

SAMPLE TEST 6
 PHYS 111, FALL 2011, SECTION 1

Name: _____

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

A few potentially useful equations

Moment of Inertia, discrete definition

$$I = \sum m_i r_i^2$$

Moment of Inertia, integral definition

$$I = \int r^2 dm$$

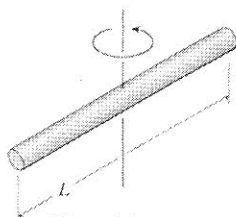
Parallel Axis Theorem

$$I = I_{cm} + Md^2$$

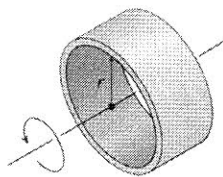
Superposition

$$I_{Total} = \sum I_i$$

TABLE 10.2 Rotational Inertias

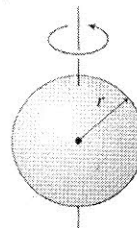


Thin rod about center
 $I = \frac{1}{12} ML^2$

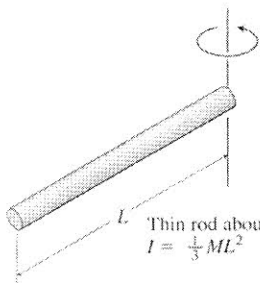
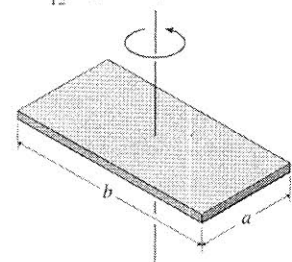


Thin ring or hollow cylinder about its axis
 $I = MR^2$

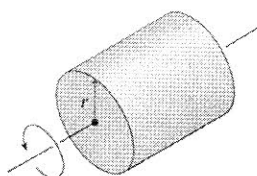
Solid sphere about diameter
 $I = \frac{2}{5} MR^2$



Flat plate about perpendicular axis
 $I = \frac{1}{12} M(a^2 + b^2)$

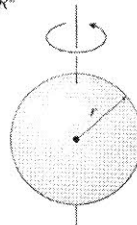


Thin rod about end
 $I = \frac{1}{3} ML^2$

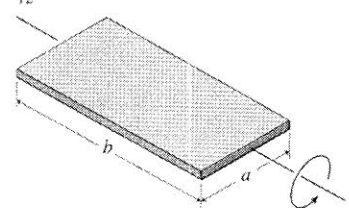


Disk or solid cylinder about its axis
 $I = \frac{1}{2} MR^2$

Hollow spherical shell about diameter
 $I = \frac{2}{3} MR^2$



Flat plate about central axis
 $I = \frac{1}{12} Ma^2$



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1) Derivations

- a) (10pts) Given a differential equation of the form $\frac{d^2 x(t)}{dt^2} = -\omega^2 x(t)$, write the general solution for $x(t)$, $v(t)$, and $a(t)$ in terms of the angular frequency ω , the amplitude A , and the phase angle ϕ .

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -\omega A \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

- b) (10pts) Given the boundary conditions $x(t_0) = x_0$ and $v(t_0) = v_0$, derive an expression for the phase angle ϕ and the amplitude A in terms of x_0 , v_0 , and ω .

Find ϕ - Divide v/x

$$\Rightarrow \frac{v_0}{x_0} = \frac{-\omega A \sin(\omega t + \phi)}{A \cos(\omega t + \phi)}$$

$$\begin{cases} x_0 = A \cos(\omega t_0 + \phi) \\ v_0 = -\omega A \sin(\omega t_0 + \phi) \end{cases}$$

$$\Rightarrow \frac{-v_0}{x_0 \omega} = \tan(\omega t + \phi) \Rightarrow \left[\phi = \tan^{-1}\left(\frac{-v_0}{x_0 \omega}\right) - \omega t \right]$$

Find A - Square both eq. divide v by ω^2 and add

$$x_0^2 = A^2 \cos^2(\omega t + \phi), \quad v_0^2 / \omega^2 = A^2 \sin^2(\omega t + \phi)$$

$$\left(x_0^2 + \frac{v_0^2}{\omega^2} \right) = A^2 \left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi) \right)$$

$$\Rightarrow \left[A = \left(x_0^2 + \frac{v_0^2}{\omega^2} \right)^{1/2} \right]$$

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2) Multiple Choice

2.1) A mass attached to a spring oscillates with a period T . If the amplitude of the oscillation is doubled, the period will be:

- A) T
- B) $1.5 T$
- C) $2T$
- D) $\frac{1}{2} T$
- E) $4T$

$$\omega = \sqrt{\frac{k}{m}}, \quad T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

independent of A

2.2) An object of mass m , oscillating on the end of a spring with spring constant k has amplitude A . Its maximum speed is:

- A) $A \sqrt{\frac{k}{m}}$
- B) $A^2 \frac{k}{m}$
- C) $A \sqrt{\frac{m}{k}}$
- D) $A \frac{m}{k}$

$$v = -\omega A \sin(\omega t + \phi)$$
$$v_{\max} = \omega A, \quad \omega = \sqrt{\frac{k}{m}}$$

2.3) In simple harmonic motion, the magnitude of the acceleration is greatest when:

- A) the displacement is zero
- B) the displacement is maximum
- C) the speed is maximum
- D) the force is zero
- E) the speed is between zero and its maximum

← greatest restoring force

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2.4) The displacement of an object oscillating on a spring is given by $x(t) = A \cos(\omega t + \phi)$.
 If the initial displacement is zero and the initial velocity is in the negative x direction, then the phase constant ϕ is:

- A) 0 radians
- B) $\pi/2$ radians
- C) π radians
- D) $3\pi/2$ radians
- E) 2π radians

$$0 = A \cos(\phi) \Rightarrow \phi = \left[\frac{\pi}{2} \right], \frac{3\pi}{2}$$

$$v = -\omega A \sin(\phi)$$

↑
 to keep minus sign

2.5) A simple pendulum of length L and mass M has frequency f. To increase its frequency to 2f:

- A) increase its length to 4L
- B) increase its length to 2L
- C) decrease its length to L/2
- D) decrease its length to L/4
- E) decrease its mass to $< M/4$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\omega_2 = 2\omega_1$$

$$\frac{\omega_1}{\omega_2} = \left(\frac{L_2 \cdot g}{g \cdot L_1} \right)^{1/2}$$

$$\frac{\omega_1}{2\omega_1} = \left[\frac{L_2}{L_1} \right]^{1/2} \Rightarrow \frac{1}{2} = \frac{L_2}{L_1} \Rightarrow \boxed{L_1 = 4L_2}$$

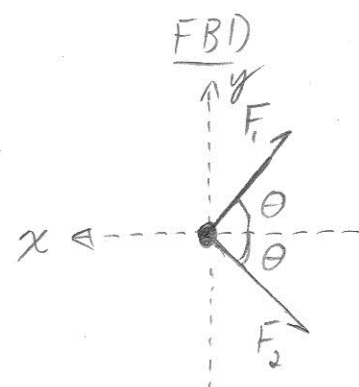
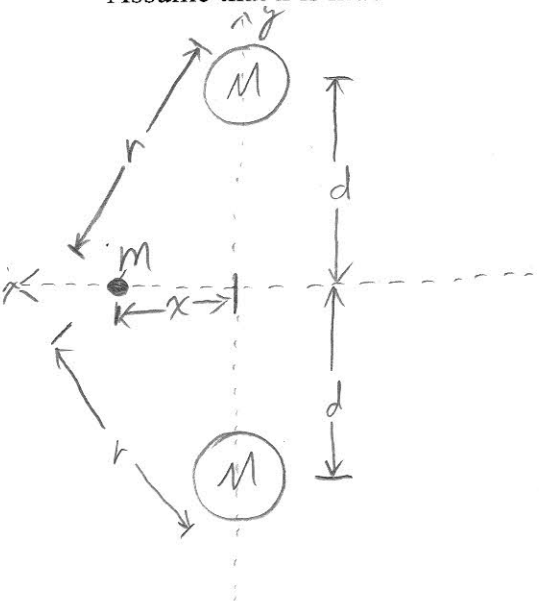
$$\Rightarrow \boxed{L_2 = \frac{1}{4}L_1} \quad *$$

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Two identical objects of mass M are held rigidly in space and separated by a distance of $2d$. A small mass is released from rest as shown below. The masses interact via gravity. The small mass oscillates back and forth with a period of τ .

Assume that x is much smaller than d and find the mass M in terms of π , d , G and τ .



NSL

$x: -F_1 \cos \theta - F_2 \cos \theta = ma_x$

Net Force on y -axis is zero.

<u>Given</u>	<u>want</u>
d, T	M

Force is Gravity: $F_G = \frac{GMm}{r^2}$

So: $-\frac{GMm}{r^2} \cos \theta - \frac{GMm}{r^2} \cos \theta = ma_x$

acceleration in x

$\Rightarrow -2 \frac{GM}{r^2} \cos \theta = \frac{d^2 x}{dt^2}$

Let's translate $\cos \theta$ and r into x :

$\cos \theta = \frac{x}{r}, \quad r = (x^2 + d^2)^{1/2} \Rightarrow \cos \theta = \frac{x}{(x^2 + d^2)^{1/2}}$

So: $-\frac{2GM}{x^2 + d^2} \frac{x}{(x^2 + d^2)^{1/2}} = \frac{d^2 x}{dt^2}$

continued
↓

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$$\Rightarrow -2GM \frac{x}{(x^2+d^2)^{3/2}} = \frac{d^2x}{dt^2}$$

and if $x \ll d$, then $x^2 + d^2 \approx d^2$

$$\text{so: } -2GM \frac{x}{d^3} = \frac{d^2x}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} = -\left[\frac{2GM}{d^3}\right]x}$$

And plugging in the general solution for a SHO:

$$\frac{d^2}{dt^2}[A \cos(\omega t + \phi)] = -\left[\frac{2GM}{d^3}\right]A \cos(\omega t + \phi)$$

$$\Rightarrow +\omega^2 A \cos(\omega t + \phi) = +\left[\frac{2GM}{d^3}\right]A \cos(\omega t + \phi)$$

$$\Rightarrow \omega = \left[\frac{2GM}{d^3}\right]^{1/2} \Rightarrow T = \left[\frac{4\pi^2 d^3}{2GM}\right]^{1/2}$$

$$\Rightarrow \boxed{M = \frac{4\pi^2 d^3}{2GT^2}}$$

