Energy Problems – Set 3

A brick is lifted to a certain height and then dropped to the ground. Next, a second identical brick is lifted twice as high as the first and also dropped to the ground. When the second brick strikes the ground, it has

A. half as much kinetic energy as the first.
B. square root of 2 as much kinetic energy as the first.
C. twice as much kinetic energy as the first.
D. four times as much kinetic energy as the first.

$mgh_1 = \Delta K_1$

$\Rightarrow \frac{\cancel{mg}h_1}{\cancel{mg}h_2} = \frac{\Delta K_1}{\Delta K_2}$, $h_2 = 2h_1$

$\Rightarrow \frac{\cancel{h_1}}{2\cancel{h_1}} = \frac{\Delta K_1}{\Delta K_2} \Rightarrow \Delta K_2 = 2\Delta K_1$

Explain:

A bottle dropped from a balcony strikes the sidewalk with a particular speed. To double the speed of impact, you would have to drop the bottle from a balcony

A. twice as high.
B. three times as high.
C. four times as high.
D. five times as high.
E. six times as high.

$mgh = \frac{1}{2}mV^2$, $V_2 = 2V_1$

$\Rightarrow \frac{\cancel{mg}h_1}{\cancel{mg}h_2} = \frac{\cancel{\frac{1}{2}m}V_1^2}{\cancel{\frac{1}{2}m}V_2^2} \Rightarrow \frac{h_1}{h_2} = \frac{\cancel{V_1^2}}{4\cancel{V_1^2}}$

Explain:

$\boxed{h_2 = 4h_1}$

A car is going 10 mph. The driver hits the brakes. The car travels 3 feet after the brakes are applied. A while later, the same car is going 20 mph. The driver hits the brakes. About how far does the car go after the brakes are applied?

A. 3 ft.
B. 6 ft.
C. 9 ft.
D. 12 ft.
E. 15 ft.

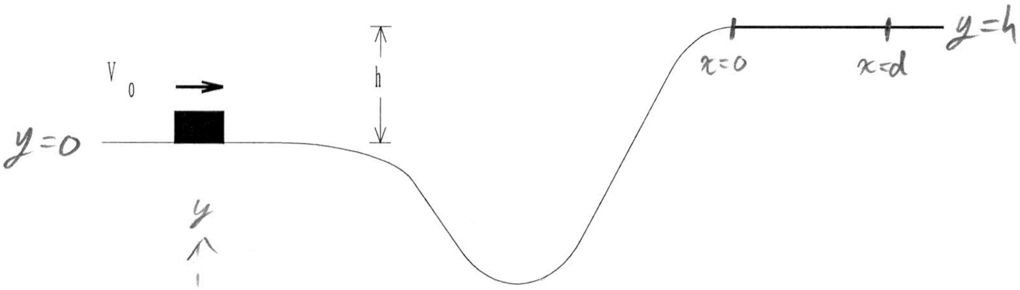
$-\mu_k \cancel{m} g d = \frac{1}{2}\cancel{m}V^2$

$\frac{\cancel{-\mu_k g}d_1}{\cancel{-\mu_k g}d_2} = \frac{\cancel{\frac{1}{2}}V_1^2}{\cancel{\frac{1}{2}}V_2^2} \Rightarrow \frac{3}{d_2} = \frac{(10)^2}{(20)^2}$

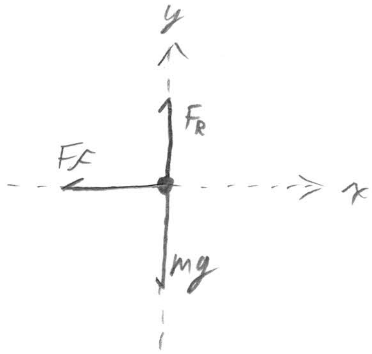
Explain:

$\Rightarrow \boxed{d_2 = 12}$

A block slides along the track shown below. The track is frictionless until the block reaches the level portion at the top of the hill, where the coefficient of friction is μ_k.



$U_g = mgy$



$F_f = -\mu_k F_R \hat{x} = -\mu_k m g \hat{x}$

$U_I = 0$ $\qquad$ $U_F = mgh$

$K_I = \frac{1}{2} m v_0^2$ $\qquad$ $K_F = 0$

$$W_F = \int_0^d \vec{F}_f \cdot d\vec{s} = \int_0^d (-\mu_k m g \hat{x}) \cdot (dx\,\hat{x}) = -\mu_k m g d$$

$$U_I + K_I + W_F = U_F + K_F$$

$$0 + \frac{1}{2} m v_0^2 - \mu_k m g d = mgh$$

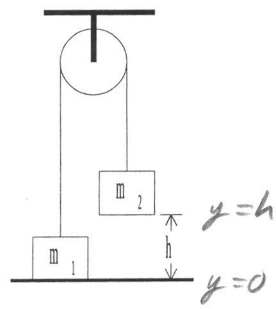
$$\boxed{v_0 = \left[2g(\mu_k d + h)\right]^{1/2}}$$

Energy Problems – Set 3

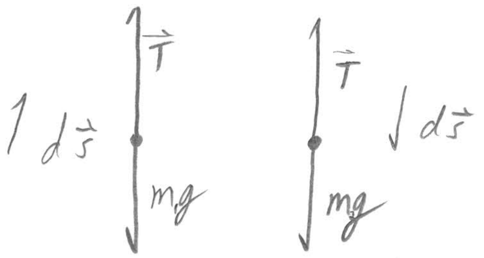
Use work-energy techniques to solve the following problem.

Two masses are connected by a light string passing over a light frictionless pulley. The mass m_2 is released from rest at a height of 4.0 m above the ground.

Determine the speed of m_1 just as m_2 hits the ground and the maximum height m_1 rises above the ground.



$m_1 = 3.0$ kg
$m_2 = 5.0$ kg



T and T do work
No pot. Func.

mg does work, conservative

consider Work done on the s[illegible]em by tension.

on mass 1 : $W_T = \int_0^h (\vec{T} \cdot d\vec{s}) = \int_0^h (T\hat{j}) \cdot (dy\hat{j}) = \underline{Th}$

on mass 2 : $W_T = \int_0^h (\vec{T} \cdot d\vec{s}) = \int_0^h (T\hat{j}) \cdot (-dy\hat{j}) = \underline{-Th}$

Net work on the <u>system</u> by T is <u>zero</u>

Now consider the <u>entire</u> system and conserve energy

a) $U_I = m_2 g h$ $\qquad$ $U_F = m_1 g h$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2$

Same velocity since they are linked by the rope.

continued ↓

EP3,6 - continued

$$m_2 gh = m_1 gh + \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2$$

$$\tfrac{1}{2}(m_1 + m_2) v^2 = (m_2 - m_1) gh$$

$$\boxed{v = \left[\frac{(m_2 - m_1)}{m_1 + m_2} 2gh\right]^{1/2}} = \boxed{4.4 \text{ m/s}}$$

b) Find max height of m_1. The floor has interfered.
Some energy ($\tfrac{1}{2} m_2 v^2$) is lost.
consider just m_1

$U_I = m_1 gh$ $\quad$ $U_F = m_1 g h_{max}$

$K_I = \tfrac{1}{2} m_1 v^2$ $\quad$ $K_F = 0$

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$m_1 gh + \tfrac{1}{2} m_1 v^2 = m_1 g h_{max} + 0$$

$$\Rightarrow h_{max} = h + \frac{v^2}{2g} = h + \frac{1}{\cancel{2g}} \frac{m_2 - m_1}{m_1 + m_2} \cancel{2g} h = h\left[1 + \frac{m_2 - m_1}{m_1 + m_2}\right]$$

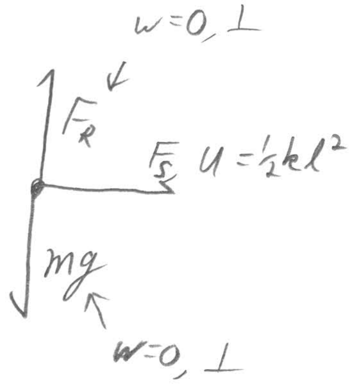
$$= h\left[\frac{\cancel{m_1} + m_2 + m_2 - \cancel{m_1}}{m_1 + m_2}\right] = \boxed{\frac{2m_2}{m_1 + m_2} h} = 5$$

yay!

A block of mass m is pushed against a spring of spring constant k and the spring is compressed a distance l. The block is released and slides across a frictionless surface for a short distance before encountering a surface with a coefficient of friction μ_k.

a. Use conservation of energy to find an expression for the velocity of the block after it leaves the spring.

b. Use conservation of energy to find an expression for how far it slides on the surface with friction before coming to a stop.



a) $U_I = \frac{1}{2}kl^2$ $\quad U_F = 0$

$K_I = 0$ $\quad K_F = \frac{1}{2}mv^2$

$$\cancel{\tfrac{1}{2}}kl^2 = \cancel{\tfrac{1}{2}}mv^2 \Rightarrow \boxed{v = \sqrt{\frac{k}{m}}\,l}$$

b) $U_I = 0$ $\quad U_F = 0$

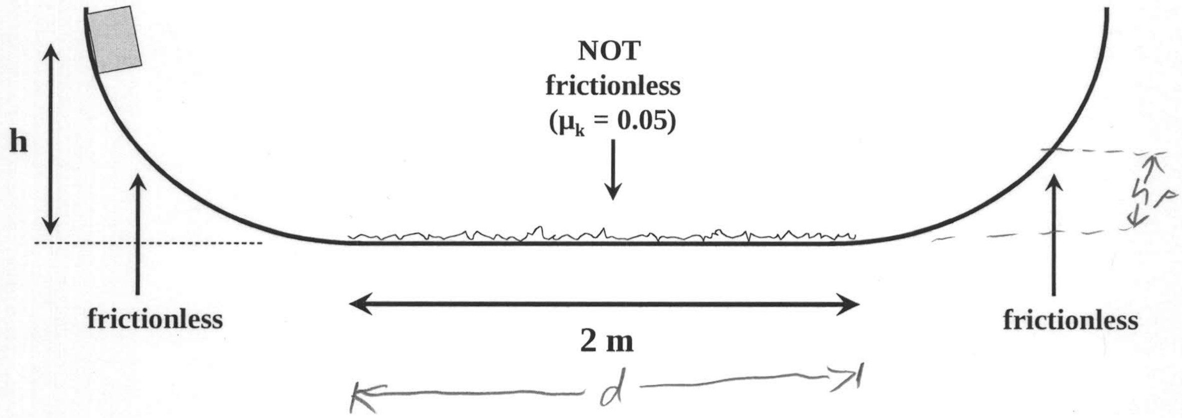
$K_I = \frac{1}{2}mv^2$ $\quad K_F = 0$

$$W_F = \int_0^d \vec{F}_f \cdot d\vec{r} = \int_d^d (-\mu_k mg\hat{x})\cdot(dx\hat{x}) = -\mu_k mgd$$

$$\tfrac{1}{2}\cancel{m}v^2 - \mu_k \cancel{m}gd = 0 \Rightarrow \frac{1}{2}\frac{k}{m}l^2 = \mu_k gd$$

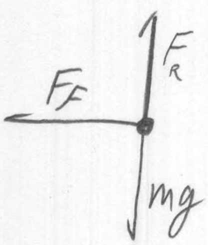
$$\boxed{d = \frac{kl^2}{2\mu_k mgd}}$$

An 8.75-kg block starts at rest, at height h = 1.0 m, and slides down a frictionless ramp onto a horizontal plane where μ_k = 0.05. If the block has enough energy after passing the plane, it will rise onto another frictionless ramp, and so forth.



(a) The block is released, makes its first trip to the right hand side, returns to the left hand side, and then returns once more to the right. On this second excursion to the right side, how high up the ramp does the block go?

This is a conservation of Energy problem with Friction.
The simplest solution is to consider the two end points.
It starts a height h on the right and ends a height h_F on the right after crossing the friction 3 times.



F_R does no work, $\vec{F}_R \perp d\vec{s}$

F_F does work on the flat bottom, non-conservative

mg does work, conservative.

$* \; U_I = mgh$ $\quad * \; U_F = mgh_F$ solve for h_F

$* \; K_I = K_F = 0$

$W_F = \int_0^d \vec{F}_F \cdot d\vec{s} = -\mu_k mgd$ for one trip

continued ↓

EP4, 6 - Continued

Conserve energy accounting for 3 trips across the friction patch.

$$U_I + K_I + W_F = U_F + K_F$$

$$\cancel{mg}h + 0 + -3\mu_k \cancel{mg} d = \cancel{mg} h_F + 0$$

$$\Rightarrow \boxed{h_F = h - 3\mu_k d}$$

$$h_F = 1.0 - (3)(0.05)(2)$$

$$\boxed{h_F = 0.7\,m}$$

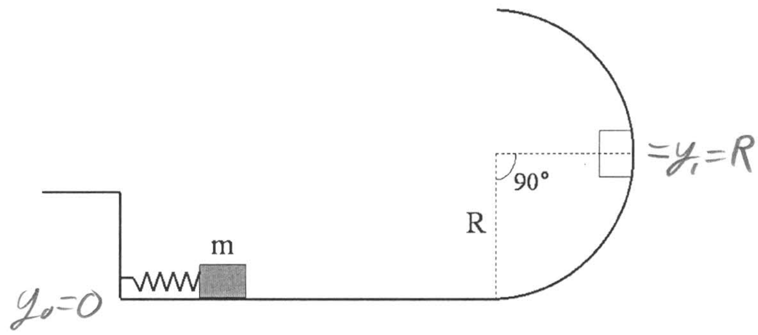
①

Energy Problems – Set 3

#5

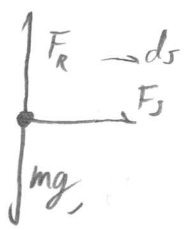
A mass *m* rests on a frictionless horizontal track while compressing a horizontal spring of spring constant *k*. The mass is released and it along a frictionless horizontal track before sliding up a frictionless circular surface of radius *R*.

a. Find an expression for the compression *d* such that the mass just comes to rest at a radius position of θ=90° as shown in the picture?



b. Now include friction in the problem. If the block stops at a radius position of θ=35°, how much work was done by the frictional force acting on the block?

a)



$W_{F_R} = 0, \; \vec{F}_R \perp d\vec{s}$

$U_g = mgy$

$U_s = \frac{1}{2}kd^2$

$U_I = \frac{1}{2}kd_0^2 + \cancel{mgy_0}^{\,0} \qquad U_F = \cancel{\frac{1}{2}kd_1^2}^{\,0} + mgy_1$

$K_I = 0 \qquad K_F = 0$

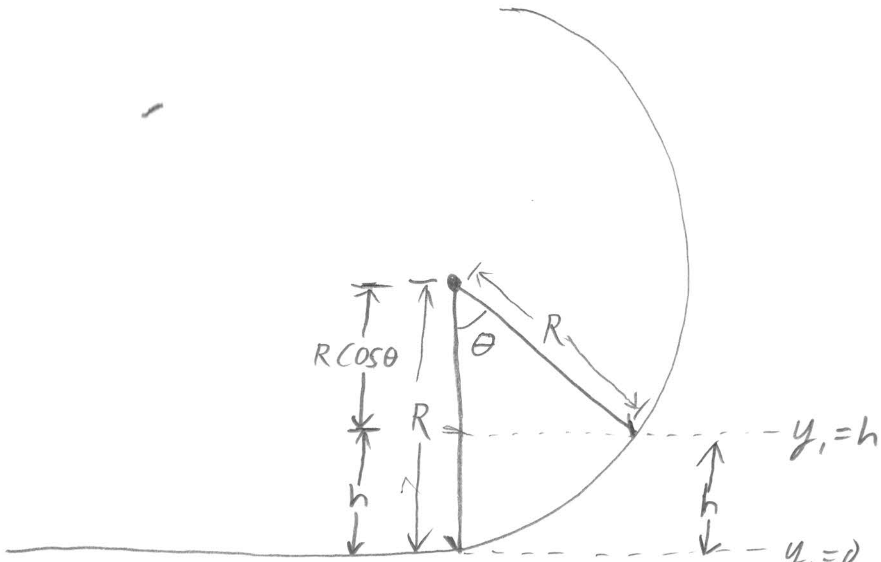
$U_I + K_I + W_{NCF} = U_F + K_F$

$\frac{1}{2}kd^2 + 0 + 0 = mgR + 0$

$$\boxed{d = \left[\frac{2mgR}{k}\right]^{1/2}}$$

EP3, ~~8~~ 5 - continued

b)



$h = R - R\cos\theta$

$h = R(1-\cos\theta)$

With friction, we don't reach as high. We only reach $h = R(1-\cos\theta)$.

So, let's write the energy balance.

$U_I = \frac{1}{2}kd_0^2$ $\qquad U_F = mgh$

$K_I = 0$ $\qquad K_F = 0$

Work done by friction

$$U_I + K_I + W_F = U_F + K_F$$

$$\frac{1}{2}kd^2 + 0 + W_F = mgh + 0$$

$$\Rightarrow W_F = mgh - \frac{1}{2}kd^2$$

$$= mgh - \frac{1}{2}k\frac{2mgR}{k} = mgR(1-\cos\theta) - mgR$$

$$\boxed{W_F = -mgR\cos\theta}$$