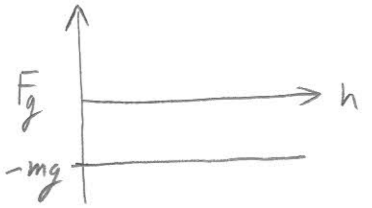
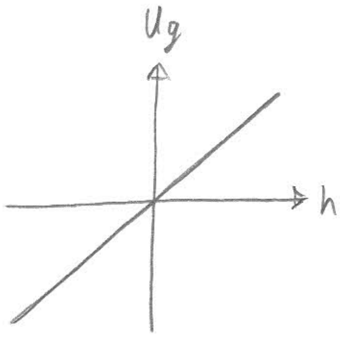
Energy Problems – Set 4

Consider the force due to gravity near the surface of the Earth: $F_g = -mg$

a) Sketch a plot of the Force versus Height (Force on the vertical axis, height on the horizontal axis)



b) Sketch a plot of the potential due to gravity near the surface of the Earth. Let the zero point be a distance h_0 above the ground.



c) Given the Potential Function, $U_g = mgh$, show using calculus how to derive an expression for F_g.

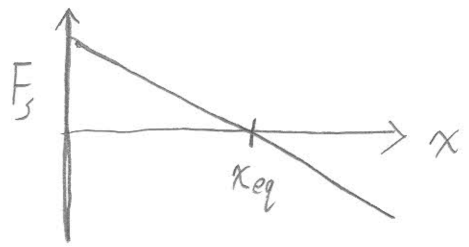
The potential Function is linear in h, so its derivative is constant:

$$\frac{dU_g}{dh} = \frac{d}{dh} mgh = mg, \qquad \text{But, } F_g = -mgh$$

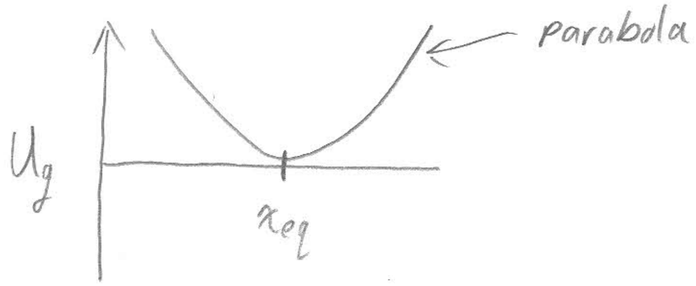
So, it must be that: $\boxed{\frac{dU_g}{dh} = -F_g}$

Consider the force due to a spring: $F_s = -k(x - x_{eq})$

a) Sketch a plot of the Force versus the spring compression x (Force on the vertical axis, height on the horizontal axis)



b) Sketch a plot of the Potential due to a spring.



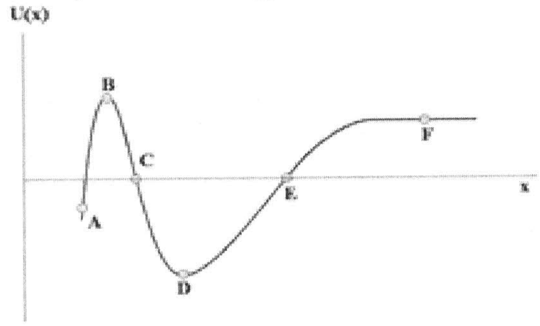
c) Given the Potential Function for a spring, $U_g = \frac{1}{2}k(x - x\,eq)^2$, show using calculus how to derive an expression for F_s.

$$\frac{dU_s}{dx} = \frac{d}{dx}\frac{1}{2}k(x - x_{eq})^2 = k(x - x_{eq}) = -F_s$$

So: $\boxed{\frac{dU_s}{dx} = -F_s}$

Energy Problems – Set 4

The figure below shows the plot of a potential-energy function for a particle moving along the *x*-axis.



a) At each point indicated, state whether the corresponding force F_x acting on the particle is positive, negative, or zero.

A: −
B: 0
C: +
D: 0
E: −
F: 0

b) At which point does the force have the greatest magnitude? Explain.

Point A. The slope of the potential curve is greatest there.

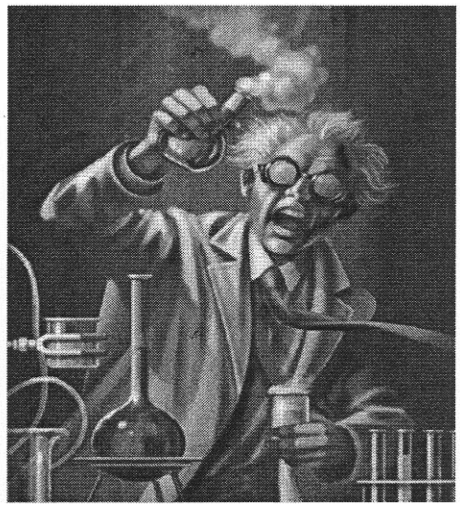
c) Identify all points corresponding to stable, unstable, and neutral equilibrium.

Unstable: B
Stable: D
Neutral: F

d) Assuming the particle starts at point A with a large positive velocity, identify the points where the particle's speed is a maximum, minimum, and constant. Explain. (Remember, in order for there to be a potential energy, the force must be conservative.)

Speed is minimum at point B. Maximum potential, minimum K
Speed ~~is~~ max at point D. Min U, max K
Speed is constant at point F. Potential is not changing.

After getting your BS in Physics, you find yourself working as a lab assistant in the stronghold of a mad scientist in a hollowed out volcano in the middle of a remote tropical island.



He is designing a new Mega-Death Ray, but he needs your help with some calculations. The transmogrifier field of the death ray could either have a potential energy function $U_1 = Ax^4$ or $U_2 = Ax^3 - Bx$. A and B are constants and x is the distance as measured from the reaction chamber.

a) Find the force, F_1, associated with U_1.

So: $F = -\frac{dU}{dx}$

$\Rightarrow F_1 = \frac{d}{dx} Ax^4 \Rightarrow \boxed{F_1 = 4Ax^3}$

b) Find the force F_2 associated with U_2.

$F_2 = \frac{d}{dx}(Ax^3 - Bx) = \boxed{3Ax^2 - B}$

c) If there are any points where the force goes to zero, the entire Island will explode. Are either of the two force fields safe, or are you in mortal danger?

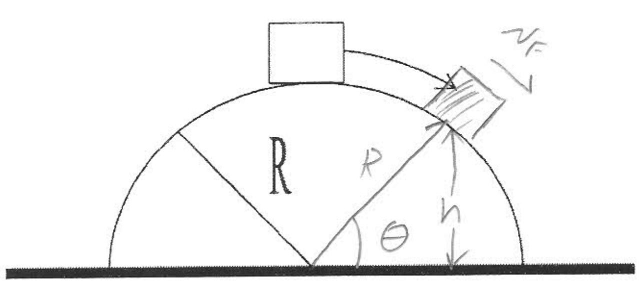
$F_1 = 0 \Rightarrow 4Ax^3 = 0$, which is true when $x = 0$

$F_2 = 0 \Rightarrow 3Ax^2 - B = 0 \Rightarrow x = \pm\sqrt{\frac{B}{3A}}$

which is only impossible if A or B are negative

2. Use *Conservation of Energy* to solve this problem

A block is seated on the top of a hemispherical mound of ice. The block is given a *slight* push (assume $V_0 = 0$) and starts sliding down the ice. If the ice is frictionless, show that the block will lose contact with the ice at a height of $\frac{2}{3}R$, where R is the radius of the ice mound.



Given: $V_0 = 0$, R

Want: h

From NSL, we can find V_{max} Before the block leaves the surface.

$y: -F_N + mg\,SIN\theta = m\frac{V_F^2}{R}$

Centripedal acceleration

when $F_N = 0$, $V_F = V_{max}$

$\Rightarrow \boxed{V_{max}^2 = gR\,SIN\theta}$

From Conservation of Energy, we can calculate V_F as a function of h:

$U_I = mgR$ $K_I = 0$

$U_F = mgh$ $K_F = \frac{1}{2}mV_F^2$

$\Rightarrow \cancel{m}gR = \cancel{m}gh + \frac{1}{2}\cancel{m}V_F^2$

$\Rightarrow \boxed{V_F^2 = 2g(R-h)}$

When $V_F = V_{max}$, The block loses contact

$2\cancel{g}(R-h) = \cancel{g}R\,SIN\theta$, $SIN\theta = \frac{h}{R}$

$2R - 2h = \cancel{R}\frac{h}{\cancel{R}} \Rightarrow 2R = 3h$

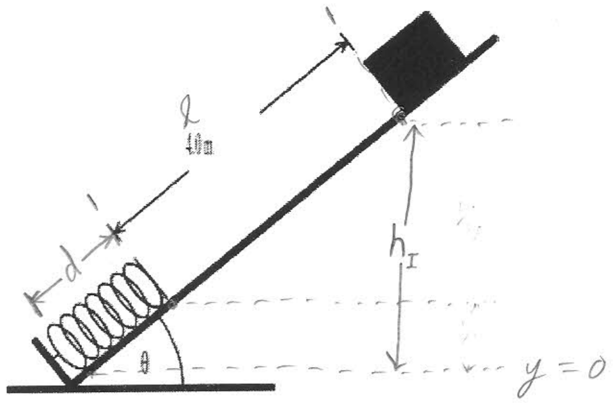
$\Rightarrow \boxed{h = \frac{2}{3}R}$

3) Use *Conservation of Energy* to solve this problem

A block of mass m is released from rest on an incline that makes an angle θ with the horizontal. It starts a distance l from a spring with spring constant k that is attached to the bottom of the incline. The coefficient of friction between the block and the incline is μ_k.

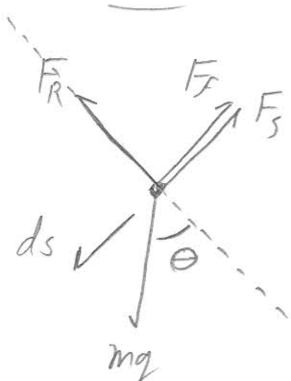
Find an expression for the maximum compression, d, of the spring.

If it's quadratic, put it into standard form: $ad^2+bd+c=0$ You don't need to apply the quadratic formula.



Given	Want
m	d
θ	
l	
k	
μ_k	

FBD



Conservation of Energy

$U_I = mgh_I + \cancel{\tfrac{1}{2}kd_I^2}^{\,0} \quad K_I = 0$

$U_F = mg\cancel{h_F}^{\,0} + \tfrac{1}{2}kd_F^2 \quad K_F = 0$

$h_I = (l+d)\sin\theta$

$\boxed{mg(l+d)\sin\theta - \mu_k(l+d)mg\cos\theta = \tfrac{1}{2}kd^2}$

Friction

$W_f = \int_0^{l+d} \vec{F}_f \cdot d\vec{s} = -\mu_k F_R(l+d)$

By NSL: $F_R - mg\cos\theta = 0$

$\Rightarrow F_R = mg\cos\theta$

$\Rightarrow \boxed{W_F = -\mu_k(l+d)mg\cos\theta}$

$mgl\sin\theta + mgd\sin\theta - \mu_k mgl\cos\theta - \mu_k mgd\cos\theta = \tfrac{1}{2}kd^2$

$\Rightarrow \boxed{\frac{k}{2mg}d^2 + (\sin\theta - \mu_k\cos\theta)d - \mu_k l\cos\theta = 0}$

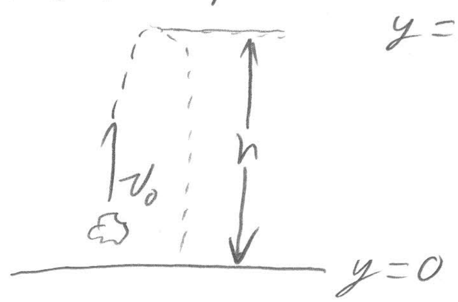
Energy Problems – Set 4

Use work energy to solve the following problem.

A stone of mass m is thrown vertically upward into the air from ground level with an initial speed of v_o. If a constant drag force equal to 20% of the stone's weight acts on the stone throughout its flight, what is the speed of the stone in terms of v_o when it returns to the ground?

Let's handle the trip up and the trip down separately.

Trip up:



$U_I = 0$ $\qquad U_F = mgh$

$K_I = \frac{1}{2} m v_o^2$ $\qquad K_F = 0$

$$W_{NCF} = \int_0^h \vec{F}_d \cdot d\vec{s}, \quad \vec{F}_d = -0.2\,mg\,\hat{j}, \quad d\vec{s} = dy\,\hat{j}$$

$$= \int_0^h (-0.2\,mg\,\hat{j}) \cdot (dy\,\hat{j}) = -\frac{1}{5} mgh$$

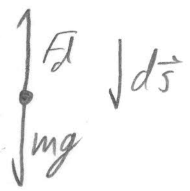
$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$0 + \frac{1}{2} m v_o^2 - \frac{1}{5} mgh = mgh + 0$$

$$\Rightarrow \boxed{\frac{1}{2} m v_o^2 = \frac{6}{5} mgh} \quad (1)$$

EP 4,5

Trip down



$U_I = mgh$ $\qquad K_I = 0$

$U_F = 0$ $\qquad K_F = \frac{1}{2} m v^2$

$W_{FCF} = \underline{-0.2mgh}$

$\cancel{m}gh - \frac{1}{5}\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$\frac{4}{5}gh = \frac{1}{2}v^2$

$\underline{v^2 = \frac{8}{5}gh}$

Plug in h from eq 1 : $h = \frac{5}{12}\frac{v_0^2}{g}$

$v^2 = \cancelto{2}{\frac{8}{5}} \cancel{g} \cdot \frac{\cancel{5}}{\cancelto{3}{12}} \frac{v_0^2}{\cancel{g}}$

$v^2 = \frac{2}{3}v_0^2$

$$\boxed{v = \sqrt{\frac{2}{3}}\, v_0}$$