Sample Test 3
Phys 111 Spring 2010

Name: ______________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

Show your work on all of the problems – your approach to the problem is as important as (if not MORE) important than) your final answer.

1) Starting with the definition of work, derive the **Work Energy Theorem**.

$W = \int \vec{F} \cdot d\vec{s}$, $\quad \vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$W = \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow W = \int (F_x\,dx + F_y\,dy + F_z\,dz) \Rightarrow \boxed{W = \int F_x\,dx + \int F_y\,dy + \int F_z\,dz}$$

$$\boxed{W_{net} = \Sigma W} = \Sigma\left(\int F_x\,dx + \int F_y\,dy + \int F_z\,dz\right)$$

$$\Rightarrow W_{net} = \Sigma\int F_x\,dx + \Sigma\int F_y\,dy + \Sigma\int F_z\,dz$$

$$\Rightarrow W_{net} = \int \Sigma F_x\,dx + \int \Sigma F_y\,dy + \int \Sigma F_z\,dz$$

Now: $\Sigma F_x = ma_x$ (NSL) so we can subst.

consider just the x-axis.

$$\int_{x_0}^{x} \Sigma F_x\,dx = \int_{x_0}^{x} ma_x\,dx = \int_{x_0}^{x} m\frac{dv_x}{dt}\,dx \overset{\text{change variable}}{=} \int_{v_{0x}}^{v_x} m\frac{dx}{dt}\,dv = \int_{v_{0x}}^{v_x} mv\,dv$$

$$= \frac{1}{2}mv_x^2 - \frac{1}{2}mv_{0x}^2 = \boxed{\Delta K_x}$$

So: $\boxed{W_{net} = \Delta K}$ QED.

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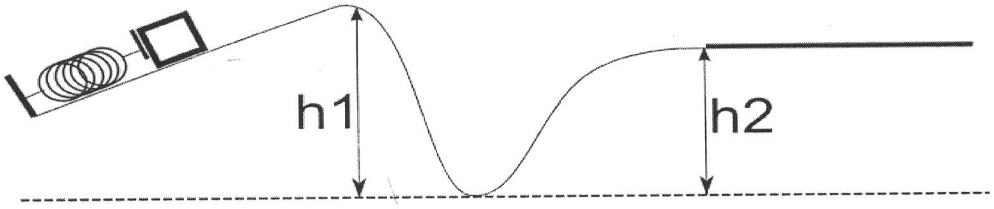
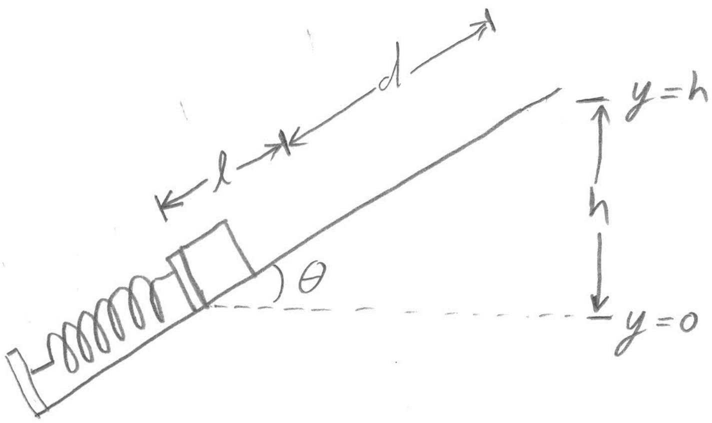
In the system below, the spring is compressed and a 3 kg block resting against it is released from rest. The surface is frictionless except for the dark horizontal surface at the end where the coefficient of friction is $\mu_k = 0.15$. The distance between the top of the *uncompressed* spring and the top of the ramp is 2 m. The spring constant is k = 200 N/m. The ramp makes a 30 degree angle with the horizontal, $h_1 = 3$ m, and $h_2 = 2$ m.

a) How much does the spring have to be compressed for the block to **just** clear the top of the ramp.

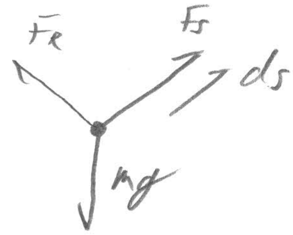
b) If the block **just** clears the top of the ramp (v is zero there but it just makes it over), how far across the frictional surface will it slide.



a)



ℓ = Spring compression
$d = 2m$
$\theta = 30°$
$m =$



Spring uncoils a dist. ℓ and block stops at the top.

$U_I = \underset{\text{spring}}{\tfrac{1}{2}k\ell^2} + \underset{\text{grav.}}{0}$ $\qquad$ $U_F = \underset{\text{spring}}{0} + \underset{\text{grav}}{mg(\ell+d)\sin\theta}$

$K_I = 0$ $\qquad$ $K_F = 0$

$W_{NCF} = 0$

$\tfrac{1}{2}k\ell^2 = mg(\ell+d)\sin\theta$ aww... Quadratic!

continued ↓

ST3, Spring Ramp continued

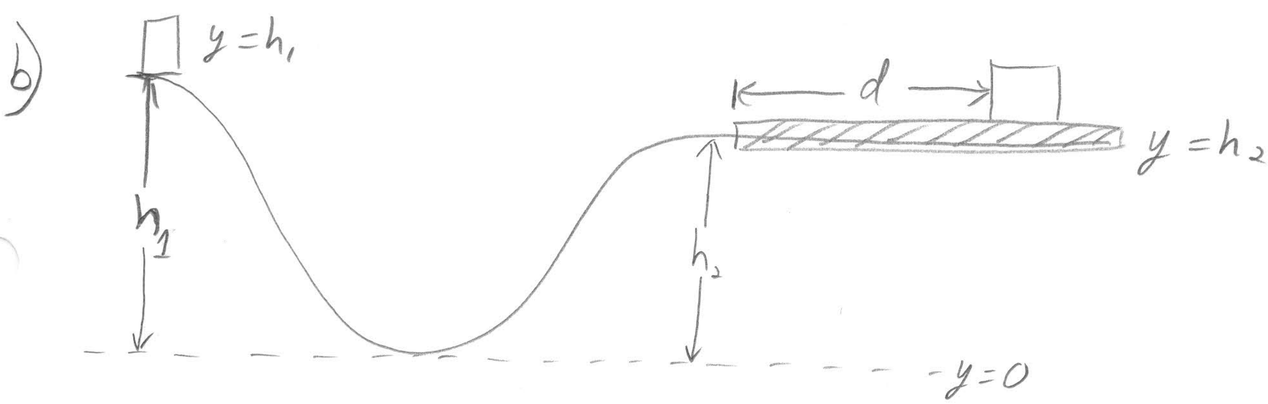
$$\tfrac{1}{2}k\ell^2 - mg\ell\sin\theta - mgd\sin\theta = 0$$

$$\boxed{\ell = \frac{1}{k}\left[mg\sin\theta \pm \left((mg\sin\theta)^2 + 2kmgd\sin\theta\right)\right]^{\frac{1}{2}}}$$

Choose positive root, otherwise ℓ is negative which is a stretch instead of a compress.

Stretching the right distance would give the correct energy, but the spring force would be the wrong direction.

b)



$U_I = mgh_1$ $\qquad$ $U_F = mgh_2$

$K_I = 0$ $\qquad$ $K_F = 0$

'Just' clears $\qquad$ Slides to a stop

$$W_f = \int \vec{F}_f \cdot d\vec{s} = \int_0^d -\mu_k mg\, dx = -\mu_k mgd$$

$$\cancel{mg}h_1 - \mu_k \cancel{mg}d = \cancel{mg}h_2 \Rightarrow \boxed{d = h_1 - h_2}$$

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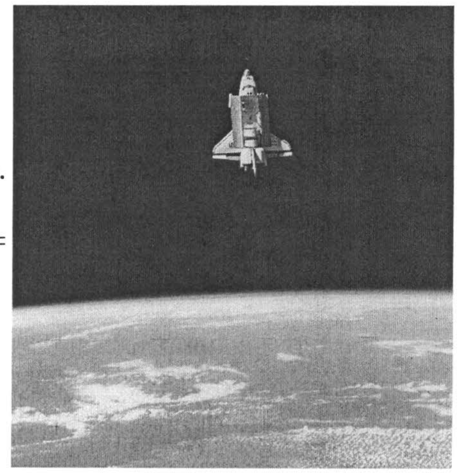


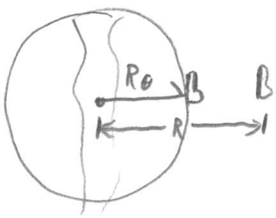
a) Derive a general expression for the amount of energy required, E, to put the space shuttle into an orbit of radius R. Ignore the Earth's rotation. (Let $K_i = 0$)

b) Now let's use the equation from part a to compare two orbital energies.

Let $R_1 = (1.05)R_{Earth}$ (the normal orbit of the space shuttle) and let $R_2 = \infty$. Show that, in terms of energy, low Earth orbit is halfway across the Universe.

In other words, show that: $\frac{E_1}{E_2} = 0.52$

a)



$$U_I = -\frac{GM_\oplus m_s}{R_\oplus} \qquad U_F = -\frac{GM_\oplus m_s}{R}$$

$$K_I = \emptyset \qquad K_F = \frac{1}{2}m_s v^2$$

What's v? have to use $F = ma$ to get it

$$F = ma$$

$$\frac{GM_\oplus \cancel{m_s}}{R^{\cancel{2}}} = \cancel{m_s}\frac{v^2}{\cancel{R}} \Rightarrow v = \sqrt{\frac{GM_\oplus}{R}}$$

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$-\frac{GM_\oplus m_s}{R_\oplus} + 0 + W_{NCF} = -\frac{GM_\oplus m_s}{R} + \frac{1}{2}\frac{GM_\oplus m_s}{R}$$

$$W_{NCF} = +\frac{GM_\oplus m_s}{R_\oplus} - \frac{1}{2}\frac{GM_\oplus m_s}{R}$$

Work by rocket engines

continued ↓

ST 3, Shuttle Orbit - continued.

$$W_{NCF} = \boxed{E = GM_{\oplus}m\left(\frac{1}{R_{\oplus}} - \frac{1}{2R}\right)}$$

Energy to Orbit

b) Compare two Energies

$$E_1 = GM_{\oplus}m_s\left(\frac{1}{R_{\oplus}} - \frac{1}{2(1.05)R_{\oplus}}\right)$$

$$= \frac{GM_{\oplus}m_s}{R_{\oplus}}\left(1 - \frac{1}{2.1}\right) = 0.52\,\frac{GM_{\oplus}m_s}{R_{\oplus}}$$

$$E_2 = GM_{\oplus}m_s\left(\frac{1}{R_{\oplus}} - \frac{1}{\infty}\right) = \frac{GM_{\oplus}m_s}{R_{\oplus}}$$

$$\frac{E_1}{E_2} = \frac{0.52\,\cancel{\frac{GM_{\oplus}m_s}{R_{\oplus}}}}{\cancel{\frac{GM_{\oplus}m_s}{R_{\oplus}}}} = \boxed{0.52}$$

SAMPLE TEST 3

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Use work-energy techniques to solve the following problem.

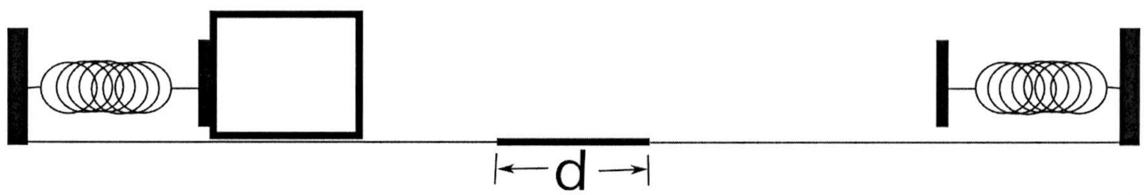
You are digging around in the physics stock room and find a box of springs. In attempting to measure the spring constants, you find one that doesn't seem to follow Hook's law. After extensive testing, you conclude that the its force law is quadratic and is of the form:

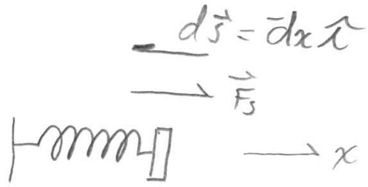
$$\vec{F} = -ax^2\hat{s}$$

where x is the displacement of the spring from equilibrium and $\hat{s}$ is a unit vector in the direction of the stretch.

a. Calculate a potential function for this spring.

b. A spring that follows Hook's with a spring constant k law is placed opposite the "strange" spring as in the figure below. The "strange" spring is compressed a distance l and a block of mass m is placed against it. All surfaces are frictionless except the dark patch between the two springs. How many times does the block cross the space between the springs?





a = 38.4 N/m²
l = 1.0 m
d = 0.5 m
m = 2 kg
k = 100 N/m
μ_k = 0.60

a) $U = -\int_l^0 \vec{F}_s \cdot d\vec{s}$

$= -\int_l^0 -(ax^2(-\hat{\imath}))\cdot(dx\hat{\imath})$ 4 minus signs! ug!

$\boxed{U = \frac{1}{3}ax^3}$ * But you know the pot. has to be positive...

b) We start with some initial potential from the spring compression. If there were no friction, that same potential would get swapped from spring 1 to kinetic to spring 2 to kinetic etc...

continued ↓

ST3, Two springs continued.

With Friction, Energy is lost at each pass until the block stops.

$U_I = \frac{1}{3}al^3$

$U_F = 0$ → no energy left in the end

$K_F = 0$

$K_I = 0$

not initially moving

Friction

$$W_{NCF} = \int_0^{nd} \vec{F}_F \cdot d\vec{s} = \int_0^{nd} -\mu_k mg\, dx = -\mu_k mgnd$$

where n = # of times across the frictiony patch.

$$U_I + K_I + W_{NCF} = U_F + K_F$$

$$\frac{1}{3}al^3 + 0 - \mu_k mgnd = 0$$

$$\Rightarrow \boxed{n = \frac{1}{3}\frac{al^3}{\mu_k mgd}} = \frac{1}{3}\frac{(38.4)(1.0)^3}{(0.6)(2)(9.8)(0.5)} =$$

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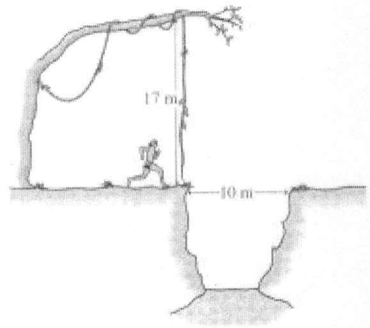
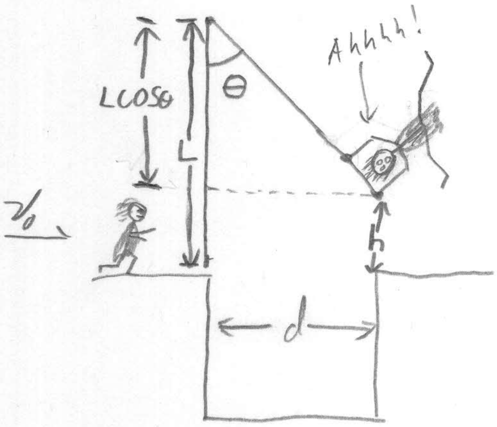
Use work-energy techniques to solve the following problem.

Tarzan is late for a date with Jane and is running as fast as he can to meet her. On the way, he has to get over a 10m wide pit of dangerous croc-a-gators. A 17m vine is hanging vertically from a tree at one side of the pit. Tarzan is going to run up, grab the vine, swing across, and drop vertically to the ground on the other side.

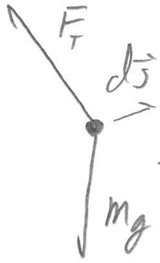


What must his minimum speed be to make it across?



$h = L - L\cos\theta$

$d = L\sin\theta$



$W_{F_T} = 0,\ \vec{F}_T \perp d\vec{s}$

some point mid swing

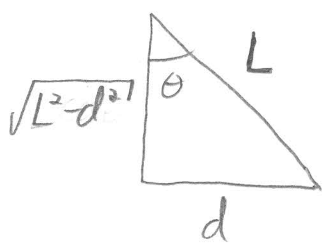
$U_I = 0$ $\qquad$ $U_F = mgL(1-\cos\theta)$

$K_I = \frac{1}{2}mv_0^2$ $\qquad$ $K_F = 0$ ← Stops to drop Vertically

$W_{NCF} = 0$

$\frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gL(1-\cos\theta)$

$v_0 = \left(2gL(1-\cos\theta)\right)^{1/2}$



$\cos\theta = \frac{L}{\sqrt{L^2-d^2}}$

$$\boxed{v_0 = \left(2gL\left(1 - \frac{L}{\sqrt{L^2-d^2}}\right)\right)^{1/2}}$$