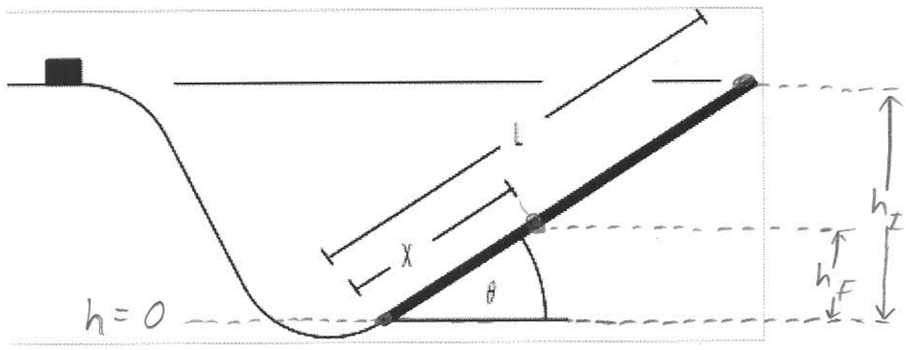
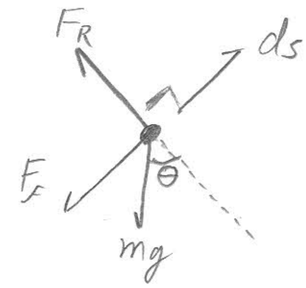
# Energy Problems – Set 3.5



In the system shown below all surfaces are frictionless except the dark part of the ramp, where the coefficient of friction is $\mu_k$. The top of the top of the ramp, which has a length $L$, is level with the surface where the block starts. After being given a very small push (assume $V_0=0$), the block slides down the left side and stops a distance $x$ from the bottom of the dark part of the ramp. Find an expression for $\theta$ in terms of $L$, $x$, and $\mu_k$.




FBD




$U_I = mgh_I$ $\qquad$ $K_I = 0$

| Given | Want |
|---|---|
| $\mu_K$ | $\theta$ |
| $L$ | |
| $V_0 = 0$ | |
| $X$ | |
| $V_F = 0$ | |

$U_F = mgh_F$ $\qquad$ $K_F = 0$

$$W_F = \int \vec{F}_F \cdot d\vec{s} = -\int_0^X F_F\, ds = -\mu_K F_R X$$

according to NSL: $F_R - mg\cos\theta = 0 \Rightarrow F_R = mg\cos\theta$

$$\Rightarrow \boxed{W_F = -\mu_K mgX\cos\theta}$$

According to the picture: $\boxed{h_I = L\sin\theta}$ and $\boxed{h_F = X\sin\theta}$

continued ↓

Energy Set 3.5, P1 continued

Now, conserve Energy:

$$U_I + \cancel{K_I}^{\,0} + W_{NC} = U_F + \cancel{K_F}^{\,0}$$

$$mgL\sin\theta - \mu_k mgX\cos\theta = mgX\sin\theta$$

$$\Rightarrow mgL\sin\theta - mgX\sin\theta = \mu_k mgX\cos\theta$$

$$\Rightarrow \cancel{mg}(L-X)\sin\theta = \mu_k \cancel{mg}X\cos\theta$$

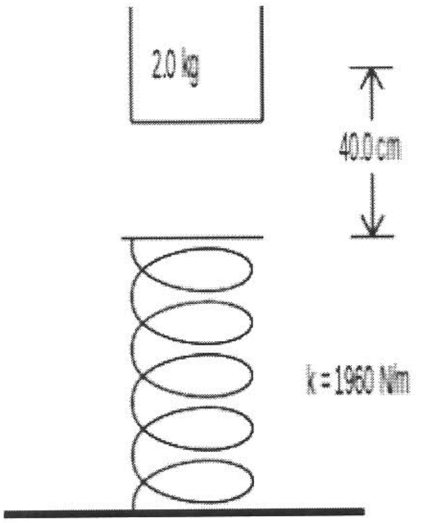
$$\Rightarrow \tan\theta = \frac{X}{L-X}\mu_k$$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left[\frac{X}{L-X}\mu_k\right]}$$

# Energy Problems – Set 3.5



A 2.0 kg block is dropped from a height of 40 cm onto a spring of spring constant k = 1960 N/m. Find the maximum distance the spring is compressed.




**Given**

$m = 2.0\ kg$

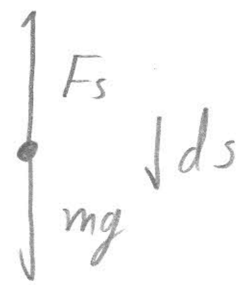
$h = 40\ cm$

$k = 1960\ N/m$

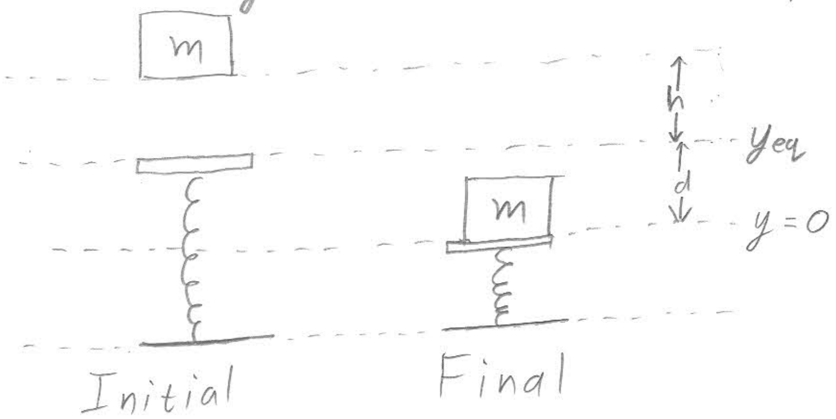
**Want**

$d$

**FBD**




Drawing Initial and Final positions




$$U_I = mgh_I + \tfrac{1}{2}k(y_I - y_{eq})^2$$

$$U_F = mgh_F + \tfrac{1}{2}k(y_F - y_{eq})^2$$

$$K_I = 0, \quad K_F = 0$$



continued ↓

so:

$$mgh_I + \tfrac{1}{2}k(y_I - y_{eq})^2 = mgh_F + \tfrac{1}{2}k(y_F - y_{eq})^2$$

From my picture:

$\boxed{h_I = h+d}$ and $\boxed{h_F = 0}$

and (because spring potential tracks the position of the end of the spring...)

$\boxed{(y_I - y_{eq})^2 = 0}$ and $\boxed{(y_F - y_{eq})^2 = d^2}$

$$\Rightarrow mg(h+d) = \tfrac{1}{2}kd^2$$ ug! quadratic...

$$-\tfrac{1}{2}kd^2 + mgd + mgh = 0$$

$$\Rightarrow d = \frac{-mg \pm \left[(mg)^2 + 2kmh\right]^{1/2}}{-k}$$

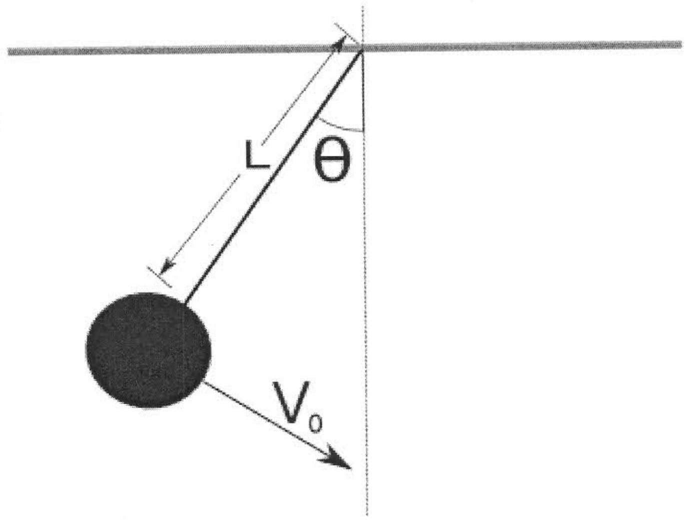
$$d = -0.08, \boxed{0.1}$$

The negative root is spring extension.

We want the positive root.

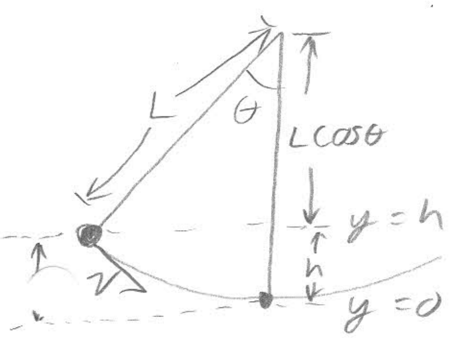
# Energy Problems – Set 4

The picture shows a pendulum with a weight of mass m attached to a light (massless) string of length L. The mass has a speed $v_0$ when the cord makes an angle θ with the vertical.

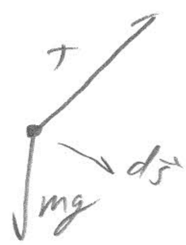



a) Derive an expression for the speed of the mass when it is in its lowest position.

b) What is the minimum value of v0 for the chord to make an angle of 90 on the pendulum's upswing?




$h = L - L\cos\theta$
$= L(1-\cos\theta)$




$W_T = 0,\ \vec{T} \perp d\vec{s}$

$U_I = mgh \qquad U_F = 0$

$K_I = \frac{1}{2} m v_0^2 \qquad K_F = \frac{1}{2} m v^2$

$\cancel{m}gh + \frac{1}{2}\cancel{m}v_0^2 = \frac{1}{2}\cancel{m}v^2$

$\Rightarrow v = \left(v_0^2 + 2gh\right)^{1/2} = \boxed{\left(v_0^2 + 2gL(1-\cos\theta)\right)^{1/2}}$

EP4,4 continued

b) When $\theta = 90$, $h = L(1-\cos(90)) = L$

$U_I = mg(L - L\cos\theta)$ $\quad$ $U_F = mgL$

$K_I = \frac{1}{2}mV_0^2$ $\quad$ $K_F = 0$

$\cancel{m}g(L - L\cos\theta) + \frac{1}{2}\cancel{m}V_0^2 = \cancel{m}gL$

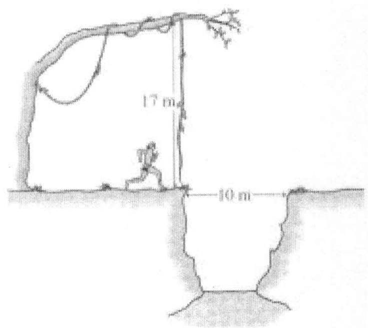
$\frac{1}{2}V_0^2 = \cancel{gL} + gL\cos\theta - \cancel{gL}$

$\boxed{V_0^2 = 2gL\cos\theta}$

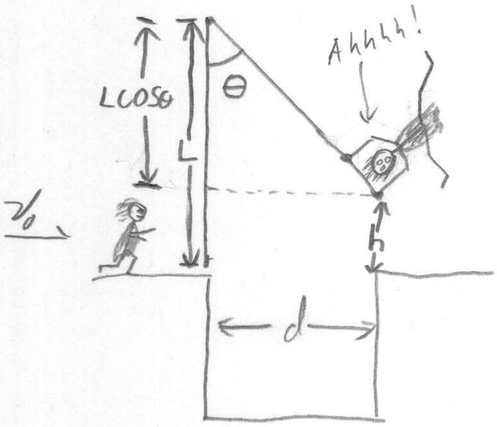
Sample Test 3
Phys 111 Spring 2010

*Use work-energy techniques to solve the following problem.*

Tarzan is late for a date with Jane and is running as fast as he can to meet her. On the way, he has to get over a 10m wide pit of dangerous croc-a-gators. A 17m vine is hanging vertically from a tree at one side of the pit. Tarzan is going to run up, grab the vine, swing across, and drop vertically to the ground on the other side.

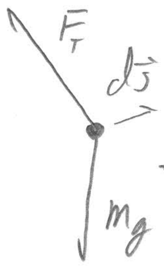



What must his minimum speed be to make it across?




$h = L - L\cos\theta$

$d = L\sin\theta$




$W_{F_T} = 0,\ \vec{F}_T \perp d\vec{s}$

some point mid swing

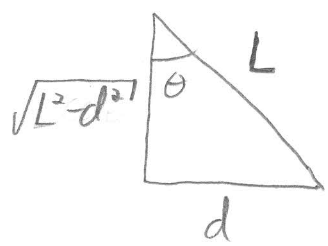
$U_I = 0$ $\qquad$ $U_F = mgL(1-\cos\theta)$

$K_I = \frac{1}{2} m v_0^2$ $\qquad$ $K_F = 0$ ← Stops to drop Vertically

$W_{NCF} = 0$

$\frac{1}{2} \cancel{m} v_0^2 = \cancel{m} gL(1-\cos\theta)$

$v_0 = \left(2gL(1-\cos\theta)\right)^{1/2}$




$\cos\theta = \frac{L}{\sqrt{L^2-d^2}}$

$$\boxed{v_0 = \left(2gL\left(1 - \frac{L}{\sqrt{L^2-d^2}}\right)\right)^{1/2}}$$