SAMPLE TEST 6

PHYS 111, FALL 2011, SECTION 1

Name: ______________________________

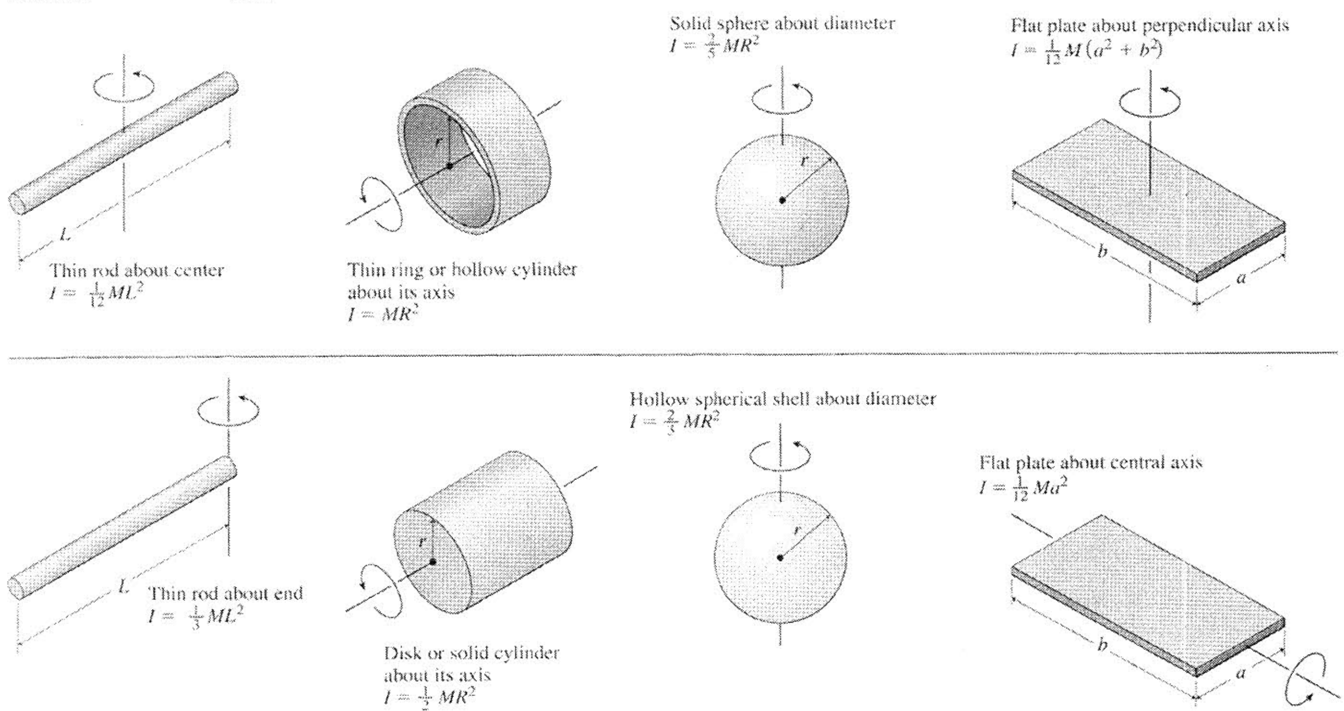
By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

A few potentially useful equations

Moment of Inertia, discrete definition	$I = \sum m_i r_i^2$
Moment of Inertia, integral definition	$I = \int r^2\, dm$
Parallel Axis Theorem	$I = I_{cm} + Md^2$
Superposition	$I_{Total} = \sum I_i$

TABLE 10.2 Rotational Inertias



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1) Derivations

a) (10pts) Given a differential equation of the form $\frac{d^2 x(t)}{dt^2} = -\omega^2 x(t)$, write the general solution for $x(t)$, $v(t)$, and $a(t)$ in terms of the angular frequency ω, the amplitude A, and the phase angle φ.

$$x(t) = A\cos(\omega t + \phi)$$
$$v(t) = -\omega A\sin(\omega t + \phi)$$
$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

b) (10pts) Given the boundary conditions $x(t_0) = x_0$ and $v(t_0) = v_0$, derive an expression for the phase angle φ and the amplitude A in terms of x_0, v_0, and ω.

Find ϕ - Divide v/x

$$x_0 = A\cos(\omega t_0 + \phi)$$
$$v_0 = -\omega A\sin(\omega t_0 + \phi)$$

$$\Rightarrow \frac{v_0}{x_0} = \frac{-\omega \cancel{A}\sin(\omega t + \phi)}{\cancel{A}\cos(\omega t + \phi)}$$

$$\Rightarrow \frac{-v_0}{x_0\omega} = \tan(\omega t + \phi) \Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{-v_0}{x_0\omega}\right) - \omega t}$$

Find A - Square both eq. divide v by ω^2 and add

$$x_0^2 = A^2\cos^2(\omega t + \phi), \quad v_0^2/\omega^2 = A^2\sin^2(\omega t + \phi)$$

$$\left(x_0^2 + v_0^2/\omega^2\right) = A^2\left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right)$$

$$\Rightarrow \boxed{A = \left(x_0^2 + \frac{v_0^2}{\omega^2}\right)^{1/2}}$$

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2) Multiple Choice

2.1) A mass attached to a spring oscillates with a period T. If the amplitude of the oscillation is doubled, the period will be:

A) T
B) 1.5 T
C) $2T$
D) ½ T
E) $4T$

$\omega = \sqrt{\frac{k}{m}}$, $T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{m/k}}$

independent of A

2.2) An object of mass m, oscillating on the end of a spring with spring constant k has amplitude A. Its maximum speed is:

A) $A\sqrt{\frac{k}{m}}$

B) $A^2\frac{k}{m}$

C) $A\sqrt{\frac{m}{k}}$

D) $A\frac{m}{k}$

$v = -\omega A \sin(\omega t + \phi)$

$v_{max} = \omega A$, $\omega = \sqrt{k/m}$

2.3) In simple harmonic motion, the magnitude of the acceleration is greatest when:

A) the displacement is zero
B) the displacement is maximum ← greatest restoring Force
C) the speed is maximum
D) the force is zero
E) the speed is between zero and its maximum

2.4) The displacement of an object oscillating on a spring is given by $x(t) = A\cos(\omega t + \phi)$.
If the initial displacement is zero and the initial velocity is in the negative x direction, then the phase constant φ is:

A) 0 radians
B) π/2 radians
C) π radians
D) 3π /2 radians
E) 2 π radians

$0 = A\cos(\phi) \Rightarrow \phi = \boxed{\pi/2}, \; 3\pi/2$

$v = -\omega A \sin(\phi)$

To keep minus sign

2.5) A simple pendulum of length L and mass M has frequency f. To increase its frequency to 2f:

A) increase its length to 4L
B) increase its length to 2L
C) decrease its length to L/2
D) decrease its length to L/4
E) decrease its mass to < M/4

$\omega = \sqrt{g/l}$

$\omega_2 = 2\omega_1$

$$\frac{\omega_1}{\omega_2} = \left(\frac{l_2}{\cancel{g}} \cdot \frac{\cancel{g}}{l_1}\right)^{1/2}$$

$$\frac{\cancel{\omega_1}}{2\cancel{\omega_1}} = \left[\frac{l_2}{l_1}\right]^{1/2} \Rightarrow \frac{1}{4} = \frac{l_2}{l_1} \Rightarrow \boxed{l_1 = 4 l_2}$$

$$\Rightarrow \boxed{l_2 = \tfrac{1}{4} l_1} *$$

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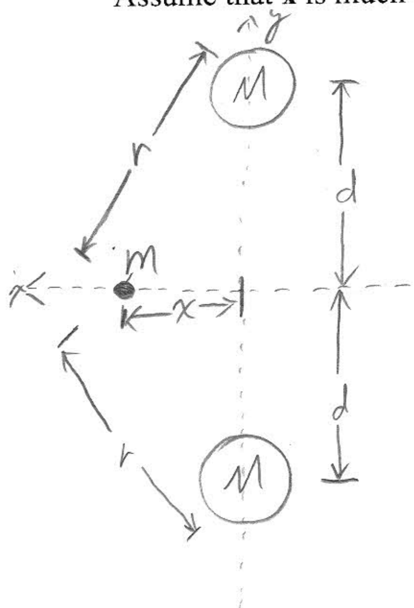
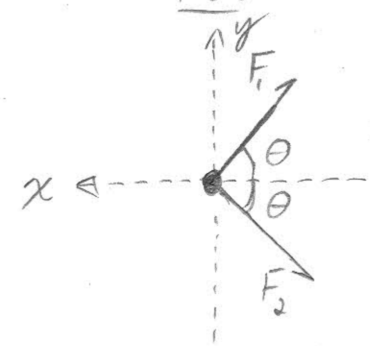
Two identical objects of mass **M** are held rigidly in space and separated by a distance of **2d**. A small mass is released from rest as shown below. The masses interact via gravity. The small mass oscillates back and forth with a period of $\boldsymbol{\tau}$.

Assume that **x** is much smaller than **d** and find the mass **M** in terms of $\boldsymbol{\pi}$, **d**, **G** and $\boldsymbol{\tau}$.



FBD



NSL

$x: \boxed{-F_1\cos\theta - F_2\cos\theta = ma_x}$

Net Force on y-axis is zero.

Given	want
d, T	M

Force is Gravity: $F_G = \dfrac{GMm}{r^2}$

acceleration in x

So: $-\dfrac{GM\cancel{m}}{r^2}\cos\theta - \dfrac{GM\cancel{m}}{r^2}\cos\theta = \cancel{m}a_x$

$$\Rightarrow \boxed{-2\frac{GM}{r^2}\cos\theta = \frac{d^2x}{dt^2}}$$

Let's translate $\cos\theta$ and r into x:

$$\cos\theta = \frac{x}{r}, \quad r = (x^2+d^2)^{1/2} \Rightarrow \cos\theta = \frac{x}{(x^2+d^2)^{1/2}}$$

$$\text{so: } \frac{-2GM}{x^2+d^2}\,\frac{x}{(x^2+d^2)} = \frac{d^2x}{dt^2}$$

continued

↓

Sample Test 6 - continued

$$\Rightarrow -2GM \frac{x}{(x^2+d^2)^{3/2}} = \frac{d^2x}{dt^2}$$

and if $x \ll d$, then $x^2 + d^2 \cong d^2$

So: $$-2GM \frac{x}{d^3} = \frac{d^2x}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} = -\left[\frac{2GM}{d^3}\right] x}$$

And plugging in the general solution for a SHO:

$$\frac{d^2}{dt^2}\left[A\cos(\omega t + \phi)\right] = -\left[\frac{2GM}{d^3}\right] A\cos(\omega t + \phi)$$

$$\Rightarrow +\omega^2 \cancel{A\cos(\omega t + \phi)} = +\left[\frac{2GM}{d^3}\right] \cancel{A\cos(\omega t + \phi)}$$

$$\Rightarrow \omega = \left[\frac{2GM}{d^3}\right]^{1/2} \Rightarrow T = \left[\frac{4\pi^2 d^3}{2GM}\right]^{1/2}$$

$$\Rightarrow \boxed{M = \frac{4\pi^2 d^3}{2GT^2}}$$

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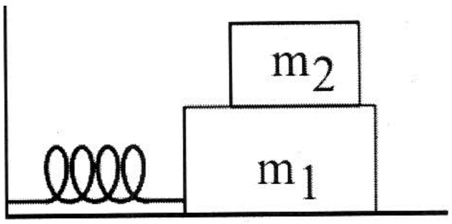
A large block m_1 executes simple harmonic motion as it slides across a frictionless surface with a frequency of $f = 1.50$ Hz. Block m_2 rests on m_1, as shown in the figure below. The coefficient of static friction between the two blocks is $\mu_s = 0.600$. What maximum amplitude of oscillation can the system have if block m_2 is not to slip?

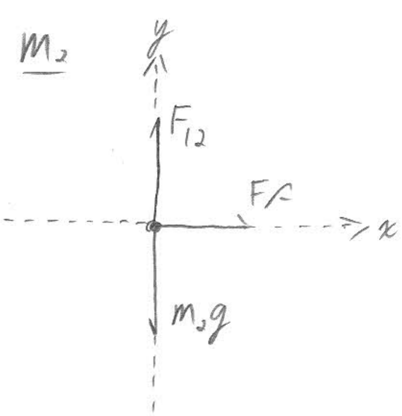


1) Find max a for slippage of block 2

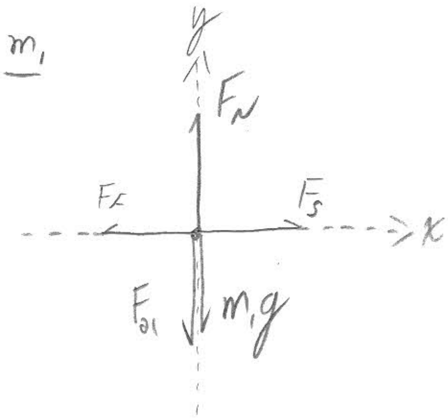
FBD

Given	Want
f, μ_s	A_{max}

m_2



m_1



NSL For block 2:

x: $\boxed{F_f = m_2 a}$ (1)

y: $\boxed{F_{12} - m_2 g = 0}$ (2)

NSL For block 1

x: $\boxed{F_s - F_f = m_1 a}$ (3)

y: $\boxed{F_N - F_{21} - m_1 g = 0}$ (4)

max a is when $F_f = \mu_s F_{12}$, the moment of slippage.

continued ↓

From ①: $a_{max} = \frac{F_f}{m_2} \Rightarrow \boxed{a_{max} = \frac{\mu_s F_{12}}{m_2}}$

and From ②: $F_{12} = m_2 g \Rightarrow a_{max} = \frac{\mu_s \cancel{m_2} g}{\cancel{m_2}}$

So: $\boxed{a_{max} = \mu_s g}$

2) Find max A for slippage of block 2

Oscillator acceleration: $a(t) = -\omega^2 A \cos(\omega t + \phi)$

maximum acceleration: $\boxed{a_{max} = \omega^2 A}$

So, the block slips when: $\omega^2 A = \mu_s g$

$\Rightarrow A_{max} = \frac{\mu_s g}{\omega^2}$ and $f = 2\pi\omega \Rightarrow \omega = \frac{f}{2\pi}$

$\Rightarrow \boxed{A_{max} = \frac{4\pi^2 \mu_s g}{f^2}}$

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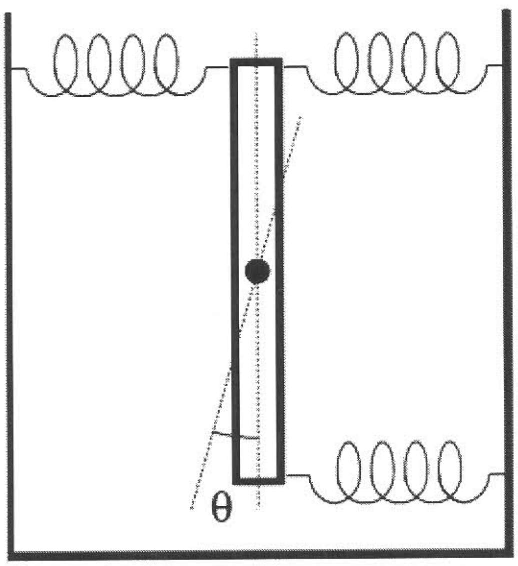
In the system shown below each of the three springs have a spring constant of 50 N/m and the bar is mounted on a frictionless pivot at its midpoint. The period of small oscillations is found to be 2.0 s.

The moment of inertia for the beam is $I=\frac{1}{12}ml^2$.

What is the mass of the bar?

Given	want
k, T	m



* Assuming that the pivot is at $\frac{l}{2}$ from the end and the springs are at the ends.

I'll use energy to get ω and then solve for m.

$$E_T = \underbrace{\tfrac{1}{2}I\omega^2}_{\text{rotating bar}} + \underbrace{3\left[\tfrac{1}{2}kx^2\right]}_{\text{3 Identical springs at equal displacements}}$$

continued ↓

Sample Test 6 continued:

Find ω by taking a derivative

$$\frac{dE_T}{dt} = I\omega\frac{d\omega}{dt} + 3kx\frac{dx}{dt}$$

$$\Rightarrow 0 = I\omega\alpha + 3kxv$$

Let's go rotation: $x = \frac{l}{2}\theta$, $v = \frac{l}{2}\omega$

$$\Rightarrow 0 = I\cancel{\omega}\alpha + 3k\frac{l}{2}\theta\frac{l}{2}\cancel{\omega}$$

$$\Rightarrow \alpha = -\frac{3kl^2}{4I}\theta \Rightarrow \alpha = -\frac{3k\cancel{l^2}}{\cancel{4}\cdot\frac{1}{\cancel{12}_{3}}m\cancel{l^2}}\theta$$

$$\Rightarrow \alpha = -\left[\frac{9k}{m}\right]\theta \Rightarrow \omega = \left[\frac{9k}{m}\right]^{1/2}$$

and $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\left[\frac{m}{9k}\right]^{1/2}$

$$\Rightarrow \boxed{m = \frac{T^2}{4\pi^2}9k}$$