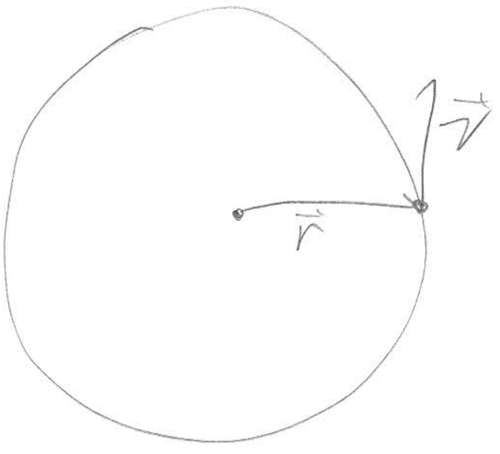
Energy Problems Set 5, P2



uniform Circular motion!

$$\boxed{a = \frac{v^2}{r}}$$

a) FBD

NSL

$$\Sigma \vec{F} = m\vec{a}$$

$$\Rightarrow F_G = m\frac{v^2}{r}$$

$$\Rightarrow \frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$\Rightarrow \boxed{v_{orb} = \left(\frac{GM}{r}\right)^{1/2}} \quad *$$

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b) So... The period of the Geosynchronous orbit is 24 hrs.
So we know P...
The velocity is distance over time

$$v_{orb} = \frac{dist.}{time} = \frac{2\pi r}{P} \quad \text{and} \quad v_{orb} = \left(\frac{GM}{r}\right)^{1/2}$$

$$\Rightarrow \frac{2\pi r}{P} = \left(\frac{GM}{r}\right)^{1/2} \Rightarrow \frac{4\pi^2 r^2}{P^2} = \frac{GM}{r}$$

$$\Rightarrow \boxed{r^3 = \frac{GM}{4\pi^2}P^2}$$

b) continued

$$r = \left[\frac{GM}{4\pi^2} P^2\right]^{1/3} = \left[\frac{(6.67\times10^{-11})(5.97\times10^{24})}{4\pi^2}\left((24)(3600)\right)^2\right]^{1/3}$$

$$r = 4.2\times10^{7}\,m = \underline{4.2\times10^{4}\,km}$$

c) Now that I have r:

$$V_{orb} = \frac{2\pi r}{P} = \frac{2\pi}{P}\left[\frac{GM}{4\pi^2}P^2\right]^{1/3} = \left[\frac{\cancel{8}^{2}\cancel{\pi^3}^{\pi}}{\cancel{P^3}^{P}}\cdot\frac{GM}{\cancel{4}\cancel{\pi^2}}\cancel{P^2}\right]^{1/3}$$

$$\boxed{V_{orb} = \left[\frac{2GM}{P}\right]^{1/3}}$$

$$V_{orb} = \left[\frac{2(6.67\times10^{-11})(5.97\times10^{24})}{(24)(3600)}\right]^{1/3} = \underline{2.1\times10^{3}\,m/s}$$

Energy Problems #5, P1

Given

a) $M_p = 5.0\times10^{23}$ kg

$R_p = 3.0\times10^{6}$ m

$M_s = 10$ kg

$V_0 = 3{,}000$ m/s

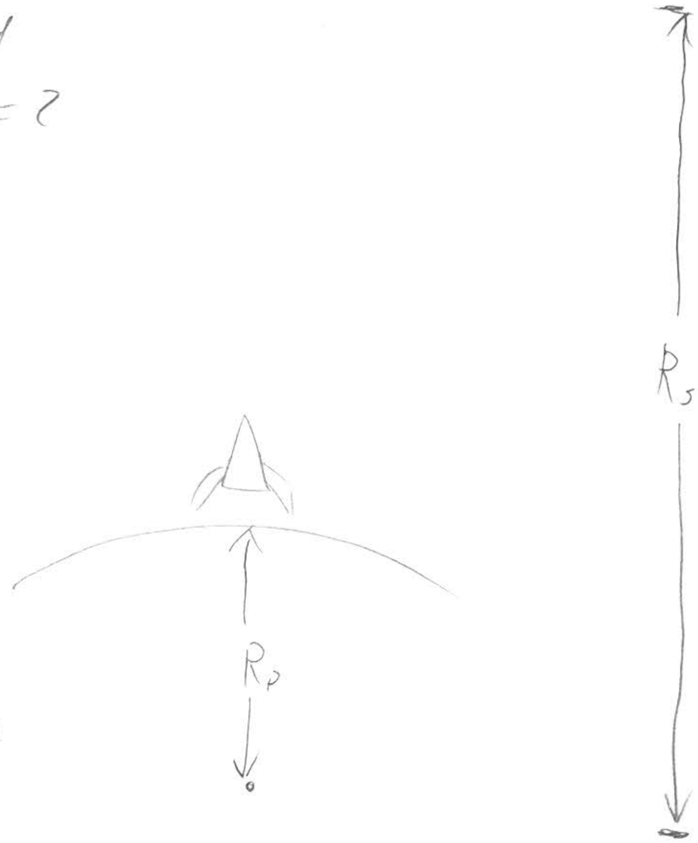
$R_s = 4\times10^{6}$ m

$G = 6.67\times10^{-11}\ m^3/kg\cdot s^2$

Only G gravity

Wanted

$V_s = ?$



$$U_I = -\frac{G M_p M_s}{R_p} \qquad K_I = \frac{1}{2} M_s V_0^2$$

$$U_F = -\frac{G M_p M_s}{R_s} \qquad K_F = \frac{1}{2} M_s V_s^2$$

$$-\frac{G M_p \cancel{M_s}}{R_p} + \frac{1}{2}\cancel{M_s} V_0^2 = -\frac{G M_p \cancel{M_s}}{R_s} + \frac{1}{2}\cancel{M_s} V_s^2$$

$$\Rightarrow 2 G M_p \underbrace{\left[\frac{1}{R_s} - \frac{1}{R_p}\right]}_{\text{negative \#}} + V_0^2 = V_s^2$$

But $R_s > R_p \Rightarrow \frac{1}{R_s} < \frac{1}{R_p}$

$$\Rightarrow \boxed{V_s^2 = V_0^2 - 2 G M_p \underbrace{\left[\frac{1}{R_p} - \frac{1}{R_s}\right]}_{\text{Positive \#}}}$$

continued ↓

a) continued

Plug in numbers

$$V_s = \left[\left(3\times10^{3}\,\tfrac{m}{s}\right)^2 - (2)\left(6.67\times10^{-11}\,\tfrac{m^3}{kg\cdot s^2}\right)\left(5.0\times10^{23}\,kg\right)\left[\frac{1}{3.0\times10^{6}\,m} - \frac{1}{4.0\times10^{6}\,m}\right]\right]^{1/2}$$

$$\boxed{V_s = 1.86\times10^{3}\,m/s}$$

b) Now, Given V_0, Find H_{max}

$$U_I = -\frac{GM_pM_s}{R_p} \qquad K_I = \tfrac{1}{2}M_sV_0^2$$

$$U_F = -\frac{GM_pM_s}{H_{max}} \qquad K_F = 0$$ ← stops and turns around at H_{max}

$$-\frac{GM_p\cancel{M_s}}{R_p} + \tfrac{1}{2}\cancel{M_s}V_0^2 = -\frac{GM_p\cancel{M_s}}{H_{max}}$$ ← Need this upstairs

I want to invert the entire equation, so I'll put the left side under a common denominator. After I divide by GM_p

$$-\frac{1}{R_p} + \frac{V_0^2}{2GM_p} = -\frac{1}{H_{max}} \Rightarrow \frac{2GM_p - V_0^2R_p}{2GM_pR_p} = \frac{1}{H_{max}}$$

$$\boxed{\Rightarrow H_{max} = \frac{2GM_pR_p}{2GM_p - V_0^2R_p}}$$ yay!

continued ↓

Energy Problems #5, P1 - continued

c) If it never stops and comes back, H_{max} will be infinity.

$$H_{max} = \frac{2GM_pR_p}{\boxed{2GM_p - V_0^2 R_p}}$$

$H_{max} = \infty$ when this goes to zero.

$$2GM_p - V_0^2 R_p = 0 \Rightarrow \boxed{V_{esc} = \left(\frac{2GM_p}{R_p}\right)^{1/2}}$$

Escape Velocity

Alternate method to find V_{esc}

$$U_I = -\frac{GMm}{R_p} \qquad K_I = \tfrac{1}{2} m V_{esc}^2$$

$$U_F = \emptyset \qquad K_F = \emptyset \leftarrow \text{stops at } \infty$$

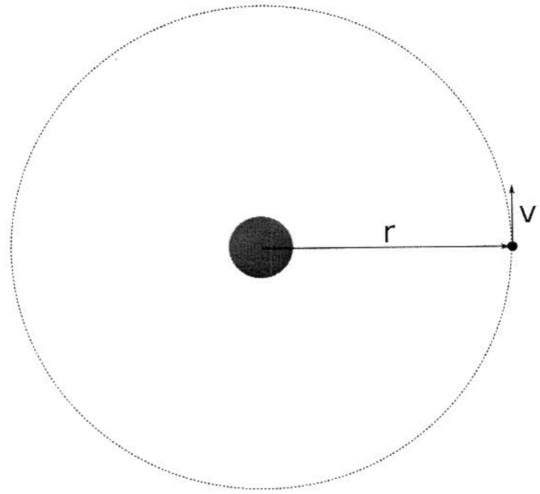
$r \to \infty,\ U \to 0$

$$\Rightarrow -\frac{GM\cancel{m}}{R_p} + \tfrac{1}{2}\cancel{m} V_{esc}^2 = 0$$

$$\Rightarrow \boxed{V_{esc} = \left(\frac{2GM_p}{R_p}\right)^{1/2}}$$

Energy Problems – Set 4

Consider a satellite in a circular orbit of radius r about the Earth.



a) Calculate the Total Mechanical Energy of the Satellite (Potential Energy plus Kinetic Energy)

b) If the satellite is in low Earth orbit, 200 km above the Earth's surface, how much work must the satellite thrusters do to boost it to the Geosynchronous orbit?

The radius of the Earth is: $R_{Earth} = 6.37\text{x}10^6$ m.
The mass of the Earth is : $M_{Earth} = 5.97\text{x}10^{24}$ kg
The gravitaional constant: $G = 6.67\text{x}10^{-11}\ m^3/(kg\ s^2)$

a) Given: M_E, r — Want: E_T

$$E_T = K + U \Rightarrow E_T = \frac{1}{2}mv^2 - \frac{GM_Em}{r}$$

But I need NSL to get $\underline{v}$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow E_T = \frac{1}{2}m\frac{GM}{r} - \frac{1}{2}\frac{GMm}{r}$$

$$\boxed{E_T = -\frac{1}{2}\frac{GMm}{r}}$$

Energy Set 4 P3 continued

b) Conservation of Energy

Given
$d = 200km$

$$\underbrace{U_I + K_I}_{E_I} + \underbrace{W_{NC}}_{\Delta E} = \underbrace{U_F + K_F}_{E_F}$$

ΔE is the change in energy

$$E_I = -\frac{1}{2}\frac{GM_E m}{r_I}, \quad E_F = -\frac{1}{2}\frac{GM_E m}{r_F}$$

$$r_I = d + R_E, \quad r_F = r_{Geo}$$

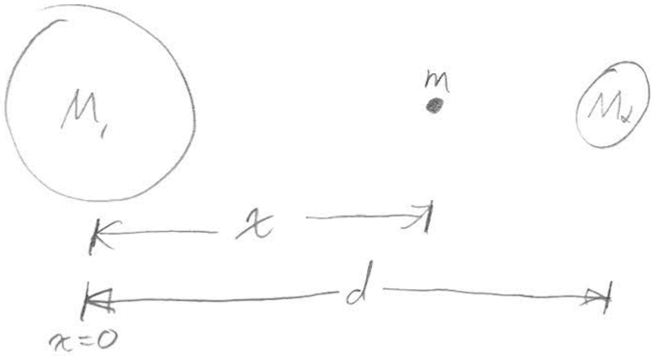
$$\boxed{\Delta E = -\frac{1}{2}\frac{GM_E m}{r_{Geo}} + \frac{GM_E m}{d + R_E}}$$

Then plug in r_{Geo} from page

$$\Rightarrow \boxed{\Delta E = -\frac{1}{2}GM_E m\left[\frac{4\pi^2}{GMP^2}\right]^{1/3} + \frac{1}{2}\frac{GM_E m}{d + R_E}}$$

Energy Problems, Set 5, P3

a)



Looking for $\sum \vec{F} = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$

$$\Rightarrow -\frac{GM_1 m}{x^2} + \frac{GM_2 m}{(d-x)^2} = 0$$

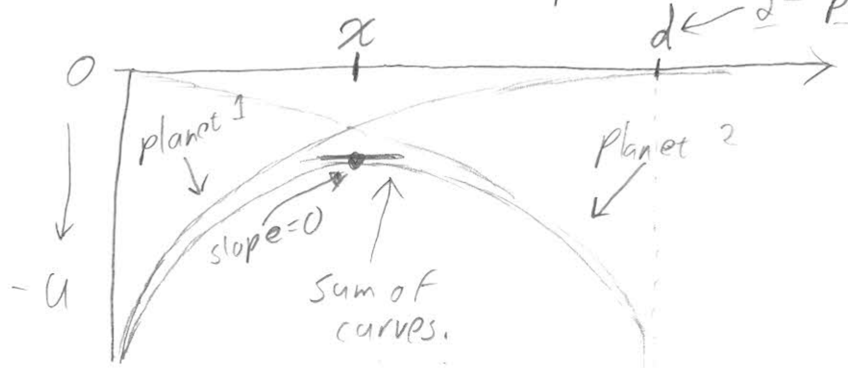
$$\Rightarrow \frac{M_1}{x^2} = \frac{M_2}{(d-x)^2} \Rightarrow \frac{\sqrt{M_1}}{x} = \frac{\sqrt{M_2}}{(d-x)}$$

$$\Rightarrow \sqrt{M_1}\,d - \sqrt{M_1}\,x = \sqrt{M_2}\,x$$

$$\Rightarrow x\left(\sqrt{M_1} + \sqrt{M_2}\right) = \sqrt{M_1}\,d$$

$$\Rightarrow \boxed{x = \frac{\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}\,d}$$

b) The total potential is due to both masses. We can simply add them. $U_G = -\frac{GMm}{r}$ ↙ goes as $-\frac{1}{r}$, always negative.



equilibrium when slope = 0

Energy Problems, set 5, P3 – Continued

(2)

c) Now mathematically:

$$U_T = U_{M1} + U_{M2} \Rightarrow U_T = -\frac{GM_1 m}{x} - \frac{GM_2 m}{(d-x)}$$

Let's find the extrema, when $\frac{dU_T}{dx} = 0$,
That's the equilibrium point.

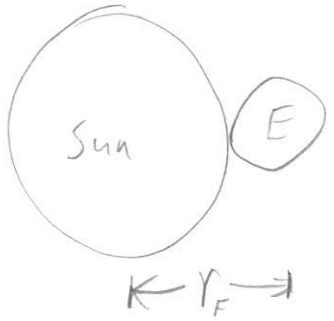
$$\frac{dU_T}{dx} = \frac{d}{dx}\left[-\frac{GM_1 m}{x} - \frac{GM_2 m}{(d-x)}\right] = 0$$

$$= -GM_1 m \frac{d}{dx}\left(\frac{1}{x}\right) - GM_2 m \frac{d}{dx}\frac{1}{(d-x)} = 0$$

$$= \boxed{+\frac{GM_1 m}{x^2} - \frac{GM_2 m}{(d-x)^2} = 0}$$

Same as part a!!

$$\boxed{x = \frac{\sqrt{M_1}}{\sqrt{M_2} + \sqrt{M_1}} d}$$



Given

$M_\odot = 1.99 \times 10^{30}$ kg

$R_\odot = 6.955 \times 10^{5}$ km

$R_\oplus = 6.3781 \times 10^{3}$ km

$M_\oplus = 5.9742 \times 10^{24}$ kg

$v_1 = 0$

Want

v_F

$$U_I = -\frac{G M_\odot m_\oplus}{r_I} \qquad K_I = 0 \qquad r_I = 1.49 \times 10^{8} \text{ km}$$

$$r_F = R_\odot + R_\oplus$$

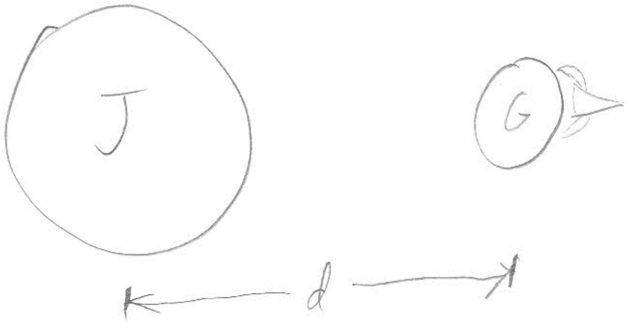
$$U_F = -\frac{G M_\odot m_\oplus}{r_F} \qquad K_F = \tfrac{1}{2} m_\oplus v_F^2$$

$$-\frac{G M_\odot \cancel{M_\oplus}}{r_I} = -\frac{G M_\odot \cancel{m_\oplus}}{r_F} + \tfrac{1}{2} \cancel{m_\oplus} v_F^2$$

$$v_F^2 = 2 G M_\odot \left[\frac{1}{r_F} - \frac{1}{r_I} \right] = \left[2(6.67 \times 10^{-11})(1.99 \times 10^{30}) \left[\frac{1}{7.02 \times 10^{5}} - \frac{1}{1.49 \times 10^{8}} \right] \right]^{1/2}$$

$$\boxed{v_F = 1.94 \times 10^{7} \text{ m/s}}$$

Energy Problems Set 5, P5 -



Given	Want
$R_G = 2.64 \times 10^6$ m	V_{esc}
$M_G = 1.495 \times 10^{23}$ kg	
$M_J = 1.900 \times 10^{27}$ kg	
$d = 1.071 \times 10^9$ m	

Escape when $U_F = 0$ $(r_F = \infty)$ and $K_F = 0$

$$U_I = -\underbrace{\frac{G M_J m_R}{d + R_G}}_{\text{Jupiter}} - \underbrace{\frac{G M_G m_R}{R_G}}_{\text{Ganymede}} \qquad K_I = \tfrac{1}{2} m_R V_{esc}^2$$

$$K_F = 0$$

$$U_F = 0$$

$$-\frac{G M_J \cancel{m_R}}{d + R_G} - \frac{G M_G \cancel{m_R}}{R_G} + \tfrac{1}{2} \cancel{m_R} V_{esc}^2 = 0$$

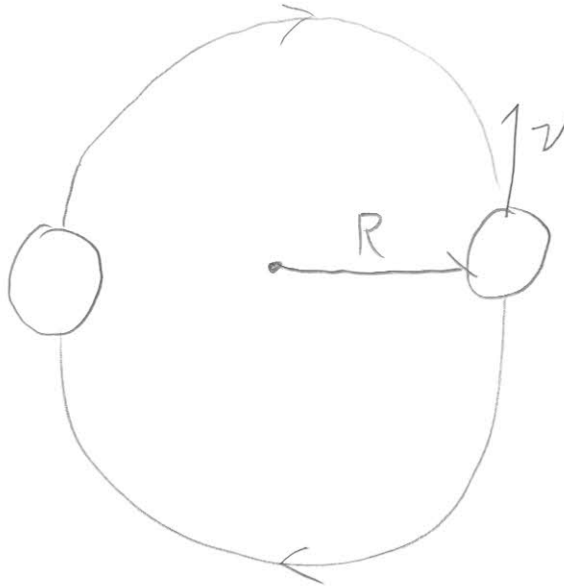
$$\boxed{V_{esc} = \left[G \left(\frac{M_J}{d + R_G} + \frac{M_G}{R_G} \right) \right]^{1/2}}$$

Energy Problems Set 5, P6

a) Given: M, R Want: F

$$\vec{F}_G = -\frac{GM_1M_2}{r^2}\hat{r} \Rightarrow |\vec{F}_G| = \frac{GM^2}{4R^2}$$

b)



$$F_G = M\frac{v^2}{R}$$

$$\Rightarrow \frac{GM^{\cancel{2}}}{4R^{\cancel{2}}} = \cancel{M}\frac{v^2}{\cancel{R}}$$

$$\Rightarrow \boxed{v = \left[\frac{GM}{4R}\right]^{1/2}}$$

2 stars

c) $U_I = -\frac{GM^2}{2R}$ $K_I = \frac{1}{2}Mv_{orb}^2 + \frac{1}{2}Mv_{orb}^2$

$U_F = 0$ $K_F = 0$; $r \to \infty$

at $r = \infty$

continued ↓

Energy Problems Set 5, P6 — continued

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$-\frac{GM^2}{2R} + M v_{orb}^2 + W = 0$$

Energy to separate

$$W = \frac{GM^2}{2R} - M\boxed{\frac{GM}{4R}} \leftarrow v_{orb} \text{ from part b}$$

$$W = \frac{GM^2}{R}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$W = \frac{1}{4}\frac{GM^2}{R}$$