

SAMPLE TEST 5

PHYS 111, FALL 2010, SECTION 1

1) Derivations

a) (10pts) Starting with the definition of linear Kinetic energy ($K = \frac{1}{2}mV^2$), show that rotational kinetic

energy of a rigid body is $K = \frac{1}{2}I\omega^2$ where $I = \int r^2 dm$.

b) (10pts) Starting with the definition of angular momentum ($L = m(\vec{r} \times \vec{V})$), show that the angular momentum of a rigid body is $L = I\omega$ where $I = \int r^2 dm$.

Proofs given in another post.

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2) Multiple Choice, 4 points each.

2.1) A wad of clay with mass m_c is thrown at a **thin rod** whose length is L and whose mass is $2m_c$. The rod is allowed to rotate about a pivot a distance $d = L/4$ from its center as in the picture below. What is the moment of inertia of the clay, stick combination after the impact?

a) $\frac{7}{6}m_c L^2$

b) $\left(\frac{1}{6} + \frac{1}{8} + \frac{9}{16}\right)m_c L^2$

c) $\left(\frac{1}{12} + \frac{9}{16}\right)m_c L^2$

d) $\pi m_c L^2$

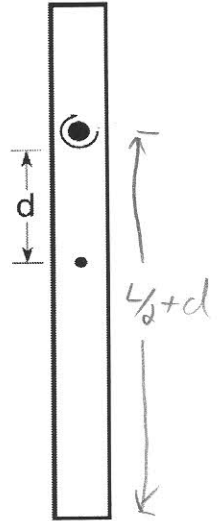
$$I = I_R + I_c$$

$$= [I_{cmR} + M_R d^2] + m_c \left(\frac{L}{2} + d\right)^2$$

$$= \left[\frac{1}{12} M_R L^2 + M_R \left(\frac{L}{4}\right)^2 + m_c \left(\frac{L}{2} + \frac{L}{4}\right)^2\right]$$

$$= \frac{1}{12} 2m_c L^2 + 2m_c \frac{1}{16} L^2 + \frac{9}{16} m_c L^2$$

$$= \left[\frac{1}{6} + \frac{1}{8} + \frac{9}{16}\right] m_c L^2$$



2.2) A disk, a hoop, a solid sphere, and a hollow sphere, all with the same mass and radius, are having a race down an incline plane. Rank them in the order that they will arrive at the bottom of the ramp, 1 = winner, 4 = loser.

2 Disk $\frac{1}{2}$

4 Hoop 1

1 Solid Sphere $\frac{2}{5}$

3 Hollow Sphere $\frac{2}{3}$

Because: $T = I\alpha = \alpha = \frac{T}{I}$

T is constant, α is inverse to I

so, rank in reverse I order

2.3) Some children are riding on the outside edge of a merry-go-round. All of the children simultaneously move towards the center. Ignore friction in the rotation of the merry-go-round. When they move:

- a) the moment of inertia of the system stays constant.
- b) the angular momentum of the system stays constant.
- c) the angular velocity of the system stays constant.
- d) the merry-go-round slows down.

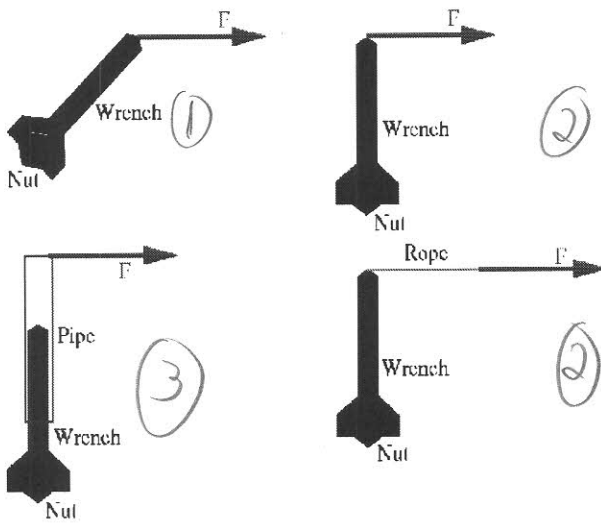
No External Torque,

$$L = \text{constant}$$

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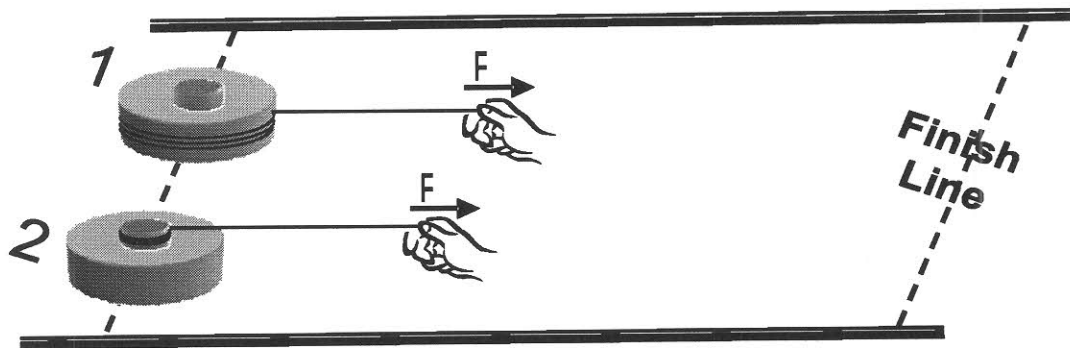
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2.4) You are trying to turn a nut with a wrench. The same force is applied in each picture. Rank the pictures by torque, 1 = smallest. If any of the torques are the same, give them the same ranking.



2.5) Strings are wound around two identical pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force F . Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?

- A) Puck 1
- B) Puck 2
- C) They arrive at the same time
- D) There is not enough information to tell.



$$F_T = ma$$

$$RF_T = I\alpha$$

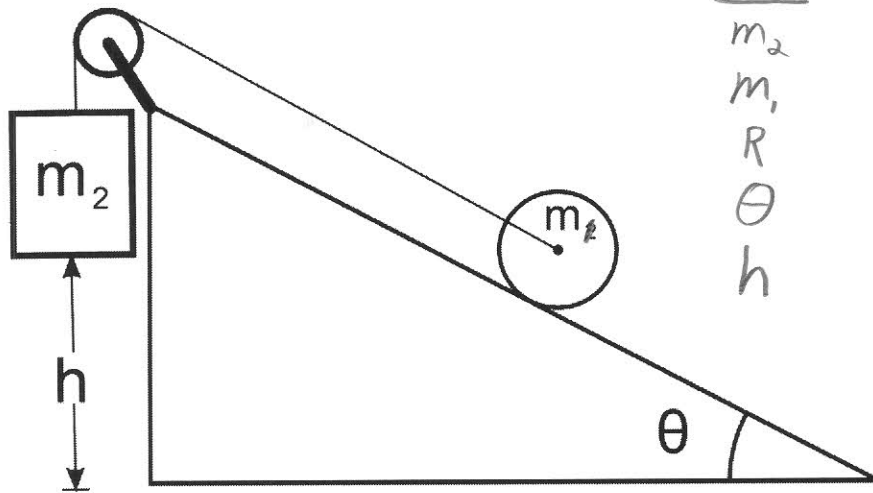
$$|a| = \frac{F_T}{m}$$

a is the same for both

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- 4) Mass m_2 is attached to a string that passes over a massless pulley. The other end of the string is attached to the central axle of a cylinder of mass m_1 and radius R . Assuming that $m_2 \gg m_1$, the cylinder rolls without slipping up a slope that makes an angle θ with the horizontal.

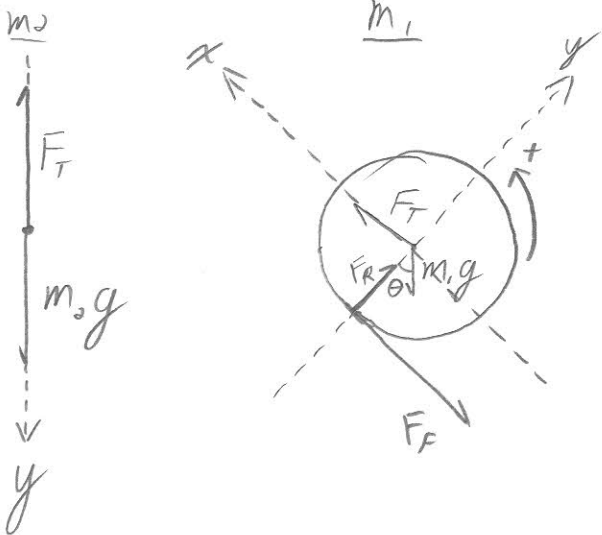
Assuming that the system starts from rest, use **Torque and Kinematics**, to find an expression for the velocity, v , of m_2 after it falls a distance h .



Given
 m_2
 m_1
 R
 θ
 h

Want
 v

FBD



NSL

$$m_2: m_2 g - F_T = m_2 a \quad (1)$$

$$m_1: x: F_T - F_F - m_1 g \sin \theta = m_1 a \quad (2)$$

$$y: F_R - m_1 g \cos \theta = 0 \quad (3)$$

$$\text{Rot: } F_F R = I \alpha \quad (4)$$

continued
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Extra Space

* Eliminate F_f between eq (3) and (4):

From (4): $F_f = \frac{I}{R} \alpha$

into (2) $\boxed{F_T - \frac{I}{R} \alpha - m_1 g \sin \theta = m_1 a} \quad (5)$

* Solve (1) for F_T : $F_T = m_2 g - m_2 a$

into (5) $\boxed{m_2 g - m_2 a - \frac{I}{R} \alpha - m_1 g \sin \theta = m_1 a} \quad (6)$

* Fix α using $a = R\alpha$ and solve (6) for a

$$\Rightarrow m_2 g - m_2 a - \frac{I}{R^2} a - m_1 g \sin \theta = m_1 a$$

$$\Rightarrow (m_2 - m_1 \sin \theta) g = (m_1 + m_2 + \frac{I}{R^2}) a$$

$$\Rightarrow \boxed{a = \frac{m_2 - m_1 \sin \theta}{m_1 + m_2 + \frac{I}{R^2}} g}$$

Kinematics

$$y = y_0^0 + v_{y0}^0 + \frac{1}{2} a_y t^2$$

$$h = \frac{1}{2} a_y t^2$$

$$v = v_0^0 + a_y t$$

$$t = \frac{v}{a_y}$$

$$\boxed{v = \sqrt{2 h a}} \Rightarrow$$

$$\boxed{v = \left[2 h \frac{m_2 - m_1 \sin \theta}{m_1 + m_2 + \frac{I}{R^2}} g \right]^{\frac{1}{2}}}$$