The picture below shows a modified atwood machine composed of a large YoYo mounted on a central axle so that it spins in place. Two blocks of equal mass are attached to the system by ropes. One rope is wound around the inner axle of the YoYo the outer disk. The mass of the YoYo is the same as the mass of the two blocks and $R_2 = \frac{1}{2} R_1$.

Assume that the moment of inertia of the pulley is $I = \frac{1}{2} MR_1^2$

a) If the masses are initially at rest, which way will the pulley rotate, clockwise or counter clockwise?
b) Using **Torque and Kinematics**, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.
c) Using **Work/Energy** techniques, find an expression for the angular velocity of the pulleys after the mass attached to the large pulley has moved a distance d.
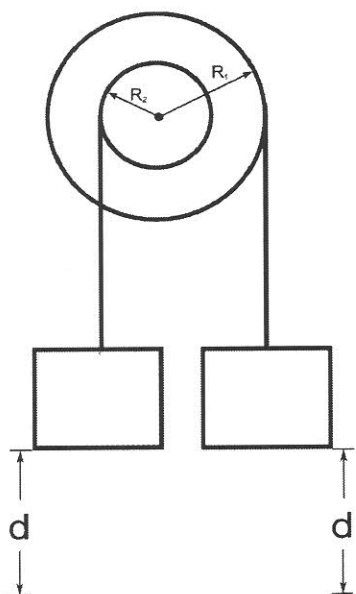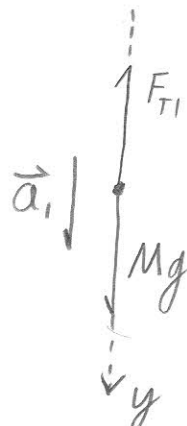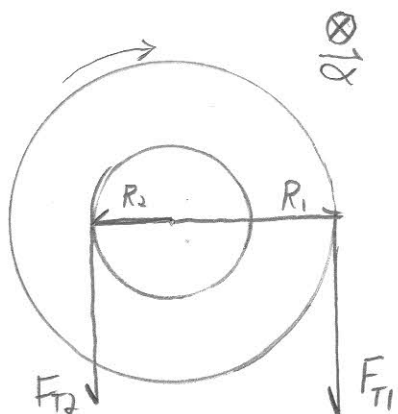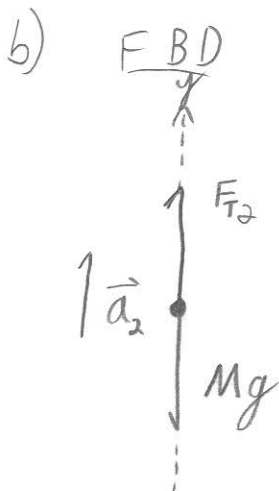
Given
M – same for all
d
I_p

Want
v_F

a) The torque due to the right most mass will be greater than that due to the left most mass. Therefore, it will rotate CW.

b) FBD

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Rotation Set 5, P1 continued.

※ NSL

① $\boxed{F_{T2} - Mg = Ma_2}$ ② $\boxed{F_{T1}R_1 - F_{T2}R_2 = I\alpha}$ ③ $\boxed{Mg - F_{T1} = Ma_1}$

Because $R_1 \neq R_2$, $a_1 \neq a_2$. But, we can relate a_1 and a_2 to α:

$$\boxed{a_1 = R_1\alpha, \quad a_2 = R_2\alpha}$$

Then we from ①: $\underline{F_{T2} = Mg + MR_2\alpha}$

and from ③: $\underline{F_{T1} = Mg - MR_1\alpha}$

plug into ②: $(Mg - MR_1\alpha)R_1 - (Mg + MR_2\alpha)R_2 = I\alpha$

$$\Rightarrow MgR_1 - MgR_2 = MR_1^2\alpha + MR_2^2\alpha + \frac{1}{2}MR_1^2\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{R_1 - R_2}{\frac{3}{2}R_1^2 + R_2^2}g}$$

※ Kinematics

$$\theta = \cancel{\theta_0} + \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2 \qquad \omega = \cancel{\omega_0} + \alpha t \Rightarrow t = \frac{\omega}{\alpha}$$

$$\frac{d}{R_1} = \frac{1}{2}\alpha t^2 \Rightarrow \frac{d}{R_1} = \frac{1}{2}\alpha\frac{\omega^2}{\alpha^2} \Rightarrow \omega = \left[\frac{2\alpha d}{R_1}\right]^{1/2}$$
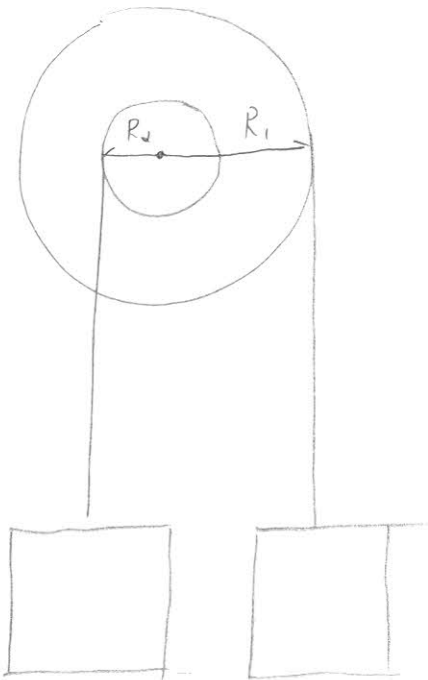
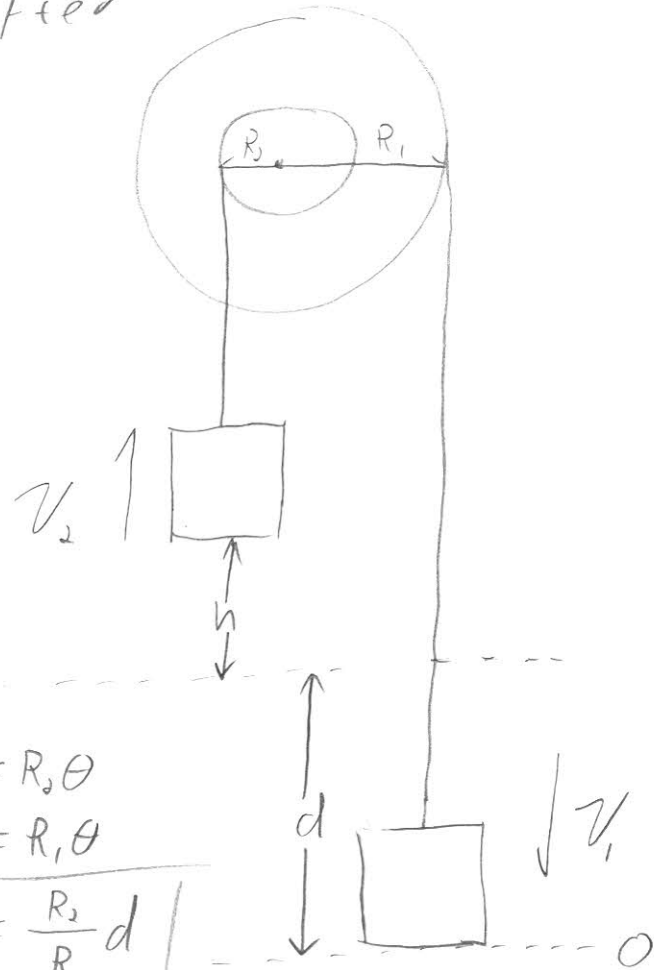$$\Rightarrow \boxed{\omega = \left[2d \frac{R_1 - R_2}{\frac{3}{2}R_1^2 + R_2^2}\frac{g}{R_1}\right]^{1/2}}$$

continued ↓

c) Work - Energy

BY

After

R_2 R_1

R_2 R_1

$v_2 \uparrow$

h

$h = R_2 \theta$

$d = R_1 \theta$

d

$\downarrow v_1$

$\Rightarrow \boxed{h = \dfrac{R_2}{R_1} d}$

$\cdots\cdots$ O

$U_I = Mgd + Mgd$

$\Rightarrow v_1 = R_1 \omega, \quad v_2 = R_2 \omega$

$U_F = Mg(d+h) + 0$

$K_I = 0$

$K_F = \tfrac{1}{2} M v_1^2 + \tfrac{1}{2} M v_2^2 + \tfrac{1}{2} I \omega^2$

$\Rightarrow 2Mgd = Mgd + Mg \dfrac{R_2}{R_1} d + \tfrac{1}{2} M R_1^2 \omega^2 + \tfrac{1}{2} M R_2^2 \omega^2 + \tfrac{1}{12} M R_1^2 \omega^2$

Rotation Set 5, P1 continued.

$$\Rightarrow \left(2 - 1 - \frac{R_2}{R_1}\right) g d = \left(\frac{3}{4} R_1^2 + \frac{1}{2} R_2^2\right) \omega^2$$

$$\Rightarrow \left(1 - \frac{R_2}{R_1}\right) g d = \frac{1}{2}\left(\frac{3}{2} R_1^2 + R_2^2\right) \omega^2$$

$$\Rightarrow \left[\frac{R_1 - R_2}{R_1}\right] 2 g d = \left(\frac{3}{2} R_1^2 + R_2^2\right) \omega^2$$

$$\Rightarrow \boxed{\omega = \left[2 d\, \frac{R_1 - R_2}{\frac{3}{2} R_1^2 + R_2^2} \frac{g}{R_1}\right]^{1/2}}$$

Use **Torque and Kinematics** to solve this problem.

A rolling object with a radius R, mass m, and moment of inertia I, starts from rest at the top of an incline plane of height h that makes an angle θ with the horizontal.

a) Find an expression for the linear and angular acceleration of the object in terms of I.

b) Using kinematics, find an expression for the linear and angular ~~acceleration~~ of the object in terms of
    I?   *Velocity*

c) Assume that the object is a disk with $I = \frac{1}{2} mR^2$ and plug I into your velocity expressions. Verify

   that your answers are the as when you solved this problem using energy.
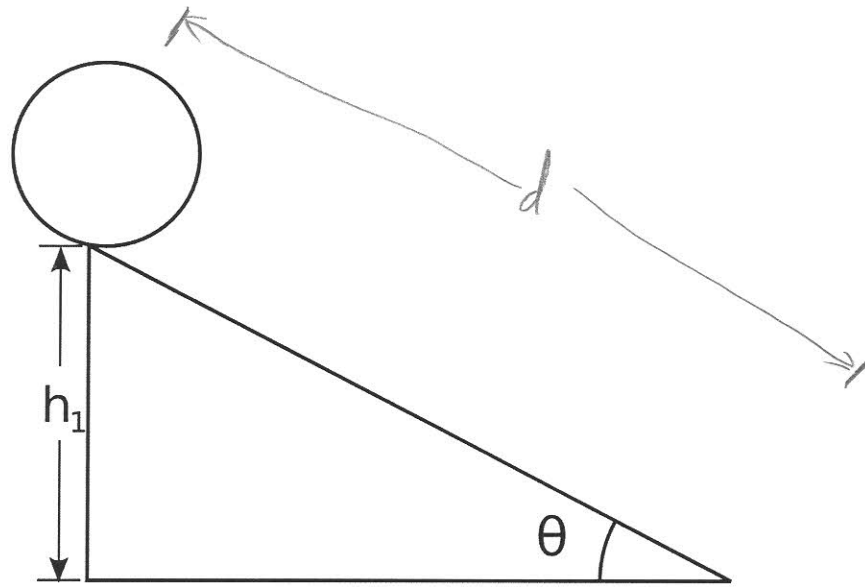
d)

Step _1_ – FBD
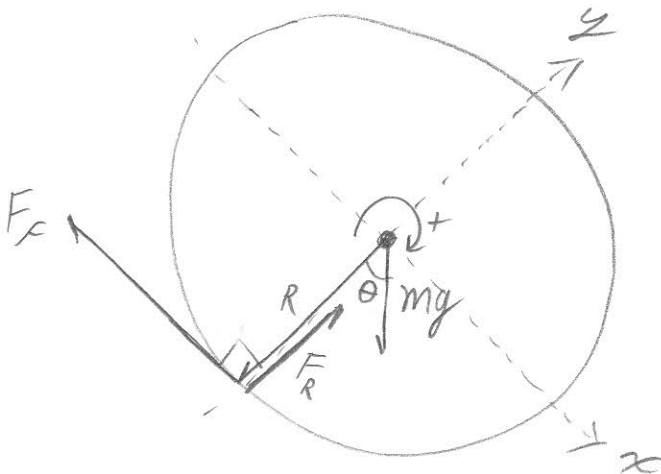
In this problem, we
need to consider **both**
Rotation and translation.
So we have positive
rotation as well as
the x-y coordinates
labeled

Continued ↓

Rotation set 5, P1 continued

step ② - NSL

One object but both translation and rotation

Rotation

$$\sum \vec{\tau} = \breve{I}\alpha$$

$$\Rightarrow \sum RF\sin\theta = I\alpha$$

$$\Rightarrow RF_x\sin(90) + \underbrace{RF_R\sin(180)}_{\sin(180)=0}{}^{=0} + \underbrace{0 \cdot mg\sin\theta}_{R=0}{}^{=0} = I\alpha$$

$$\underbrace{}_{\sin 90 = 1}$$

$$\Rightarrow RF_x = I\alpha \Rightarrow \boxed{F_x = \frac{I}{R}\alpha}$$

Translation

$$x: \boxed{mg\sin\theta - F_x = ma} \quad ②$$

$$y: F_R - mg\cos\theta = 0 \longleftarrow \text{not useful}$$

Plug ① → ②: $mg\sin\theta - \frac{I}{R}\alpha = ma$

Let $a = R\alpha$: $mg\sin\theta - \frac{I}{R^2}a = ma \Rightarrow mg\sin\theta = \left(m + \frac{I}{R^2}\right)a$

$$\boxed{a = \frac{m}{m + I/R^2} g\sin\theta} \quad ①$$

$$\boxed{\alpha = \frac{m}{m + I/R^2} \frac{g}{R}\sin\theta} \quad ②$$

Rotation set 5, P1 continued

b) kinematics

$$x = x_0^0 + v_0^0 t + \frac{1}{2}at^2 \qquad v = v_0^0 + at$$

$$t = \frac{v}{a}$$

$$d = \frac{1}{2}a\frac{v^2}{a^2}$$

$$d = \frac{1}{2}\frac{v^2}{a} \implies \boxed{v = (2da)^{1/2}} \quad ③$$

Plug in a from eq. ①

$$v = \left[2d \frac{m}{m + I/R^2} g \, SIN\theta\right]^{1/2}$$

$$\boxed{v = \left[\frac{m}{m + I/R^2} 2gd \, SIN\theta\right]^{1/2} = \left[\frac{m}{m + I/R^2} 2gh\right]^{1/2}}$$

$$\omega = \frac{v}{R} \implies \boxed{\omega = \left[\frac{m}{m + I/R^2} \frac{2gh}{R^2}\right]^{1/2}}$$

IF $I = \frac{1}{2}mR^2$, $m + I/R^2 \implies m + \frac{\frac{1}{2}mR^2}{R^2} \implies \frac{3}{2}m$

So: $\frac{m}{m + I/R^2} \implies \frac{m}{\frac{3}{2}m} \implies \frac{2}{3}$

and: $\boxed{v = \left[\frac{4}{3}gh\right]^{1/2}}$ and $\boxed{\omega = \left[\frac{4}{3}\frac{gh}{R^2}\right]^{1/2}}$

4. The picture below represents the side view of a yo-yo. The inner dashed circle represents the axle that the string is wound around. The top of the string is held stationary and the yo-yo is allowed to fall, unwinding the string as it descends.

The moment of inertia of the yo-yo is: $I_{cm} = \dfrac{1}{2} MR_1^2$

Use **Torque/Kinematics** to answer the following question.

If the yo-yo starts from rest, what is its angular velocity after a length of string, d, is unwound?

FBD

NSL

$\Sigma F = ma$

$\boxed{mg - F_T = ma} \quad ①$

$\Sigma T = I\alpha$

$\boxed{F_T R_2 - mg(0) = I\alpha} \quad ②$
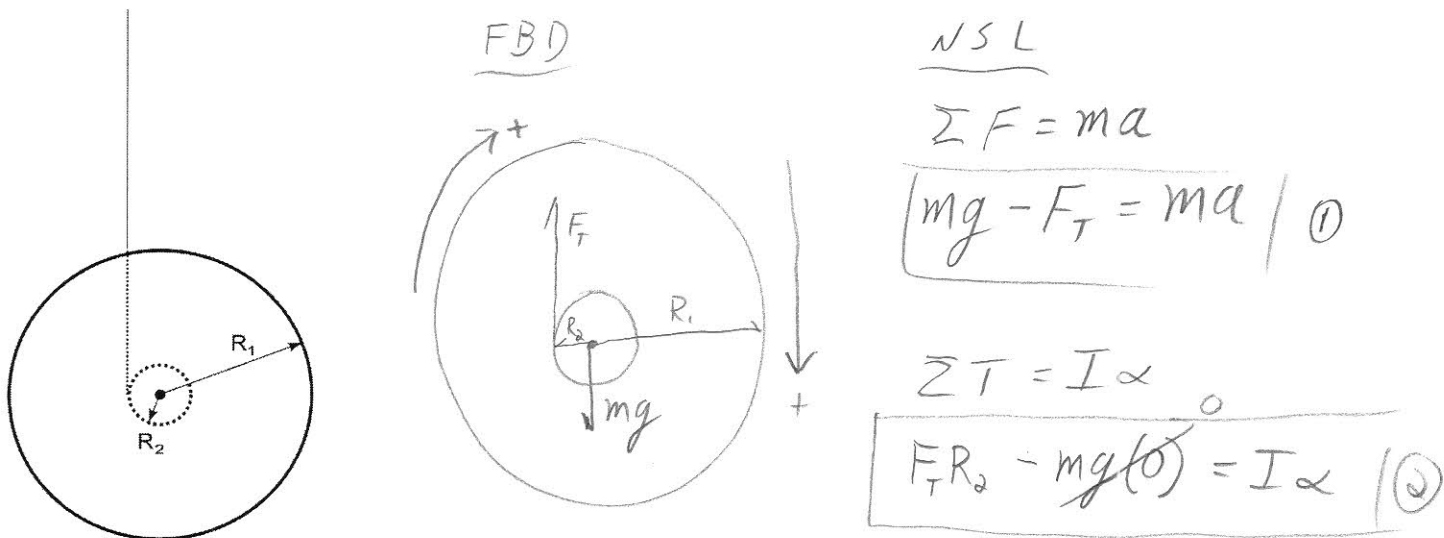
We need the linear acceleration, a, of the yo-yo. Let's eliminate F_T from equations ① and ②, then solve for a:

From ①: $F_T = mg - ma$

into ②: $\boxed{(mg - ma)R_2 = I\alpha} \quad ③$

Now, α and a are related through $\underline{R_2}$ (not R_1).

$a = R_2 \alpha$ (why? Think about it ...)

So: $(mg - ma) = \dfrac{I}{R_2}\dfrac{a}{R_2} \Rightarrow mg = \left(m + \dfrac{I}{R_2^2}\right)a \Rightarrow \boxed{a = \dfrac{m}{m + I/R_2^2}\, g}$

Plug in I: $a = \dfrac{m}{m + \frac{1}{2}M\frac{R_1^2}{R_2^2}} \Rightarrow a = \dfrac{1}{1 + \frac{1}{2}\frac{R_1^2}{R_2^2}}\, g$ continued

$$a = \frac{R^2}{\frac{1}{2}R_1{}^2 + R_2{}^2}\, g \quad \boxed{} \quad ④$$

Now, kinematics will provide the linear _velocity_ after traveling a distance d.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \qquad\qquad v = v_0 + at$$

$$d = 0 + 0 + \tfrac{1}{2}at^2 \qquad\qquad v = 0 + at$$

$$\Rightarrow d = \tfrac{1}{2}a\frac{v^2}{a^2} \Rightarrow \boxed{v = (2da)^{1/2}} \qquad\qquad \Rightarrow \boxed{t = \frac{v}{a}}$$

Now; plug in a:

$$\boxed{v = \left[2d\frac{R_2{}^2}{\frac{1}{2}R_1{}^2 + R_2{}^2}g\right]^{1/2}}$$

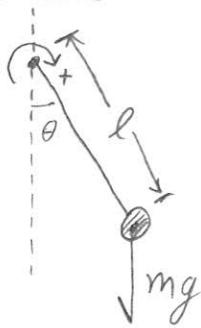And finally, convert to angular velocity: $v = R_2\omega$

$$\Rightarrow \omega = \frac{1}{R_2}\left[2dg\frac{R_2{}^2}{\frac{1}{2}R_1{}^2 + R_2{}^2}\right]^{1/2}$$

$$\boxed{\omega = \left[\frac{2dg}{\frac{1}{2}R_1{}^2 + R_2{}^2}\right]^{1/2}}$$

A small 0.75 kg ball is attached to one end of a 1.25 m long massless rod. The other end of the rod is hung from a pivot under the influence of gravity. When the resulting pendulum is 30^0 from vertical:

a) What is the magnitude of the torque about the pivot? (Draw a free body diagram)
b) What is the instantaneous angular acceleration (the acceleration at the moment when the pendulum is released)?
c) Can we use kinematics to find the angular velocity of the pendulum at the bottom of its swing? Why or why not?

FBD

NSL

$$\Sigma T = I\alpha$$

$$mg\ell SIN\theta = I\alpha$$

a) $$|\vec{T}| = |\vec{r} \times \vec{F}| = rF SIN\theta = \underline{\ell mg SIN\theta}$$

b) From above: $$mg\ell SIN\theta = I\alpha, \quad I = m\ell^2$$

$$\Rightarrow mg\ell SIN\theta = m\ell^2 \alpha$$

$$\Rightarrow \boxed{\alpha = \frac{g}{\ell} SIN\theta}$$

c) <u>No!</u> α is not constant, it depends on θ

Consider, once again, the clock pendulum pictured below. The disk has a mass m and radius r and the bar has mass M and length L.

a) Calculate the moment of inertia of the pendulum (or look it up if you've already calculated it!).
b) Calculate the center of mass of the pendulum *as measured from the axis of rotation*.
c) Assuming that the pendulum is pivoted so that it makes an angle θ with the vertical, use **Torque** to calculate the angular acceleration as a function of θ, l_{cm}, and I.
d) Assume that we then release the pendulum from rest. Can we use Kinematics to find the angular velocity of the pendulum when $\theta=0$? Why or why not?
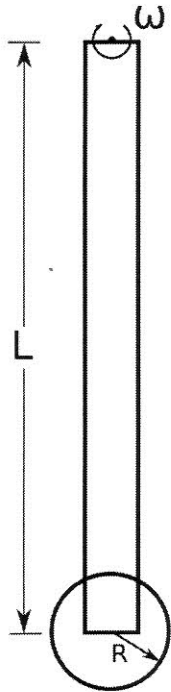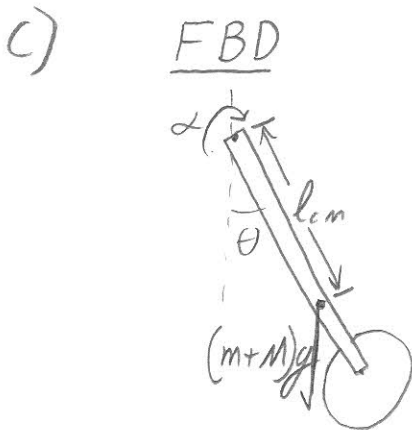
a) $I = I_R + I_D$

$I = \left[I_{cm,R} + Md_1^2 \right] + \left[I_{cm,D} + md_2^2 \right]$

$I = \left[\tfrac{1}{12}ML^2 + M\tfrac{L^2}{4} \right] + \left[\tfrac{1}{2}mR^2 + mL^2 \right]$

$I = \tfrac{1}{3}ML^2 + \tfrac{1}{2}mR^2 + mL^2$

b) $l_{cm} = \dfrac{M\tfrac{L}{2} + mL}{M + m}$

c) FBD

NSL

$\sum T = I\alpha \implies (m+M)g\, l_{cm} \sin\theta = I\alpha$

$\implies \alpha = \dfrac{m+M}{I} l_{cm}\, g \sin\theta$

d) **No!** α is not constant.
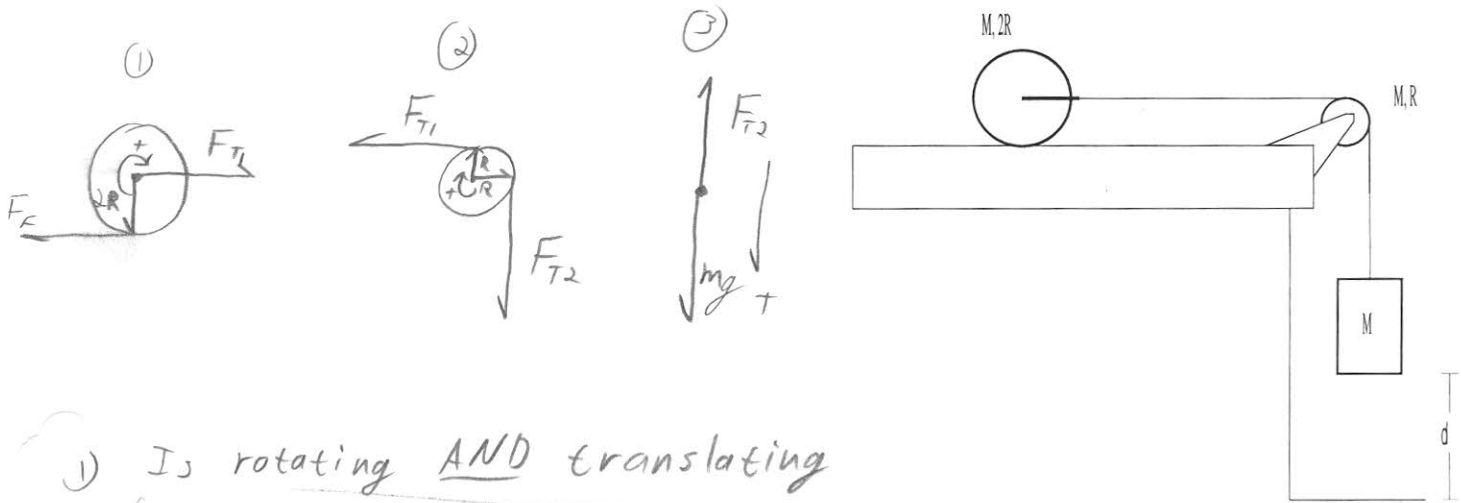
Use **Torque and Newton's Second Law** solve this problem.

A solid cylinder (radius = 2R, mass = M) rolls without slipping as it is pulled by a massless yoke attached to a string. The string goes over a frictionless pulley shaped as a solid disk (radius = R, mass = M) and is attached to a hanging weight (mass = M).

$$I_{cylinder} = \frac{1}{2}MR^2$$

What is the acceleration of the system?

1) Is rotating AND translating

$$-2\cancel{(0)}F_{T_1} + 2RF_F = \frac{1}{2}m4R^2\alpha \quad \rightarrow \text{Torque}$$

$$F_F = mR\alpha$$

$$F_{T_1} - F_F = ma \quad \rightarrow \text{Force}$$

Put these two together

$$F_{T_1} - mR\alpha = ma$$

$$\Rightarrow F_{T_1} - \frac{1}{2}ma = ma$$

$$\boxed{F_{T_1} = \frac{3}{2}ma} \quad ①$$

continued

↓

Rotation Set 5, P3 continued

② This pulley is only rotating

$$RF_{T2} - RF_{T1} = \frac{1}{2}mR^2\alpha \qquad a = v\alpha$$

$$\boxed{F_{T2} - F_{T1} = \frac{1}{2}ma} \quad ②$$

③ The mass is translating

$$mg - F_{T2} = ma$$

$$\Rightarrow \boxed{F_{T2} = m(g-a)} \quad ③$$

Plug ① and ③ into ②

$$m(g-a) - \frac{3}{2}ma = \frac{1}{2}ma$$

$$g - a - \frac{3}{2}a = \frac{1}{2}a$$

$$g = (1 + \frac{3}{2} + \frac{1}{2})a$$

$$\Rightarrow \boxed{a = \frac{1}{3}g}$$

Rotation – Set 5

⟡ 4

Use **Torque and Newton's Second Law** solve this problem.

A block of mass M rests on a rough table with $\mu_k = 0.3$. A massless string is attached to the block, wrapped around a solid cylinder having a mass M and a radius R, runs over a massless frictionless pulley, and is attached to a second block of mass M that is hanging freely.

Find the acceleration of this system.
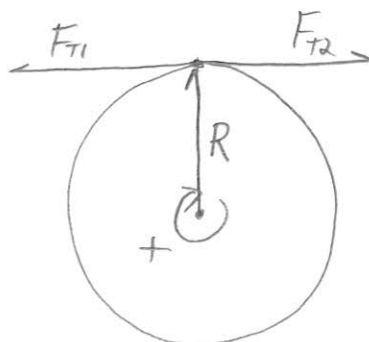
$$I_{cylinder} = \frac{1}{2}MR^2$$
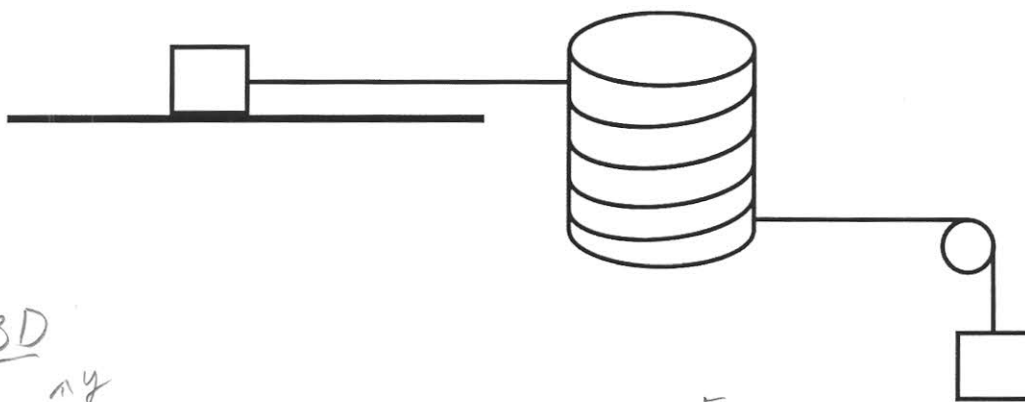
FBD
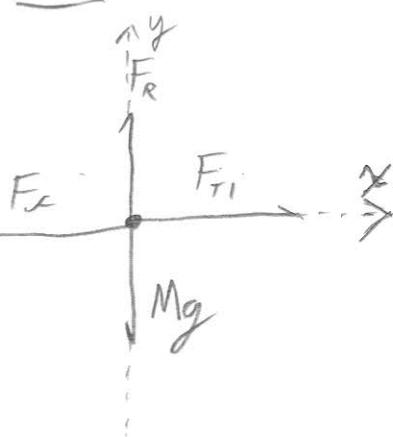
NSL $\sum \vec{F} = ma$

x: $F_{T1} - F_f = Ma$

y: $F_R - Mg = 0$

$F_R = Mg$

$\Rightarrow \boxed{F_{T1} - \mu_k Mg = Ma} \ ①$

$\sum \vec{T} = I\vec{\alpha}$

$\sum RF \sin\theta = I\alpha$

$RF_{T2} - RF_{T1} = \frac{1}{2}MR^2\alpha$

$\boxed{F_{T2} - F_{T1} = \frac{1}{2}MR\alpha} \ ②$

$\sum \vec{F} = m\vec{a}$

$\boxed{-F_{T2} + Mg = Ma} \ ③$

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Rotation Set 5, P4 continued

Eliminate F_{T_1} and F_{T_2}:

From ① : $F_{T_1} = \mu_K Mg + Ma$

From ③ : $F_{T_2} = Mg - Ma$

into ② : $\cancel{Mg} - \cancel{Ma} - \mu_K \cancel{Mg} - \cancel{Ma} = \frac{1}{2} \cancel{M} R\alpha$

$$g(1 - \mu_K) = 2a + \frac{1}{2} R\alpha \implies g(1 - \mu_K) = 2R\alpha + \frac{1}{2} R\alpha$$

$$\implies g(1 - \mu_K) = \frac{5}{2} R\alpha$$

$$\implies \boxed{\alpha = \frac{2}{5(1 - \mu_K)} \frac{g}{R}}$$

or $\boxed{a = \frac{2}{5(1 - \mu_K)} g}$