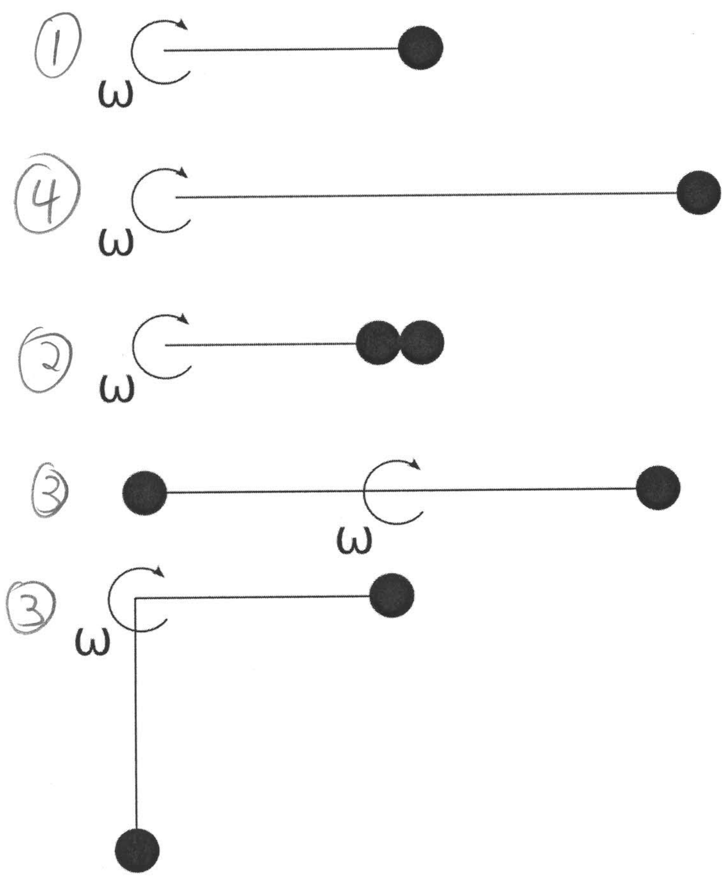
Rotation – Set 4

Below are several objects. Each circle is a point mass, and each point has the same mass. The connecting rods are massless. Rank them in order of moment of inertia, least to most. If any have the SAME moment, give them the same ranking number. **Explain your reasoning**.



$I = \sum m_i r_i^2$, So: The top object has a single mass close to the pivot

next, $2 < 3$ because the mass doubles but 2 has one mass sightly closer. to the pivot

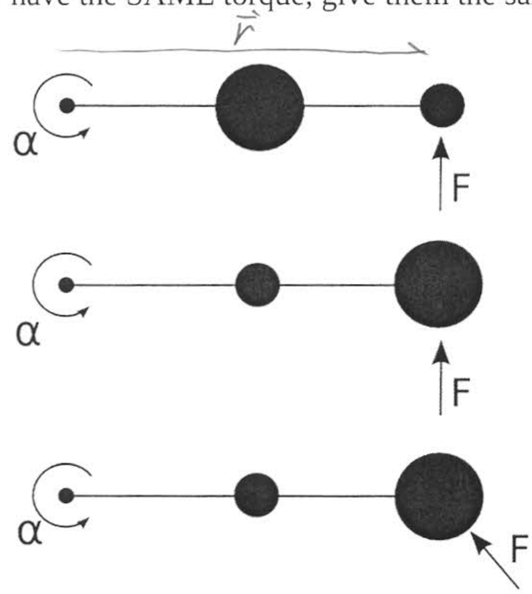
the 2 3's have twice the mass, but both at the same distance

last, 4 is a single mass twice as far but I goes as r^2, so r makes a

bigger difference than m.

In the pictures below, two masses are connected by a massless rod and the system is allowed to rotate about the pivot shown. The large circle is more massive than the small circle. A force is applied to each system as shown in the diagram.

Rank the three systems in order of the applied *torque*, least to most. **Explain your reasoning.**
If any have the SAME torque, give them the same ranking.



A 2

B 2

C 1

$T = |\vec{r}||\vec{F}| SIN\theta$

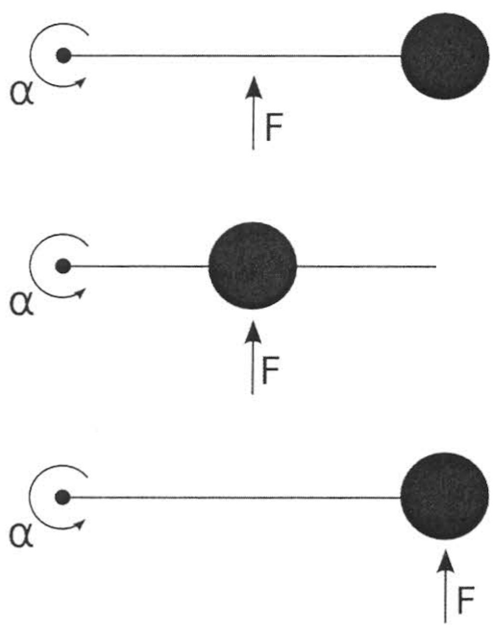
$|\vec{r}|$ is the same for each

$|\vec{F}|$ is the same for each

$\theta = 90$ for A and B

$\theta > 90$ for C

Rank the three systems in order of the applied *torque*, least to most. **Explain your reasoning.**
If any have the SAME torque, give them the same ranking.



A 1

B 1

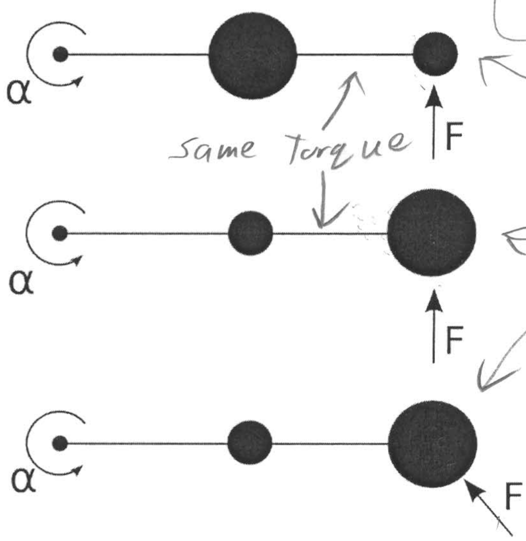
C 2

$|\vec{r}|$ is small for A and B

$|\vec{F}|$ and θ are the same for each

In the pictures below, two masses are connected by a massless rod and the system is allowed to rotate about the pivot shown. The large circle is more massive than the small circle. A force is applied to each system as shown in the diagram. Rank the three systems in order of their angular accelerations, least to most. **Explain your reasoning.**

$$\boxed{\alpha = \frac{T}{I} = \frac{r \times F}{I}}$$



(3) ← smallest I

same Torque

constant T, I↑ α↓

(2) ← same I

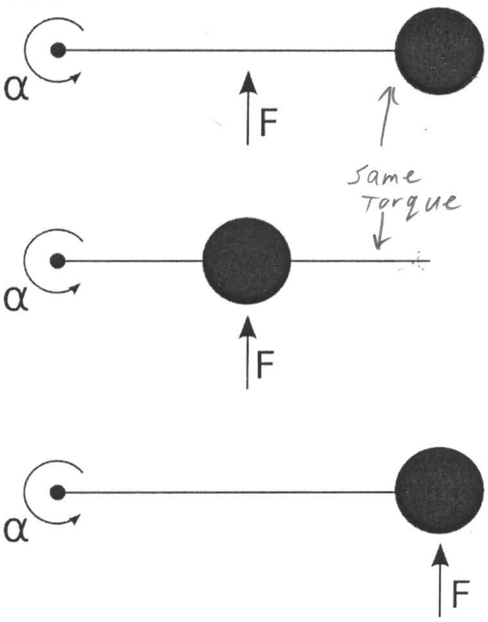
const I, T↑, α↑

(1) ← smallest Torque

In the pictures below, two masses are connected by a massless rod and the system is allowed to rotate about the pivot shown. A force is applied to each system as shown in the diagram. Rank the three systems in order of their angular accelerations, least to most. **Explain your reasoning.**

$$\boxed{\alpha = \frac{r \times F}{M L^2}}$$

Least



(1)

same Torque

(3)

same I

(2)

Torque is proportional to R from pivot

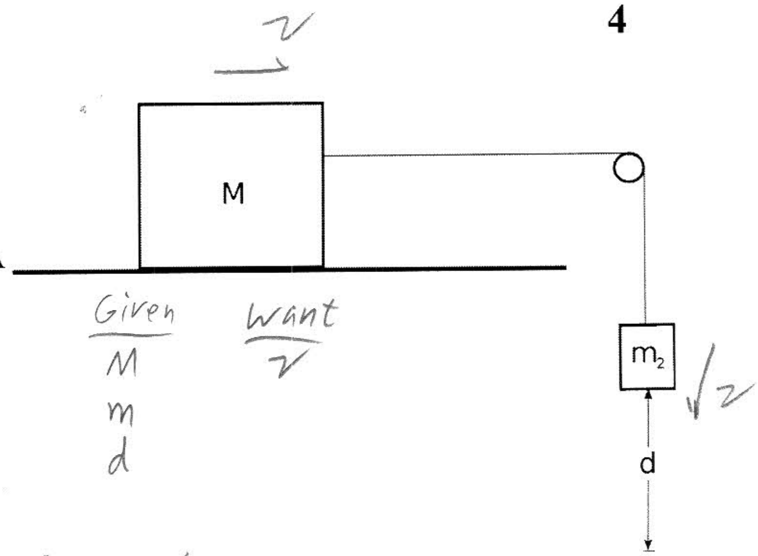
I is proportional to R^2 so moving the mass has a greater effect than moving the force.

Rotation – Set 4

Use **Newton's Second Law and Kinematics** to solve this problem.

A block of mass *M* is at rest on a frictionless surface. A massless string is attached to the block, passes over a small massless frictionless pulley, and is attached to a small mass *m*.

What will the velocity block on the surface be after the hanging mass falls through a distance d?

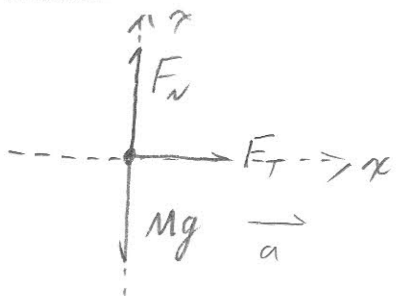


Given
M
m
d

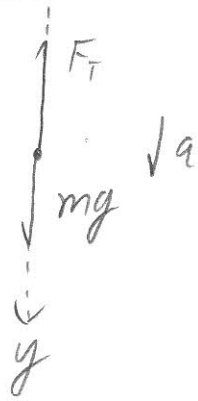
Want
v

1) Draw Free Body Diagrams, one for each mass.

Sliding mass



Hanging mass



2) Write Newton's Second Law for each object.

x: $\boxed{F_T = Ma}$ (1)

y: $\boxed{F_N - Mg = 0}$ (2)

x: $\boxed{mg - F_T = ma}$ (3)

continued ↓

Rotation Set 4, P4 continued

Solve for a, which is the same for each mass.

substitute ① → ③:

$$mg - Ma = ma \Rightarrow \boxed{a = \frac{m}{M+m} g}$$

3) Use kinematics to find v when m goes a distance d.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad v = v_0 + a_y t$$

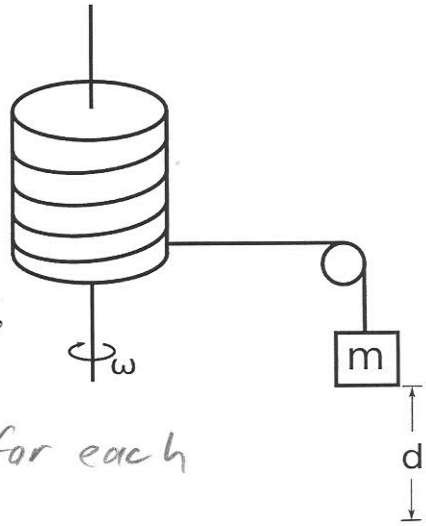
$$d = 0 + 0 + \tfrac{1}{2}at^2 \qquad v_F = at \Rightarrow t = \frac{v_F}{a}$$

$$d = \tfrac{1}{2}\cancel{a}\frac{v_F^2}{a^{\cancel{2}}} \Rightarrow v_F = \sqrt{2da}$$

$$\Rightarrow \boxed{v_F = \sqrt{\frac{m}{M+m}2dg}}$$

Rotation – Set 4

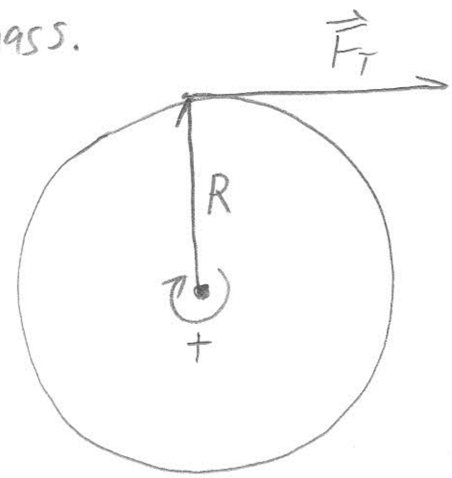
A solid cylinder of mass *M*, radius *R*, and moment of inertia $I = \frac{1}{2}MR^2$ is allowed to rotate without friction about an axis through its center as shown. A massless string is wrapped around the cylinder, passes over a small massless frictionless pulley and is attached to a small mass *m*.



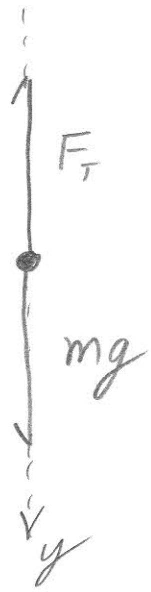
If the mass and the cylinder start from rest, what will the angular velocity of the cylinder be after the mass falls through a distance d?

Using **Torque and Kinematics**, find an expression for ω_f in terms of *d*, *M*, *m*, and *R*.

1) Draw Free-Body Diagrams, one for each mass.



Cylinder, Top View
Defining clockwise rotation as positive.



Defining down as positive y to agree with positive rotation of the cylinder

2) Write Newton's 2^{nd} law, Both torque version and translation version for each mass.

Cylinder, torque only no translation

$$\sum \vec{\tau} = I\alpha$$

$$\sum R \cdot F \sin\theta = I\alpha$$

Hanging mass, translation only no torque

$$\sum F = ma$$

continued ↓

Rotation Set 4, P7 Continued

Cylinder

$\sum RF\sin\theta = I\alpha$

$\theta = 90°$, $\sin\theta = 1$

$\Rightarrow RF_T = I\alpha$, $I = \frac{1}{2}MR^2$

$\Rightarrow \cancel{R}F_T = \frac{1}{2}MR^{\cancel{2}}\alpha$

$\Rightarrow \boxed{F_T = \frac{1}{2}MR\alpha}$ (1)

Hanging mass

$\sum F = ma$

$\boxed{mg - F_T = ma}$ (2)

Combine (1) and (2) to eliminate F_T, solve for α

$mg - \frac{1}{2}MR\alpha = ma$, $a = R\alpha$

$\Rightarrow mg - \frac{1}{2}MR\alpha = mR\alpha$

$\Rightarrow mg = (\frac{1}{2}M + m)R\alpha$

$\Rightarrow$ (3) $\boxed{\alpha = \frac{m}{\frac{1}{2}M + m}\frac{g}{R}}$ save this for later.

3) We have acceleration. Now we need to use kinematics to get ω, the angular velocity.

$\theta = \cancel{\theta_0}^{0} + \cancel{\omega_0}^{0}t + \frac{1}{2}\alpha t^2$

$\omega = \cancel{\omega_0}^{0} + \alpha t$ eliminate t

$\Rightarrow t = \frac{\omega}{\alpha}$

$\theta = \frac{1}{2}\cancel{\alpha}\frac{\omega^2}{\alpha^{\cancel{2}}} \Rightarrow \boxed{\omega = [2\theta\alpha]^{\frac{1}{2}}}$

continued ↓

Rotation Set ④, P7 continued

So we have: $\omega = [2\theta\alpha]^{1/2}$

we are given d, so we need to write $d = R\theta$

$\Rightarrow \theta = \frac{d}{R}$

Then: $\omega = \left[2\frac{d}{R}\alpha\right]^{1/2}$

And we plug in α from eq. ③

$$\omega = \left[2\frac{d}{R}\frac{m}{\frac{1}{2}M+m}\frac{g}{R}\right]^{1/2}$$

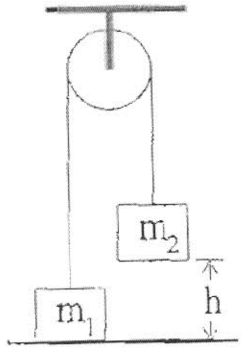
$$\boxed{\omega = \left[\frac{m}{\frac{1}{2}M+m}\frac{2dg}{R^2}\right]^{1/2}}$$

Use **Torque and Kinematics** to solve the following problem.

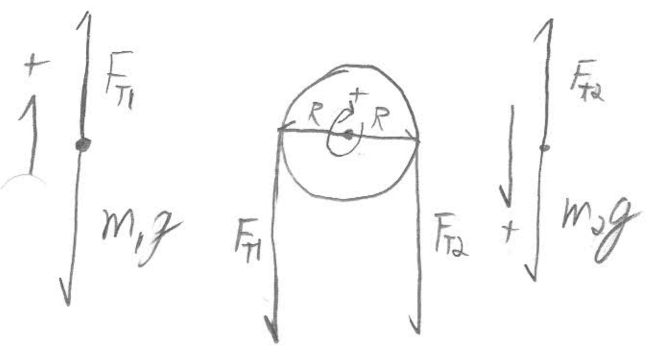
Two masses are connected by a light string passing over a frictionless pulley. the Mass m_2 is released from rest at a height of 4.0 m above the ground. You can treat the pulley as a solid disk.

Determine the speed of m_1 as m_2 hits the ground.

$m_1 = 3.0$ kg
$m_2 = 5.0$ kg
$m_{pulley} = 0.5$ kg
$r_{pulley} = 0.1$ m



FBD



Force

$$F_{T1} - m_1 g = m_1 a \quad (1)$$

$$m_2 g - F_{T2} = m_2 a \quad (2)$$

Torque

$$R F_{T2} - R F_{T1} = I\alpha \quad (3)$$

Solve (1) and (2) for F_{T1} and F_{T2} (they are NOT the same)

From (1): $F_{T1} = m_1(a + g)$

From (2): $F_{T2} = m_2(g - a)$

continued ↓

Rotation Set 5, P2 continued

① and ② → ③ :

$$R m_2(g-a) - R m_1(a+g) = I\alpha$$

want a for kinematics, so sub $\alpha = \frac{a}{R}$

and sub $I = \frac{1}{2} m_p R^2$

$$\Rightarrow \cancel{R} m_2(g-a) - \cancel{R} m_1(a+g) = \frac{1}{2} m_p R^{\cancel{2}} \frac{a}{\cancel{R}}$$

$$\Rightarrow m_2 g - m_2 a - m_1 a - m_1 g = \frac{1}{2} m_p a$$

$$\Rightarrow (m_2 - m_1) g = (m_1 + m_2 + \frac{1}{2} m_p) a$$

$$\Rightarrow \boxed{a = \frac{m_2 - m_1}{m_1 + m_2 + \frac{1}{2} m_p} g}$$

* Do kinematics, sub in a <u>last</u>

$$y = \cancel{y_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2} a t^2 \qquad v = \cancel{v_0}^{0} + at$$

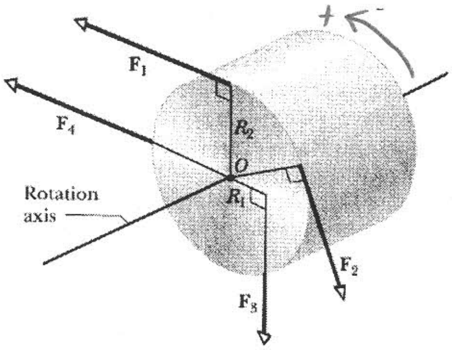
$$\Rightarrow t = \frac{v}{a}$$

$$h = \frac{1}{2} \cancel{a} \frac{v^2}{a^{\cancel{2}}}$$

$$\Rightarrow v = (2ya)^{1/2} \Rightarrow \boxed{v = \left[2h \frac{m_2 - m_1}{m_1 + m_2 + \frac{1}{2} m_p} g\right]^{1/2}}$$

Rotation – Set 4

A cylinder with a mass of 2.0 kg can rotate about its central axis through the point 0. Forces are applied as in the figure below. $F_1 = 6.0$ N, $F_2 = 4.0$ N, $F_3 = 2.0$ N, $F_4 = 5.0$ N, $R_1 = 5$ cm, $R_2 = 12$cm. Find the direction and the magnitude of the angular acceleration of the cylinder. (During rotation, the forces maintain the same angles relative to the cylinder.)



$I = \frac{1}{2} M R^2$

Given

$F_1 = 6.0\,N$ $M = 2.0\,kg$

$F_2 = 4.0\,N$

$F_3 = 2.0\,N$

$F_4 = 5.0\,N$

$R_1 = 5\,cm$

$R_2 = 12\,cm$

$\sum \vec{\tau} = I\vec{\alpha}$

$F_1 R_2 \sin(90) - F_2 R_2 \sin(90) - F_3 R_1 \sin(90) + \cancel{F_4 (R_0) \sin(0)}^{0} = I\alpha$

$\Rightarrow F_1 R_2 - F_2 R_2 - F_3 R_1 = I\alpha$

$\Rightarrow (6.0)(.12) - (4.0)(.12) - (2.0)(.05) = \frac{1}{2}(2.0)(.12)^2 \alpha$

$\Rightarrow \boxed{\alpha = 9.7\,rad/s}$

A door has a mass of 50 kg and is 0.8 m wide. The moment of inertia is $I = 1/3\ MW^2$ where W is the width of the door. I push on the door with a constant force of F = 10 N in two places; in the middle of the door a distance W/2 from the hinge and at the knob, a distance W from the hinge.

a) Draw free body diagrams of the two cases.
b) What is the magnitude of the Torque for each case?
c) What is the magnitude of the angular accelerations for each case?
d) How much time does it take the door to rotate through 90^0 in each case?
e) How much force would I have to apply at W/2 so that the door rotated through 90^0 in the same amount of time as applying 10 N to the knob?

a)

b) $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta, \quad \theta = 90^\circ \Rightarrow \sin\theta = 1$

$\tau_1 = W/2\ F \qquad \tau_2 = WF$

$\tau_1 = (0.4)(10) \qquad \tau_2 = (0.8)(10)$

$= 4\ N\cdot m \qquad = 8\ N\cdot m$

c) $\alpha = \dfrac{\tau}{I}$

$\alpha_1 = \dfrac{4}{\frac{1}{3}MW^2} = \dfrac{4}{\frac{1}{3}(50)(0.8)^2} = 0.375\ \text{rad}/s$

$\alpha_2 = \dfrac{8}{\frac{1}{3}(50)(0.8)^2} = 0.75\ \text{rad}/s$

Rotation Set 4 P6 Continued

d) $\theta = \cancel{\theta_0}^{\,0} + \cancel{\omega_0 t}^{\,0} + \frac{1}{2}\alpha t^2$

$$\theta = \frac{1}{2}\alpha t^2 \Rightarrow t = \left(\frac{2\theta}{\alpha}\right)^{1/2}, \quad t_1 = \left(\frac{(2)(\pi/2)}{0.375}\right)^{1/2} = 2.9\ s$$

$$t_2 = \left(\frac{2\,\pi/2}{0.75}\right)^{1/2} = 2.0\ s$$

e) To make the time equal, I need $\alpha_1 = \alpha_2$

And $\alpha = \frac{T}{I}$

So, $\frac{T_1}{I_1} = \frac{T_2}{I_2}$, But $I_1 = I_2$

$$\Rightarrow T_1 = T_2 \Rightarrow r_1 F_1 = r_2 F_2$$

So: $F_1 = \frac{r_2}{r_1} F_2 \Rightarrow F_1 = \frac{W}{W/2} F_2$

$$F_1 = 2F_2$$

So: $\boxed{F_1 = 20\,N}$