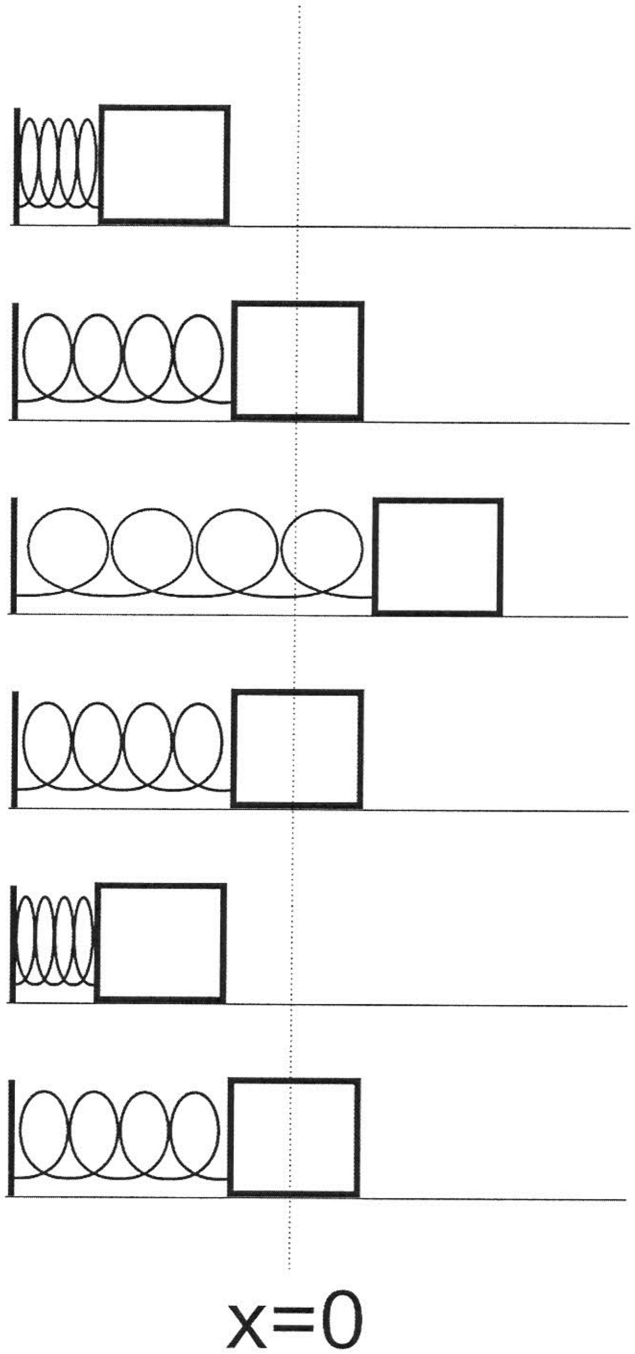
Oscillation – Set 1

Each row in the table below represents a snapshot of a mass attached to a spring. Assume that the mass starts from rest in the first row. In the second row, it is passing through x=0. In the third row, it has reached its maximum extension. In the fifth row, it has reached its maximum compression. In the cells below, mark an arrow indicating the direction of the associated force, acceleration, velocity, and position vectors for each row. If the magnitude is zero, put a zero in the cell.

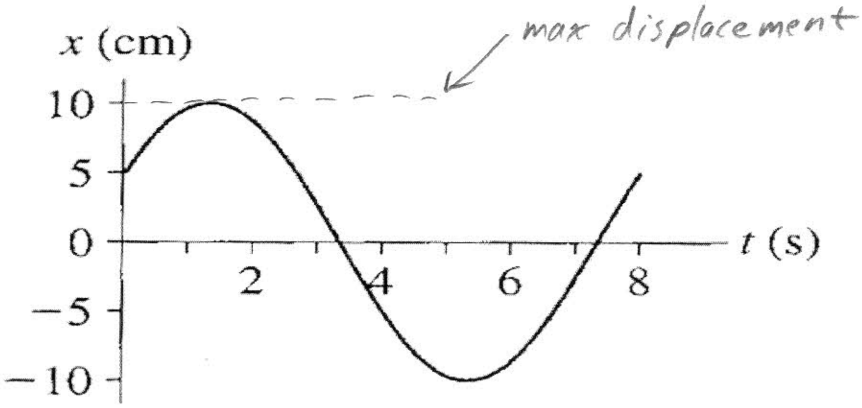
F	a	v	x
→	→	0	←
0	0	→	0
←	←	0	→
0	0	←	0
→	→	0	←
0	0	→	0



The figure below is a position versus time graph of a particle in simple harmonic motion. Assume that its position as a function of time is given by

$$x(t) = A\cos(\omega t + \varphi)$$

where A, ω and φ are constants.



a) What is the maximum displacement (amplitude) of the particle?

10cm

b) Which constant in the above equation gives the maximum displacement, or amplitude, of the oscillations? *(HINT: What's the maximum possible value of cosine?)*

max value of cosine is +1.

$x_{max} = A \cdot 1 = \boxed{A}$

c) What is the value *x(0)* (ie. *x* when *t* = 0)?

at $t=0$, $x = 5$cm. so: $x(0) = 5$

d) Given your answer to part c, solve $x(0) = A\cos(\omega t + \varphi)$ for φ (the phase constant) when t = 0?

$x(0) = A\cos(\omega t + \phi)$

$5 = 10\cos(0 + \phi) \Rightarrow \cos\phi = \frac{1}{2}$

$\boxed{\phi = \cos^{-1}(\frac{1}{2})}$

e) What is the period (T) of the oscillations?

Period is the time required for one cycle.
According to the graph, $\underline{T = 8 \text{ sec.}}$

f) What are the units of ω? (*HINT: What are the units of the input to the cosine function?)*

Cosine takes an angle, (let's use radians)
so $(\omega t + \phi)$ must have units of radians.
$\Rightarrow$, $\boxed{\omega \text{ must be rad/sec}}$

g) What is the mathematical relationship between ω and T?

$T\left(\text{sec/cycle}\right)$ and there are $2\pi\left(\text{rad/cycle}\right)$ so $\boxed{T = \frac{2\pi}{\omega}}$

$\omega\left(\text{rad/sec}\right)$ $\quad T = \dfrac{2\pi\left(\cancel{\text{rad}}/\text{cycle}\right)}{\omega\left(\cancel{\text{rad}}/\text{sec}\right)} = \dfrac{2\pi}{\omega}\left(\text{sec/cycle}\right)$

h) Good! Now calculate the numerical value of ω.

$$\omega = \frac{2\pi}{8} = \boxed{\frac{\pi}{4}\ \text{rad/s}}$$

d) What is the maximum velocity of the particle?
(HINT: What's the maximum possible value of sine?)

$$v = -\omega A \sin(\omega t + \phi)$$

when $\sin = -1$, $v = \omega A = \frac{\pi}{4} \cdot 10 = \boxed{5/2\,\pi}$

e) What is the maximum acceleration of the particle?
(HINT: What's the maximum possible value of cosine?)

$$a = -\omega^2 A \cos(\omega t + \phi)$$

when $\cos = -1$, $a = \omega^2 A = \frac{\pi^2}{16} \cdot 10 = \boxed{5/8\,\pi^2}$

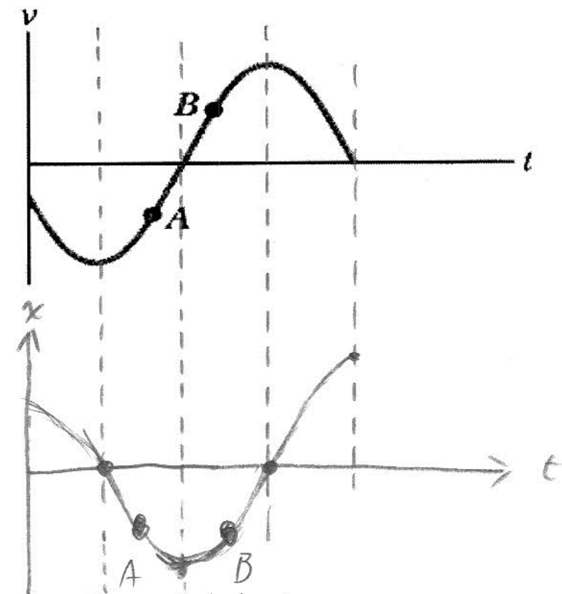
Oscillation – Set 1

If $x(t)=A\cos(\omega t+\varphi)$ is the position of a Simple Harmonic Oscillator, write expressions for the velocity and acceleration of a Simple Harmonic Oscillator.

$v(t) = -\omega A SIN(\omega t + \phi)$

$a(t) = -\omega^2 A cos(\omega t + \phi)$

The figure to the right is the velocity of a SHO. Sketch the position versus time plot



Is the particle stationary, moving towards -x, or moving towards +x when the particle is at:

Point A: moving towards -x

Point B: moving towards +x

Is x=0, x>0, or x<0 when the particle is at:

Point A: x < 0

Point B: x < 0

Is the particle's speed (the magnitude of it's velocity) increasing, decreasing, or constant when the particle is at:

Point A: Decreasing

Point B: Increasing

Oscillation – Set 2

The acceleration of a particle in Simple Harmonic Motion is plotted in the figure below.

1) Which point(s) represent the particle's acceleration when it is at $x = -x_{max}$?

Point 2

2) Which point(s) represent the particle's acceleration when it is at $x = +x_{max}$?

Point 6

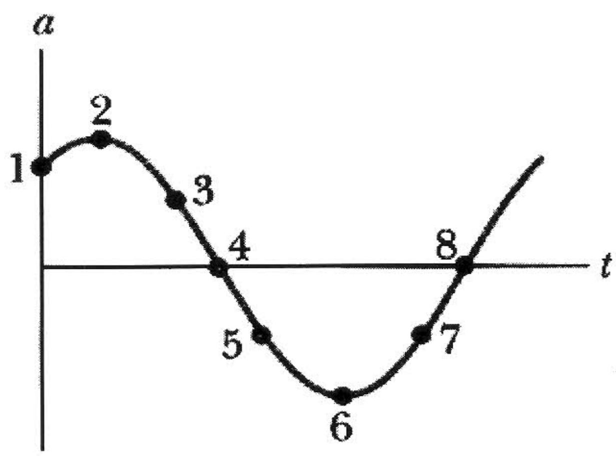
2) At point 4, is the velocity of the particle positive, negative, or zero?

Positive (just after a period of positive accel)

3) At point 5, what is the particle's position?

A) $x = 0$
B) $x = -x_{max}$
C) $x = +x_{max}$
D) $\boxed{0 < x < +x_{max}}$
E) $-x_{max} < x < 0$

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A \sin(\omega t + \phi)$
$a(t) = -\omega^2 A \cos(\omega t + \phi)$



Oscillation – Set 1

You are given the position and velocity of a simple harmonic oscillator (SHO) at some time t:

$$x(t)=x_0 \text{ and } v(t)=v_0 \text{ .}$$

Starting with the equations for position and velocity:

$$x(t)=A\cos(\omega t+\varphi) \ , \quad v(t)=-\omega A\sin(\omega t+\varphi)$$

a) find an expression for the amplitude, A, of a **S**imple **H**armonic **O**scillator in terms of x_0 and v_0.

① Square both equations:

$$x_0^2 = A^2\cos^2(\omega t+\phi) \ , \quad v_0^2 = \omega^2 A^2 \sin^2(\omega t+\phi)$$

② Divide v eq. by ω^2:

$$v_0^2/\omega^2 = A^2 \sin^2(\omega t+\phi)$$

③ Add them together

$$x_0^2 + \frac{v_0^2}{\omega^2} = A^2\cos^2(\omega t+\phi) + A^2\sin^2(\omega t+\phi) \Rightarrow x_0^2+\frac{v_0^2}{\omega^2} = A^2\left(\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi)\right)$$

$$\Rightarrow \boxed{A = \left[x_0^2 + \frac{v_0^2}{\omega^2}\right]^{1/2}}$$

b) find an expression for the phase angle, ϕ, of a **S**imple **H**armonic **O**scillator in terms of x_0 and v_0.

Divide v_0 by x_0:

$$\frac{v_0}{x_0} = \frac{-\omega \cancel{A}\sin(\omega t+\phi)}{\cancel{A}\cos(\omega t+\phi)} \Rightarrow -\frac{v_0}{\omega x_0} = \tan(\omega t+\phi)$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right) - \omega t}$$

Oscillation – Set 1

A mass attached to a spring is in simple harmonic motion. At the exact moment the mass moves through equilibrium:

its instantaneous acceleration

a) has maximum magnitude.

b) is zero.

c) has greater than zero magnitude (but not maximum).

its instantaneous speed

a) has maximum magnitude.

b) is zero.

c) has greater than zero magnitude (but not maximum).

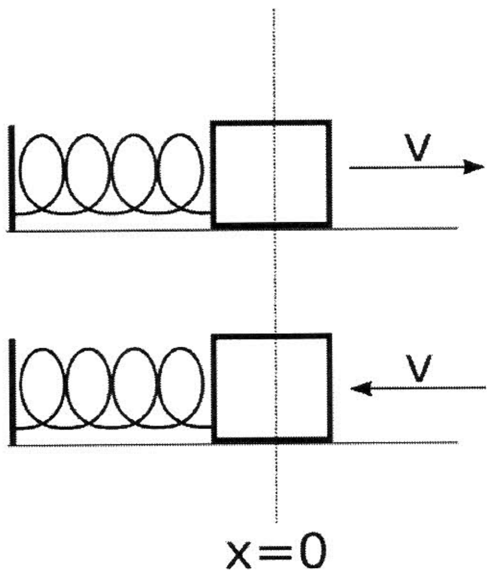
A mass attached to the free end of an ideal spring is in simple harmonic motion with an amplitude A=0.5 m and an angular frequency 18 rad/s. What is the maximum velocity of the mass?

a) 36 m/s
b) 9 m/s
c) 3 m/s
d) None of the above

$V_{max} = \omega A$

$V_{max} = 18(1/s)0.5(m) = 9\ m/s$

The two oscillators pictured below have identical springs and masses. If the position of the top oscillator is given by $x(t) = A\cos(\omega t)$, what is the phase constant, ϕ, for the second oscillator?



A) $\frac{\pi}{2}$

B) π

C) $\frac{\pi}{4}$

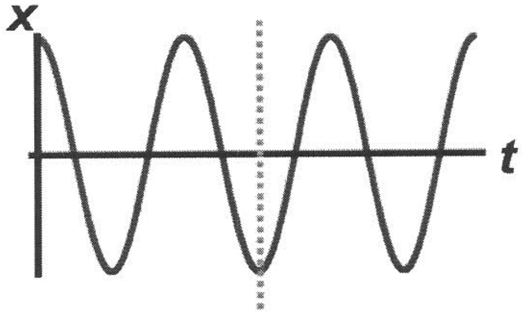
D) ω

Below is a position versus time graph of a mass on a spring. What can you say about the velocity, net force, and acceleration at the time indicated by the dotted line?

Velocity: Positive, Negative, or zero: Zero

Force: Positive, Negative, or zero: Positive

Acceleration: Positive, Negative, or zero: Positive



If the amplitude of a simple harmonic oscillator is doubled, the maximum speed of the oscillator:

A) doubles

$V_{max} = \omega A$,

B) halves

C) stays the same

If the amplitude of a simple harmonic oscillator is doubled, the period of the oscillations:

A) doubles

Period is independent of Amplitude.

B) halves

C) stays the same