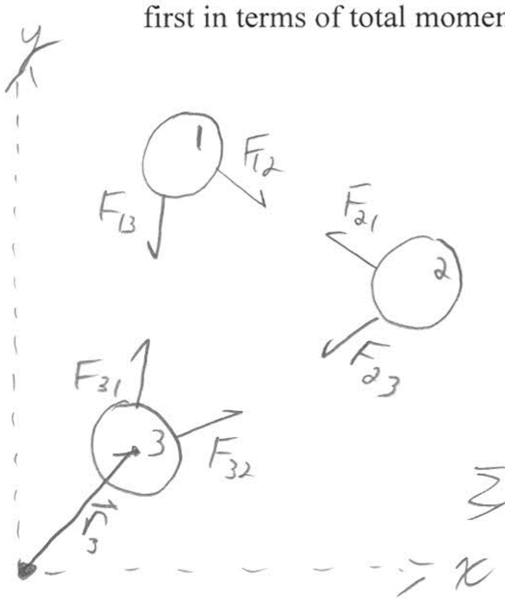
SAMPLE TEST 4
PHYS 111

Name: ______________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

1) (15pts) Starting with Newton's Second Law for a single particle, derive Newton's Second Law for systems first in terms of total momentum and again in terms of the acceleration of the center of mass.



Single particle: $\vec{F}_{net} = d\vec{P}/dt$

System:

$$\sum \vec{F} = \sum \frac{d\vec{P}}{dt}$$

$$\underbrace{\cancel{\sum \vec{F}_{int}}}_{0} + \sum \vec{F}_{ext} = \frac{d\sum \vec{P}_i}{dt} \Rightarrow \boxed{\sum \vec{F}_{ext} = \frac{d\sum \vec{P}_i}{dt}}$$

But: $\sum \vec{P}_i = \sum m_i \vec{v}_i = \sum m_i \frac{d\vec{r}_i}{dt}$

$$\Rightarrow \sum \vec{F}_{ext} = \frac{d}{dt}\sum m_i \frac{d\vec{r}_i}{dt} = \frac{d^2}{dt^2}\sum m_i \vec{r}_i = \frac{\sum m_i}{\sum m_i}\frac{d^2}{dt^2}\sum m_i \vec{r}_i$$

$$\Rightarrow \sum \vec{F}_{ext} = \sum m_i \frac{d^2}{dt^2}\frac{\sum m_i \vec{r}_i}{\sum m_i}, \quad \text{Let } M_T = \sum m_i \text{ and } \vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$\Rightarrow \sum \vec{F}_{ext} = M_T \frac{d^2 \vec{r}_{cm}}{dt^2} \Rightarrow \boxed{\sum \vec{F}_{ext} = M_T \vec{a}_{cm}}$$

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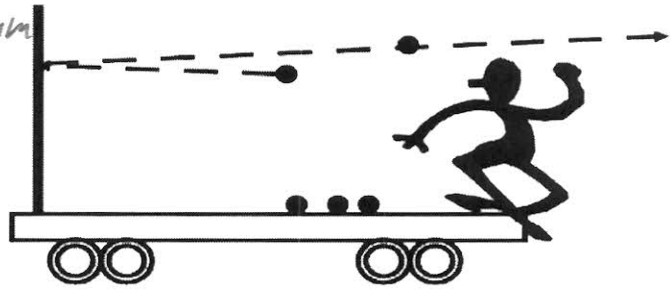
2) Multiple Choice Questions, 6 points each.

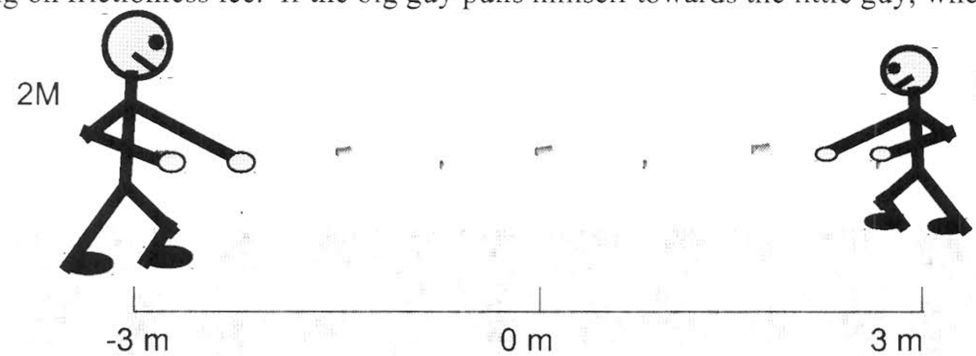
2.1) Suppose you are on a cart that is initially at rest on a frictionless track. You throw a ball at a vertical wall that is firmly attached to the cart. If the ball bounces straight back as shown in the picture, what direction will the cart move after the bounce?

Conserve Momentum



A) Left
B) Right
C) Up
D) It doesn't move.

2.2) A big guy, mass 2M, and a skinny guy, mass M, are holding opposite ends of a massless pole while standing on frictionless ice. If the big guy pulls himself towards the little guy, where will they meet?



A) -1 m
B) 0 m
C) 1 m
D) -3 m

Conserve $\vec{r}_{cm}$

$$\frac{(-3)(2M)+3M}{\cancel{2M+M}} = \frac{(2M+M)(x)}{\cancel{2M+M}}$$

$$\Rightarrow x = -\frac{3\cancel{M}}{3\cancel{M}} = -1$$

2.3) A compact car and a large truck collide head-on and stick together. Which vehicle undergoes the larger magnitude momentum change?

a. Car.
b. Truck.
c. Same for both.
d. Can't tell without knowing the final velocity of the wreck.

$\Delta P = \int \vec{F} dt$

same force, same t

$\Delta P_T = \Delta P_c$

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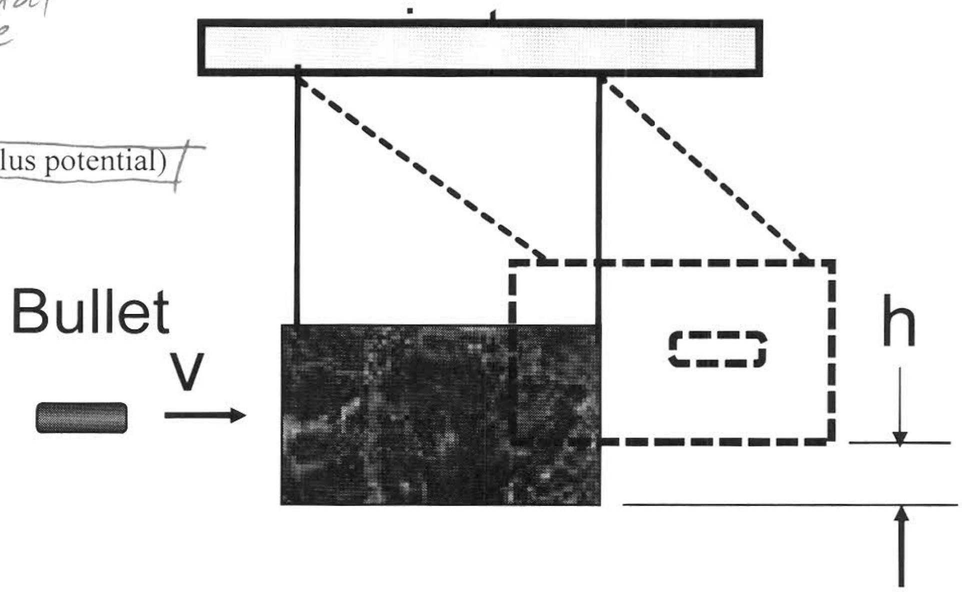
2.4) A bullet with an initial velocity v is fired into a block of wood attached to the ceiling as in the picture below. **While the block with the bullet stuck inside of it is swinging upward**, which of the following quantities is conserved?

It slows down

External Force

~~A) Momentum~~
~~B) Kinetic Energy~~
C) Total Energy (Kinetic plus potential)
D) A and B only
E) B and C only

$\frac{1}{2}mv^2 = mgh$



2.5) A car accelerates from rest. In doing so, the magnitude of the car's momentum changes by an amount Δp. At the same time, the magnitude of the Earth's momentum changes by:

A) a smaller amount
B) the same amount
C) a larger amount
D) depends on the time of year.

Earth/car system is closed

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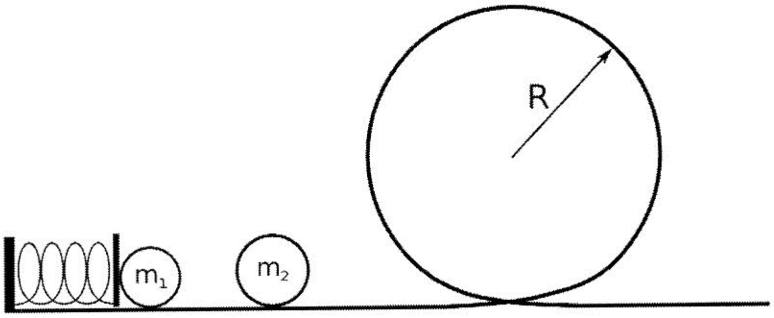
3) In the system below, a ball of mass m_1 is placed against a spring with spring constant k that has been compressed a distance d. It is released from rest and collides with a second ball of mass m_2 which then goes around the loop the loop of radius R.

Find an expression for the minimum spring compression d in terms of m_1, m_2, k, R, and g such that m_2 makes it around the loop.



Stage 1: Spring releases

$U_I = \frac{1}{2}kd^2$ $\qquad$ $U_F = 0$

$K_I = 0$ $\qquad$ $K_F = \frac{1}{2}m_1 v_1^2$

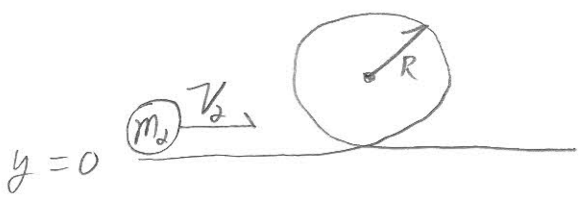
$$\cancel{\tfrac{1}{2}}kd^2 = \cancel{\tfrac{1}{2}}m_1 v_1^2 \Rightarrow \boxed{d = \sqrt{\frac{m_1}{k}}\, v_1} \quad (1)$$

Stage 2: Elastic collision

$$v_{2F} = \frac{m_2 - m_1}{\cancel{m_1 + m_2}} \cancel{v_{2I}}^{\,0} + \frac{2m_1}{m_1 + m_2} v_{1I}$$

$$\Rightarrow v_2 = \frac{2m_1}{m_1 + m_2} v_1 \Rightarrow \boxed{v_1 = \frac{m_1 + m_2}{2m_1} v_2} \quad (2)$$

Stage 3: Loop the Loop

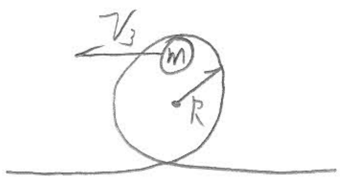


$U_I = 0$ $\qquad$ $U_F = m_2 g(2R)$

$K_I = \frac{1}{2} m_2 v_2^2$ $\qquad$ $K_F = \frac{1}{2} m_2 v_3^2$

$$\frac{1}{2} m_2 v_2^2 = 2 m_2 g R + \frac{1}{2} m_2 v_3^2$$

But, what is v_3 so that m_2 loops?



FBD

NSC

$\sum F = ma$

uniform circular motion

$$\Rightarrow m_2 g + \cancel{N}^{\,0} = m_2 \frac{v_3^2}{R}$$

minimum speed ... $N \to 0$

$$\Rightarrow \cancel{m_2} g = \cancel{m_2} \frac{v_3^2}{R}$$

$$\Rightarrow \boxed{v_3 = \sqrt{gR}}$$

So: $\frac{1}{2} \cancel{m_2} v_2^2 = 2 \cancel{m_2} g R + \frac{1}{2} \cancel{m_2} g R$

$$\Rightarrow \boxed{v_2 = \sqrt{5gR}}$$ 3

continued ↓

Sample Test 4, P3 continued

Put it all together

From ①: $d = \sqrt{\frac{m_1}{k}}\, v_1$

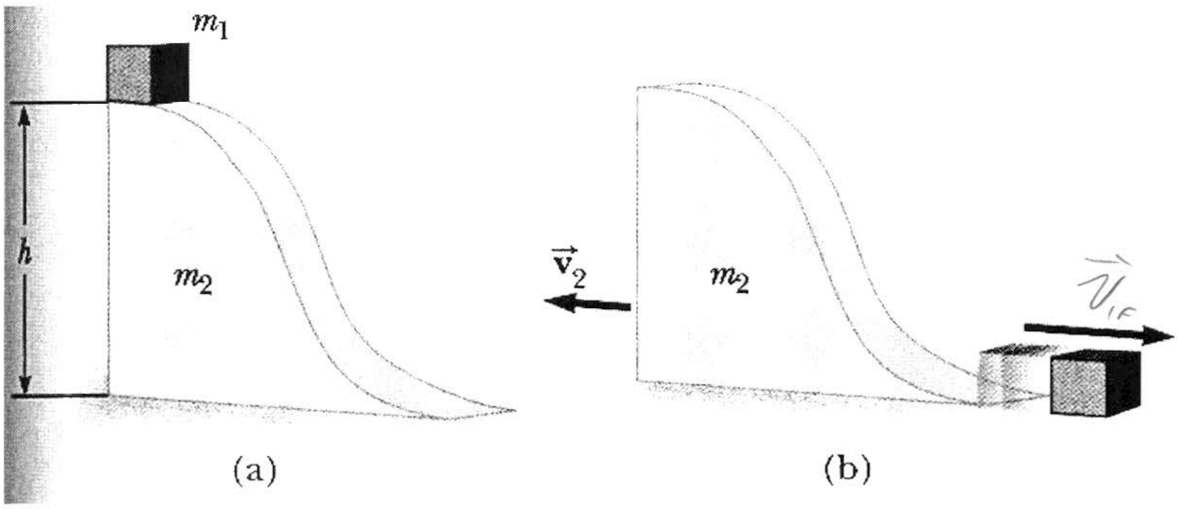
plug in ②: $d = \sqrt{\frac{m_1}{k}}\, \frac{m_1 + m_2}{2m_1}\, v_2$

plug in ③: $d = \sqrt{\frac{m_1}{k}}\, \frac{m_1 + m_2}{2m_1} \sqrt{5gR}$

$$\Rightarrow \boxed{d = \sqrt{\frac{5gR}{k}} \cdot \frac{m_1 + m_2}{2\sqrt{m_1}}}$$

4. A small block of mass m_1 is released from rest from the top of a curve-shaped frictionless wedge of mass m_2 that sits on a frictionless horizontal surface. After leaving the wedge, m_1 has a velocity v_1.

Find an expression for the height of the ramp, h, in terms of m_1, m_2, and v_1.



(a) (b)

Because there is no friction, $\sum \vec{F}_{ext} = 0$ and we can conserve momentum:

$$\cancel{m_1 v_{1I}}^{0} + \cancel{m_2 v_{2I}}^{0} = m_1 v_{1F} + m_2 v_{2F} \quad ①$$

We can also conserve Energy:

$$U_I + \cancel{K_I}^{0} + \cancel{W_{NCF}}^{0} = \cancel{U_F}^{0} + K_F$$

$$m_1 g h + 0 + 0 = 0 + \tfrac{1}{2} m_1 v_{1F}^2 + \tfrac{1}{2} m_2 v_{2F}^2 \quad ②$$

I will solve ① for v_{2F} and sub it into ②:

From ①: $v_{2F} = -\frac{m_1}{m_2} v_{1F}$

into ②: $\cancel{m_1} g h = \tfrac{1}{2} \cancel{m_1} v_{1F}^2 + \tfrac{1}{2} \cancel{m_2} \frac{m_1^{\cancel{2}}}{m_2^{\cancel{2}}} v_{1F}^2$

$$\Rightarrow \boxed{h = \frac{v_{1F}^2}{2g}\left[1 + \frac{m_1}{m_2}\right]}$$

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4. In many classic westerns, gunfighters fly backwards several meters after being shot, often crashing through windows or saloon doors. Assume that a typical bullet weights 2 g and that a typical cowboy weights 80 kg.

a) If the bullet leaves the gun at 200 m/s, what is the velocity of the cowboy/bullet system after the impact?

b) What velocity does the bullet need for the cowboy to slide 3 meters across the floor after being shot (assuming $U_k = 0.5$)?

a)

Given

$m_B = 2\times10^{-3}$ kg

$m_c = 80$ kg

$v_{BI} = 200$ m/s

$v_F = ?$

$$P_i = m_B v_{BI} + \cancel{m_c v_{CI}}^{\,0} \qquad P_F = (m_B + m_c) v_F$$

$$\Rightarrow \boxed{v_F = \frac{m_B}{(m_B + m_c)} v_{BI}}$$

$$v_F = \frac{2\times10^{-3}}{80.002}\cdot 200 = \boxed{0.4 \text{ m/s}}$$

b) Slide to a stop in a distance d

$$K_I = \tfrac{1}{2}(m_c + m_B) v_F^2 \qquad W_f = -\mu_k (m_c + m_B) g d$$

$$K_f = U_I = U_F = 0$$

$$\Rightarrow \tfrac{1}{2}\cancel{(m_c + m_B)} v_F^2 = \mu_k \cancel{(m_c + m_B)} g d$$

$$v_F = (2\mu_k g d)^{1/2}$$

continued

Sample Test 4, P4 continued

$$\Rightarrow \frac{m_B}{(m_c + m_B)} v_{BI} = (2\mu_k g d)^{1/2}$$

$$\Rightarrow \boxed{v_{BI} = \frac{m_c + m_B}{m_B}(2\mu_k g d)^{1/2}}$$

$$v_{BI} = \frac{80.002}{2\times 10^{-3}}\left((2)(0.5)(9.8)(3)\right)^{1/2}$$

$$= \boxed{7.4\times 10^{3}\ m/s} = 16{,}000\ \text{miles/hour}$$

For comparison, the M16 muzzle velocity is approximately 1,000 m/s or 2,200 miles/hour

So... Flying backwards is bogus...