a. greater than
- b. the same as
- c. less than

$$m_R v_R = -m_B v_B \Rightarrow v_R = -\frac{m_B}{m_R} v_B$$

$$\frac{K_B}{K_R} = \frac{\frac{1}{2} m_B v_B^2}{\frac{1}{2} m_R v_R^2} = \frac{m_B}{m_R} \frac{v_B^2}{\frac{m_B^2}{m_R^2} v_B^2} = \frac{m_R}{m_B} > 1 \Rightarrow K_B > K_R$$

2.3) According to Newton's Second Law for systems of particles, if the sum of the external forces is zero:

- a. The position of the center of mass must be constant
- b. The velocity of the center of mass must be zero
- c. The velocity of the center of mass must be constant.
- d. None of the above statements is true.

2.4) Which of the following systems do **NOT** conserve momentum?

- a. While slipping on a patch of ice a car collides with another car on the ice. (Ignore friction with the ice.) System: Both cars.
- b. A single car sliding across the ice. (Ignore friction with the ice.) System: The car.
- c. A ball drops to earth. (Ignore air friction.) System: The ball.
- d. A billiard ball collides with another billiard ball on a pool table. (Ignore friction with the table during the collision.) System: Both balls.

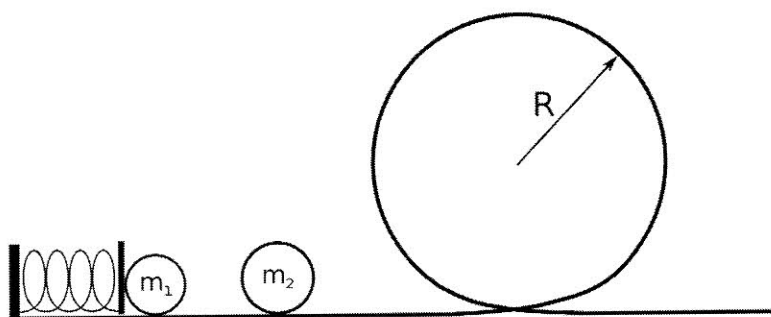
2.5) Two friends are standing on opposite ends of a canoe. The canoe is initially at rest with respect to the lake. The person on the right throws a very massive ball to the left and the person on the left catches it. After the ball is caught, the canoe is (ignore friction between the canoe and the water)

- a. stationary
- b. moving to the right
- c. moving to the left.

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3) In the system below, a ball of mass m_1 is placed against a spring with spring constant k that has been compressed a distance d . It is released from rest and collides with a second ball of mass m_2 which then goes around the loop the loop of radius R .

Find an expression for the minimum spring compression d in terms of m_1 , m_2 , k , R , and g such that m_2 makes it around the loop.

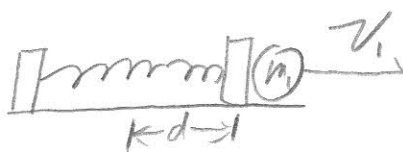


Stage 1: Spring releases



$$U_I = \frac{1}{2}kd^2$$

$$K_I = 0$$

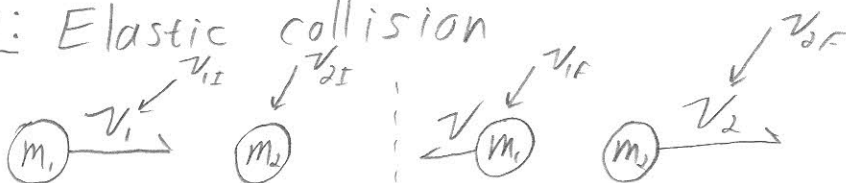


$$U_F = 0$$

$$K_F = \frac{1}{2}m_1v_1^2$$

$$\frac{1}{2}kd^2 = \frac{1}{2}m_1v_1^2 \Rightarrow d = \sqrt{\frac{m_1}{k}}v_1 \quad \textcircled{1}$$

Stage 2: Elastic collision



$$v_{2F} = \frac{m_2 - m_1}{m_1 + m_2}v_{2I} + \frac{2m_1}{m_1 + m_2}v_{1I}$$

$$\Rightarrow v_2 = \frac{2m_1}{m_1 + m_2}v_1 \Rightarrow v_1 = \frac{m_1 + m_2}{2m_1}v_2 \quad \textcircled{2}$$

Sample Test 4, P3 continued

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Put it all together

$$\text{From ①: } d = \sqrt{\frac{m_1}{k}} v_1$$

$$\text{Plug in ②: } d = \sqrt{\frac{m_1}{k}} \frac{m_1 + m_2}{2m_1} v_2$$

$$\text{plug in ③: } d = \sqrt{\frac{m_1}{k}} \frac{m_1 + m_2}{2m_1} \sqrt{5gR}$$

$$\Rightarrow d = \sqrt{\frac{5gR}{k}} \cdot \frac{m_1 + m_2}{2\sqrt{m_1}}$$

