Rotation – Set 1

An aging hippie is jammin' to his old "In-A-Gadda-Da-Vida" album. The turntable on which the record sits spins it with a constant angular speed of 33.33 revolutions per minute. The hippie notices that when he turns off the turntable motor, the record makes exactly three revolutions before stopping.

a) What's the angular deceleration of the record ~~in radians per second~~, man? (Assume it's a constant.)

b) He also notices that when he starts the turntable, it takes the record 3.0 seconds to come up to speed. After the record gets up to speed, "In-A-Gadda-Da-Vida" plays for an agonizing 3.50 minutes. What is the total angular displacement of the record at the end of the song?

a)

$\theta_0 = 0$ $\qquad \alpha = ?$

$\theta = 3 \text{ rev}$

$\omega_0 = 33.3 \text{ rev/min} = 33.3 \text{ rev/min} \cdot \frac{1}{60} \frac{\text{min}}{\text{s}} = 0.55 \text{ rev/s}$

$\omega = 0$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad \omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad 0 = \omega_0 + \alpha t$$

$$\theta = -\frac{\omega_0^2}{\alpha} + \frac{1}{2}\alpha\frac{\omega_0^2}{\alpha^2} \qquad \Rightarrow t = -\frac{\omega_0}{\alpha}$$

$$\theta = -\frac{1}{2}\frac{\omega_0^2}{\alpha} \Rightarrow \boxed{\alpha = -\frac{1}{2}\frac{\omega_0^2}{\theta}}$$

$$\alpha = -\frac{1}{2}\frac{33.3^2}{3} = -184 \text{ rev/min}^2$$

or $$\alpha = -\frac{1}{2}\frac{(0.55)^2}{3} = -0.05 \text{ rev/s}^2 = 0.314 \text{ rad/s}^2$$

Rotation Problems, Set 1, P1 continued

b) Let: $\theta_0 = 0$, $\omega_0 = 0$, $\omega = 33.3\ \text{rev/min}$, $t_1 = 3.0\,s$, $t_2 = 3.5\ \text{min}$

<u>During acceleration</u>

$$\theta = \cancel{\theta_0}^{=0} + \cancel{\omega_0 t}^{=0} + \tfrac{1}{2}\alpha t^2$$

$$\theta_1 = \tfrac{1}{2}\alpha t_1^2$$

$$\Rightarrow \theta_1 = \tfrac{1}{2}\frac{\omega}{\cancel{t}}\, t_1^{\cancel{2}}$$

$$\boxed{\theta_1 = \tfrac{1}{2}\omega_1 t_1}$$

$$\omega_1 = \omega_0 + \alpha t$$

$$\omega_1 = \alpha t$$

$$\Rightarrow \alpha = \frac{\omega}{t}$$

<u>After acceleration</u>: $\omega_0 = \omega_1 = 33.3\ \text{rev/min}$, $\theta_0 = \theta_1$

$$\theta = \theta_0 + \omega_0 t + \cancel{\tfrac{1}{2}\alpha t^2}^{=0}$$

$$\theta_2 = \theta_1 + \omega_1 t_2$$

$$t_1 = 3.0\,s \cdot \frac{1}{60}\frac{\text{min}}{s} = \underline{0.05\,m}$$

$$\Rightarrow \theta_2 = \tfrac{1}{2}\omega_1 t_1 + \omega_1 t_2$$

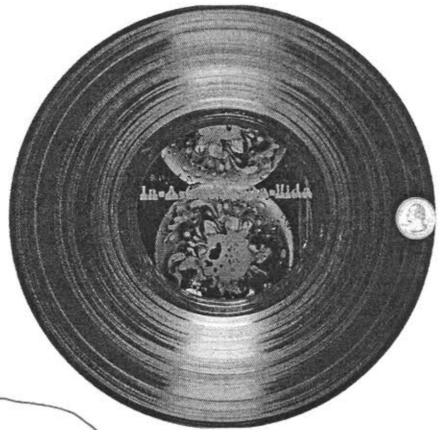
$$\Rightarrow \boxed{\theta_2 = \omega_1\left(\tfrac{t_1}{2} + t_2\right)}$$

$$\theta_2 = 33.3\ \text{rev/min}\left(\tfrac{1}{2}(0.05\,m) + 3.5\,m\right)$$

$$\boxed{\theta_2 = 117\ \text{revolutions}}$$

Rotation – Set 1

Your hippie friend is fascinated by watching a quarter on his record go around and around and around. He's somehow managed to formulate a few questions about the quarter that he needs your help answering.



a) So dude, like, if I totally put the quarter a distance *r* from the center, like, what distance, *c*, does it go in one revolution?

$c = 2\pi r$, Because $\boxed{\pi = \frac{C}{D}}$ ← It's the definition of π!

C = Circumference
D = diameter

b) Groovy, so then, like, how far does the quarter go, let's call the distance *s*, if the record rotates through an angle θ.

I like to use a ratio.

$$\frac{\theta}{\theta_1} = \frac{s}{c} \Rightarrow \boxed{\frac{\theta}{\theta_1} = \frac{s}{2\pi r}}$$

$\theta_1 = \theta$ for 1 rev

θ in degrees

$$\frac{\theta}{360} = \frac{s}{2\pi r} \Rightarrow \boxed{s = \frac{\pi}{180} r\theta}$$

θ in radians

$$\frac{\theta}{\cancel{2\pi}} = \frac{s}{\cancel{2\pi} r}$$

$\boxed{s = r\theta}$ ← My favorite!

c) Woah! Trippy... so then how FAST, is the quarter moving, call it *v*, in terms of the angular velocity ω?

If: $s = r\theta$ and $\omega = \frac{d\theta}{dt}$, then let's take the derivative

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow \boxed{v = r\omega}$$

d) Right on! So like, let's say the record totally has angular acceleration, α! What's the *tangential* acceleration, a_T, of the quarter?

Again, $v = r\omega$ and $\alpha = \frac{d\omega}{dt}$

So: $\frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow \boxed{a_T = r\alpha}$

what's a_c? oh yea! $\boxed{a_c = r\omega^2}$

Rotation – Set 1

The hippie wants to explore rolling the record across the floor now. Thankfully, this means that the terrible music has stopped playing. Assume that the record rolls in a perfectly straight line.

a) If the radius of the record is r = 30 cm, how far has it rolled after one revolution?

b) What is it's total angular displacement after it has rolled 5 meters?

c) If the initial *translational* velocity of the center of the record is 2 m/s, what is it's angular velocity?

d) If the record comes to a stop after 5 m, what was is its *angular* acceleration (assuming constant acceleration).

a) It has to roll one circumference, so:

$c = 2\pi r \Rightarrow c = 2\pi(0.30m) = \boxed{1.9\ m}$

b) From problem #2, we know:

$s = r\theta$, with θ in radians.

$\Rightarrow \theta = \frac{s}{r} \Rightarrow \theta = \frac{5m}{0.3m} = \boxed{16.7 \text{ radians}}$

$\frac{16.7 \text{ rad}}{2\pi \text{ rad/rev}} = \boxed{2.7 \text{ revolutions}}$

c) We know $v = r\omega \Rightarrow \omega = \frac{v}{r} \Rightarrow \omega = \frac{2.0 m/s}{0.3m} = \boxed{6.7 \text{ rad/s}}$

$\omega = 6.7 \text{ rad/s} \cdot \frac{1 \text{ rev}}{2\pi \cancel{\text{rad}}}$

$\boxed{\omega = 1.1 \text{ rev/s}}$

continued

d) We know $a_T = r\alpha$. But what's a_T?

① $x = \cancel{x_0}^{0} + v_0 t + \frac{1}{2}\overset{?}{\boxed{a_T}}\overset{?}{\boxed{t^2}}$ $\cancel{v}^{0} = v_0 + \overset{?}{\boxed{a_T}}\overset{?}{\boxed{t}}$ ②

2 eq. 2 unknowns!

solve ② for t and plug into ①

From ②: $t = -\frac{v_0}{a_T}$

into ①: $x = -v_0 \frac{v_0}{a_T} + \frac{1}{2}\cancel{a_T}\frac{v_0^2}{a_T^{\cancel{2}}}$

$\Rightarrow x = -\frac{v_0^2}{a_T} + \frac{1}{2}\frac{v_0^2}{a_T}$

$\Rightarrow x = -\frac{1}{2}\frac{v_0^2}{a_T} \Rightarrow a_T = -\frac{1}{2}\frac{v_0^2}{x}$

Then!

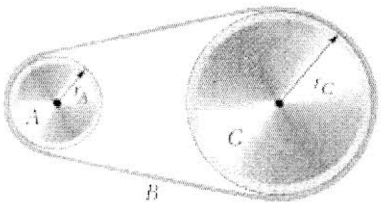
$a_T = r\alpha \Rightarrow -\frac{1}{2}\frac{v_0^2}{x} = r\alpha \Rightarrow \boxed{\alpha = -\frac{v_0^2}{2rx}}$

$\alpha = -\frac{(2\,m/s)^2}{(2)(0.3\,m)(5\,m)} = -1.3\ rad/s^2$

$= (1.3\ rad/s^2)\left(\frac{1}{2\pi}\frac{rev}{rad}\right)$

$\boxed{= -0.2\ rev/s^2}$

A pulley of radius r_A = 10 cm is coupled by a belt to a pulley of radius r_C = 25 cm. A motor is attached to the axle of pulley A giving it an angular acceleration of α_A = 1.6 rad/s^2. How long does it take pulley C to achieve an angular velocity of 100 rev/min assuming that the belt does not slip?



$\alpha_A = 1.6\,\text{rad/s}^2$

$\omega_c = 100\,\frac{\text{rev}}{\text{min}} \cdot 2\pi\,\frac{\text{rad}}{\text{rev}} \cdot \frac{1\,\text{min}}{60\,\text{s}}$

$= 10.5\,\text{rad/s}$

We need to relate α_c to α_A.

Because they are coupled by the belt, a point on the rim of A must have the same ~~linear~~ tangential acceleration as a point on the rim of C.

(1) $\boxed{a_A = r_A \alpha_A}$ and (2) $\boxed{a_C = r_C \alpha_C}$

But: $a_A = a_c = \underline{a}$

So, $r_A \alpha_A = r_C \alpha_C \Rightarrow \boxed{\alpha_C = \frac{r_A}{r_C}\alpha_A}$

Now, using kinematics:

$\omega = \cancel{\omega_0}^{\,0} + \alpha t \Rightarrow \omega_c = \alpha_c t$

$\Rightarrow t = \frac{\omega_c}{\alpha_c} \Rightarrow \boxed{t = \frac{\omega_c}{\alpha_A} \cdot \frac{r_c}{r_A}}$

$t = \frac{10.5\,\text{rad/s}}{1.6\,\text{rad/s}^2} \cdot \frac{25\,\text{cm}}{10\,\text{cm}} = 16.5\,\text{s}$