



Neo and Agent Smith are flying towards each other. They collide in mid air and grab onto each other (they stick together).

a) Assume that momentum is conserved in the Matrix and find an expression relating their initial velocities to their final velocity.

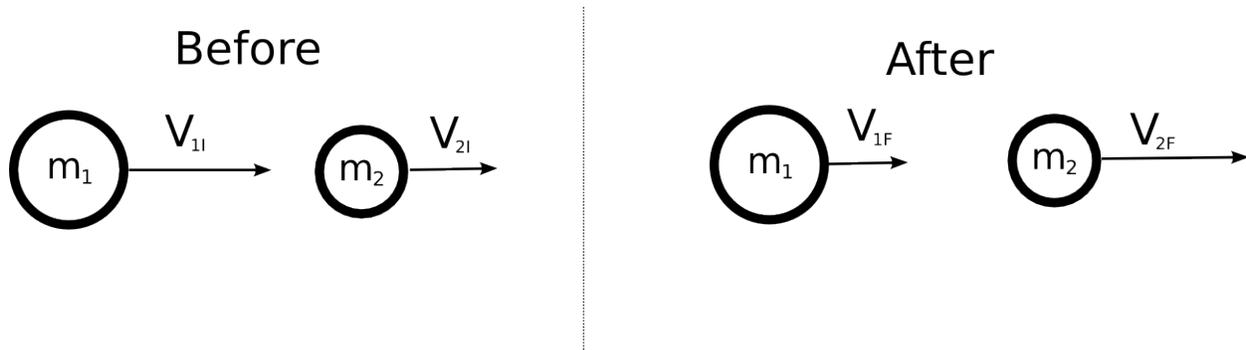
b) Let $M_N = 70$ kg, $V_{Ni} = 50$ m/s, $M_S = 100$ kg, and $V_{Si} = 35$ m/s. Put these numbers into your expression and solve for their final velocity.

c) Calculate the pre-collision and post-collision kinetic energy of the system. Does this system conserve kinetic energy through the collision?

If Neo and Agent Smith conserved energy as well as momentum, they would bounce off of each other and the collision would be *elastic*. Let's derive a general expression relating the initial and final velocities in an elastic collision.

Step 1:

Starting with the picture below, write two equations, one for the *conservation of momentum* and one for the *conservation of kinetic energy*.



Using the two equations above, work out the algebra required to get to the following equation:

$$\frac{v_{1i}^2 - v_{1f}^2}{v_{1i} - v_{1f}} = \frac{v_{2f}^2 - v_{2i}^2}{v_{2f} - v_{2i}} \quad \text{This is waypoint 1}$$

Step 2:

Starting with waypoint 1:

$$\frac{V_{1I}^2 - V_{1F}^2}{V_{1I} - V_{1F}} = \frac{V_{2F}^2 - V_{2I}^2}{V_{2F} - V_{2I}}$$

Perform the required algebra to get to waypoint 2:

$$V_{1I} + V_{1F} = V_{2I} + V_{2F}$$

The following relationship may prove useful:

$$(a+b)(a-b) = (a^2 - b^2)$$

Step 3:

Combine the results of waypoint 2, $V_{1i} + V_{1f} = V_{2i} + V_{2f}$, with the equation for conservation of momentum from part 1 to arrive at waypoint 3:

$$(m_1 - m_2)V_{1i} + 2m_2 V_{2i} = (m_1 + m_2)V_{1f}$$

Step 4:

Solve waypoint 3 to get the general expression for V_{1F} :

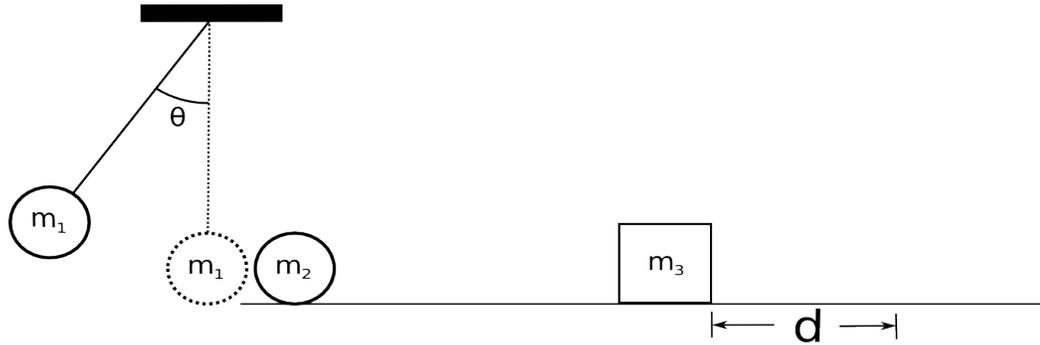
$$V_{1F} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1I} + \frac{2m_2}{m_1 + m_2} V_{2I}$$

Systems of Particles – Set 4

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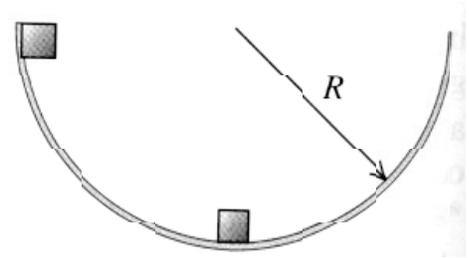
A mass $m_1 = 3$ kg is attached to a string of length $l = 4.0$ m to create a pendulum. The pendulum, initially making an angle θ with the vertical, is released from rest. At the bottom of its swing, it collides elastically with mass $m_2 = 5$ kg. Mass 2 rolls (no friction) and sticks to $m_3 = 5$ kg. The m_2, m_3 combination slides with $\mu_k = 0.3$ a distance $d = 0.2$ m before coming to rest.

What was the original value of θ ?



Extra Work Space

Two masses are released from rest in a frictionless hemispherical bowl of radius R from the positions shown in the figure. Derive an expression for their final height in the case of :



- a) An elastic collision
- b) An inelastic collision
- c) How much bigger than the second mass does the first mass have to be so that the second mass gets out of the bowl.

Systems of Particles – Set 4

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A large fish will soon make a dish of a smaller fish. What is the velocity of the large fish and his dinner immediately after he eats? Give both the magnitude and direction of the final velocity with respect to the x-axis.

$$\begin{aligned}m_{\text{large fish}} &= 4.0 \text{ kg} \\v_{0 \text{ large fish}} &= 1.0 \text{ m/s} \\ \alpha_{\text{large fish}} &= 25.0^\circ\end{aligned}$$

$$\begin{aligned}m_{\text{small fish}} &= 0.20 \text{ kg} \\v_{0 \text{ small fish}} &= 5.0 \text{ m/s} \\ \beta_{\text{small fish}} &= 50.0^\circ\end{aligned}$$

