SAMPLE TEST 5

PHYS 111, FALL 2010, SECTION 1

1) Derivations

a) (10pts) Starting with the definition of linear Kinetic energy ($K=\frac{1}{2}mV^2$), show that rotational kinetic energy of a rigid body is $K=\frac{1}{2}I\omega^2$ where $I=\int r^2 dm$.

b) (10pts) Starting with the definition of angular momentum ($L=m|\vec{r}\times\vec{V}|$), show that the angular momentum of a rigid body is $L=I\omega$ where $I=\int r^2 dm$.

Proofs given in another post.

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2) Multiple Choice, 4 points each.

2.1) A wad of clay with mass m_c is thrown at a **thin rod** whose length is L and whose mass is $2m_c$. The rod is allowed to rotate about a pivot a distance $d = L/4$ from its center as in the picture below. What is the moment of inertia of the clay, stick combination after the impact?

a) $\frac{7}{6}m_c L^2$

b) $\boxed{\left(\frac{1}{6}+\frac{1}{8}+\frac{9}{16}\right)m_c L^2}$

c) $\left(\frac{1}{12}+\frac{9}{16}\right)m_c L^2$

d) $\pi m_c L^2$

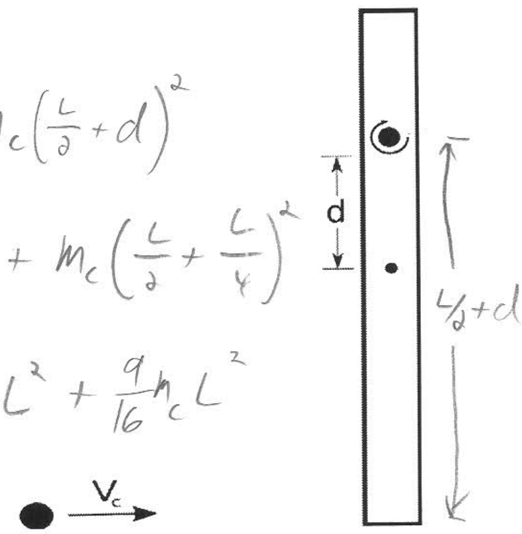
$I = I_R + I_c$

$= \left[I_{cmR} + M_R d^2\right] + m_c\left(\frac{L}{2}+d\right)^2$

$= \left[\frac{1}{12}M_R L^2 + M_R\left(\frac{L}{4}\right)^2\right] + m_c\left(\frac{L}{2}+\frac{L}{4}\right)^2$

$= \frac{1}{\cancel{12}_{6}}\cancel{2}m_c L^2 + \cancel{2}m_c\frac{1}{\cancel{16}_{8}}L^2 + \frac{9}{16}m_c L^2$

$= \left[\frac{1}{6}+\frac{1}{8}+\frac{9}{16}\right]m_c L^2$



2.2) A disk, a hoop, a solid sphere, and a hollow sphere, all with the same mass and radius, are having a race down an incline plane. Rank them in the order that they will arrive at the bottom of the ramp, 1 = winner, 4 = loser.

Because: $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$

τ is constant, α is inverse to I

so, rank in reverse I order

2 Disk 1/2

4 Hoop 1

1 Solid Sphere 2/5

3 Hollow Sphere 2/3

2.3) Some children are riding on the outside edge of a merry-go-round. All of the children simultaneously move towards the center. Ignore friction in the rotation of the merry-go-round. When they move:

a) the moment of inertia of the system stays constant.
b) the angular momentum of the system stays constant.
c) the angular velocity of the system stays constant.
d) the merry-go-round slows down.

No External Torque,

L = Constant

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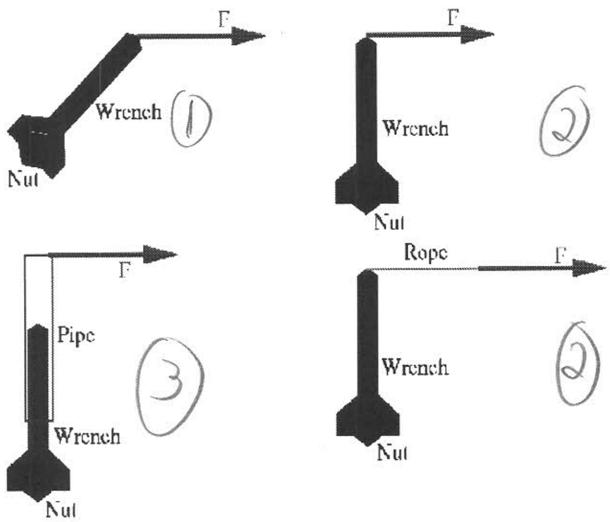
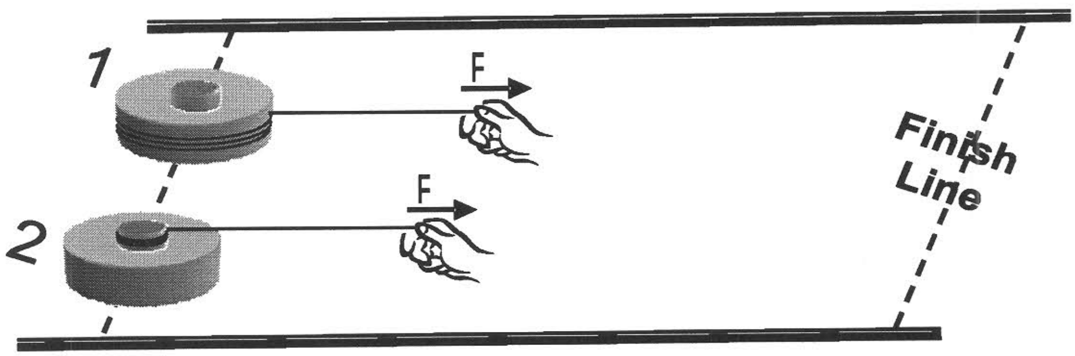
2.4) You are trying to turn a nut with a wrench. The same force is applied in each picture. Rank the pictures by torque, 1 = smallest. If any of the torques are the same, give them the same ranking.



2.5) Strings are wound around two identical pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force F. Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?

A) Puck 1
B) Puck 2
C) They arrive at the same time
D) There is not enough information to tell.



$F_T = ma$, $RF_T = I\alpha$

$\boxed{a} = \frac{F_T}{m}$ a is the same for both

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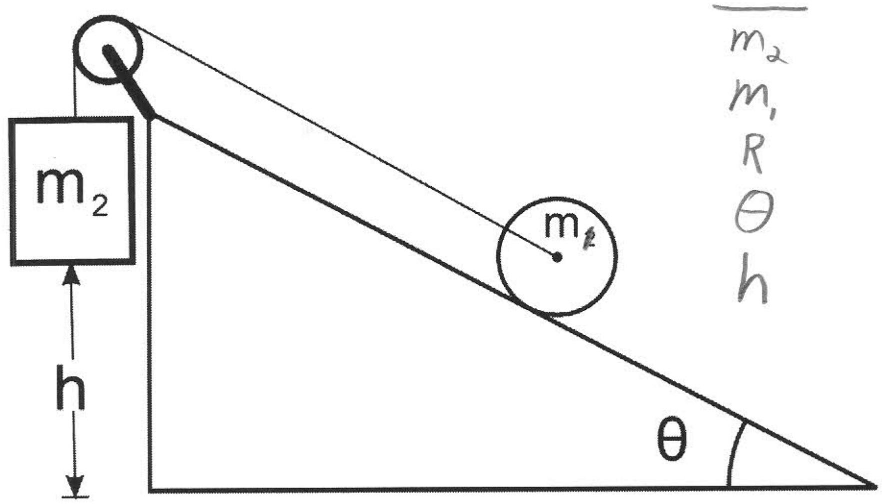
4) Mass m_2 is attached to a string that passes over a massless pulley. The other end of the string is attached to the central axle of a cylinder of mass m_1 and radius R. Assuming that $m_2 >> m_1$, the cylinder rolls without slipping up a slope that makes an angle θ with the horizontal.

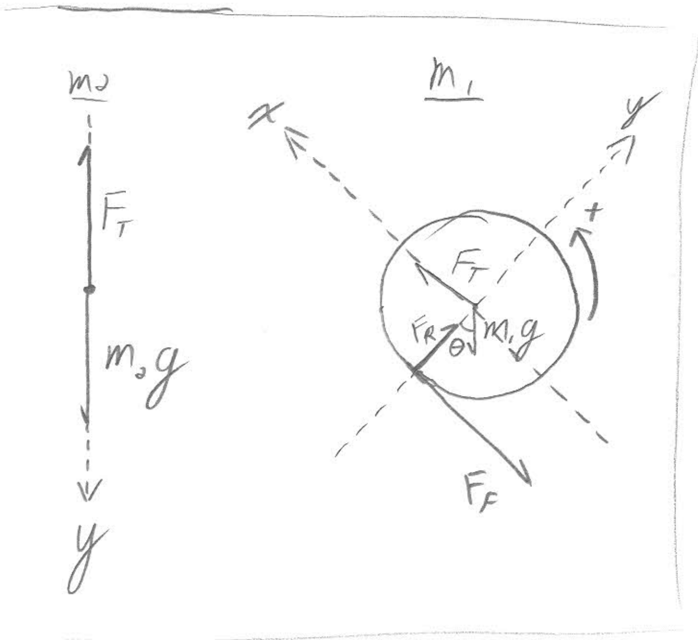
Assuming that the system starts from rest, use **Torque and Kinematics**, to find an expression for the velocity, v, of m_2 after it falls a distance h.



Given: m_2, m_1, R, θ, h

Want: v

FBD



NSL

m_2: $\boxed{m_2 g - F_T = m_2 a}$ ①

m_1: $\boxed{x: F_T - F_F - m_1 g \sin\theta = m_1 a}$ ②

$\boxed{y: F_R - m_1 g \cos\theta = 0}$ ③

Rot: $\boxed{F_F R = I\alpha}$ ④

continued

↓

Extra Space

* Eliminate F_f between eq ② and ④:

From ④: $F_f = \frac{I}{R}\alpha$

into ② $\boxed{F_T - \frac{I}{R}\alpha - m_1 g \sin\theta = m_1 a}$ ⑤

* Solve ① for F_T: $F_T = m_2 g - m_2 a$

into ⑤ $\boxed{m_2 g - m_2 a - \frac{I}{R}\alpha - m_1 g \sin\theta = m_1 a}$ ⑥

* Fix α using $a = R\alpha$ and solve ⑥ for a

$$\Rightarrow m_2 g - m_2 a - \frac{I}{R^2} a - m_1 g \sin\theta = m_1 a$$

$$\Rightarrow (m_2 - m_1 \sin\theta) g = (m_1 + m_2 + I/R^2)\, a$$

$$\Rightarrow \boxed{a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2 + I/R^2}\, g}$$

Kinematics

$$y = \cancel{y_0}^{\,0} + \cancel{v_{0y}}^{\,0} + \tfrac{1}{2} a_y t^2$$

$$v = \cancel{v_0}^{\,0} + a_y t$$

$$h = \tfrac{1}{2} a_y t^2$$

$$t = \frac{v}{a_y}$$

$$\boxed{v = \sqrt{2ha}} \Rightarrow \boxed{v = \left[2h \frac{m_2 - m_1 \sin\theta}{m_1 + m_2 + I/R^2}\, g\right]^{1/2}}$$

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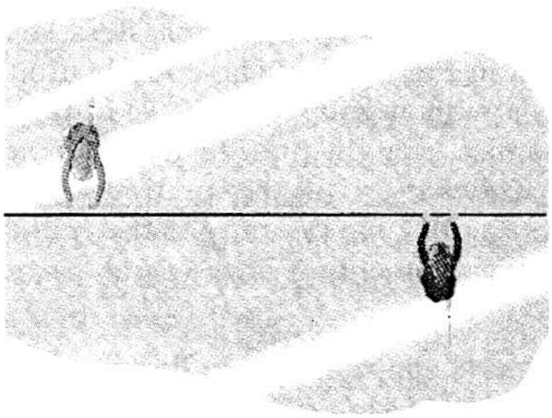
4) Two skaters, each with a mass of 50 kg, approach each other along parallel paths separated by 3.0 m. They have equal and opposite velocities of 1.4 m/s. The first skater is holding one end of a long pole with negligible mass. As the skaters pass, the second skater grabs the other end of the pole. Assume that the ice is completely frictionless.

a) What is the moment of inertia about the center of mass of the resulting skater-pole system?
b) What is the resulting angular velocity of the skater-pole system?

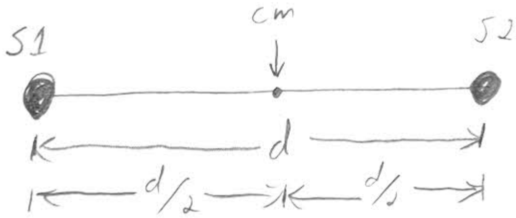


$m = 50\ kg$

$v = 1.4\ m/s$

$d = 3.0\ m$

a)



$$I = \sum m_i r_i^2$$

$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \boxed{\tfrac{1}{2} m d^2}$$

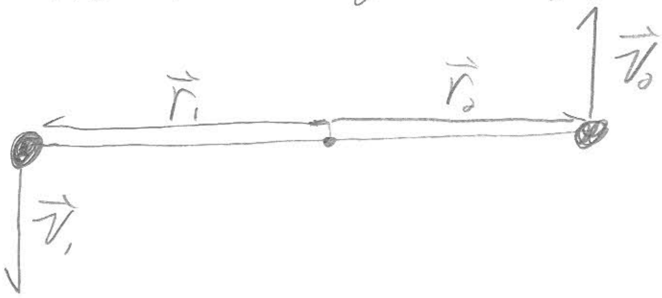
b) $L_I = L_F$

The skaters will rotate about the center of mass. But, prior to grabbing the pole, They are not a rigid body.

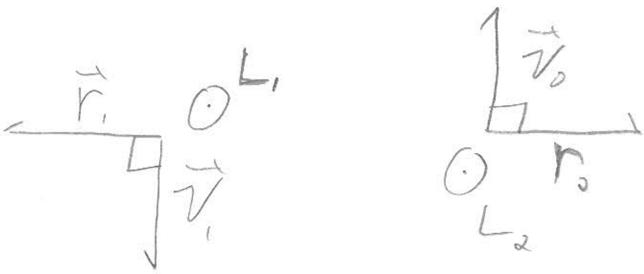
$$\vec{r}_1 \times m\vec{v}_1 + \vec{r}_2 \times m\vec{v}_2 = I\omega$$

continued ↓

At the instant of pole grabbing, it looks like this:



$\vec{r}_1 \times \vec{v}_1$ and $\vec{r}_2 \times \vec{v}_2$ give the same sign for $\vec{L}$



Both are out of the page.

So:

$$m r_1 v_1 + m r_2 v_2 = I\omega, \quad r_1 = r_2 = \frac{d}{2}$$

$$v_1 = v_2 = v$$

$$\frac{1}{2} m d v + \frac{1}{2} m d v = I\omega$$

$$m d v = I\omega \Rightarrow \omega = \frac{m d v}{I}$$

And plug in I from part a:

$$\omega = \frac{\cancel{m} d v}{\frac{1}{2}\cancel{m} d^{\cancel{2}}} \Rightarrow \boxed{\omega = 2\frac{v}{d}}$$

$$\boxed{\omega = 2\,\frac{1.4}{3} = 0.27 \text{ rad/s}}$$