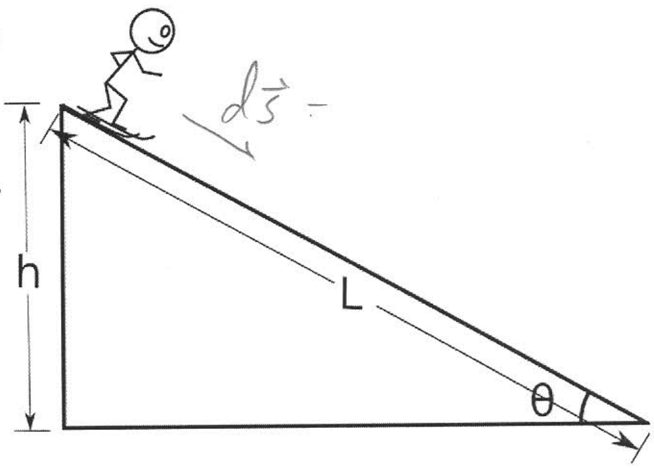
Energy Problems – Set 2

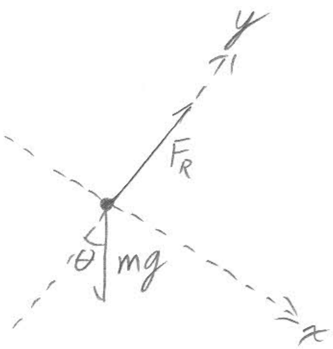
Stick Skier is back for another round. Because he knows Newton was a jerk, he wants you to calculate his velocity using this fancy new **W**ork **E**nery **T**heorem that he's heard so much about. He reluctantly tells you that he has a mass *m* (stick man is quite sensitive about his weight). The distance down the ski slope is *L* and it makes an angle θ with the horizontal. The top of the hill is a vertical distance *h* above the bottom of the hill.

a. Calculate the work done on the skier by each of the forces. (draw a free body diagram!)

b. Find an expression for the **total** work, W_{net}, done on the skier. Express your answer in terms of *m*, *g*, and *h* only.

c. Use the **W**ork **E**nergy **T**heorem to find the skier's speed, V_f, at the bottom of the hill.





$$W_{FR} = \int \vec{F}_R \cdot d\vec{s}$$

$$= \int F_R \hat{j} \cdot dx\hat{i} = 0$$

$$d\vec{s} = dx\hat{i}$$

$$m\vec{g} = mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}$$

$$\vec{F}_R = F_R\hat{j}$$

$$W_g = \int \vec{F}_g \cdot d\vec{s}$$

$$= \int \left(mg\sin\theta\,\hat{i} + \cancel{mg\cos\theta\,\hat{j}}^{\,0}\right)\cdot dx\hat{i}$$

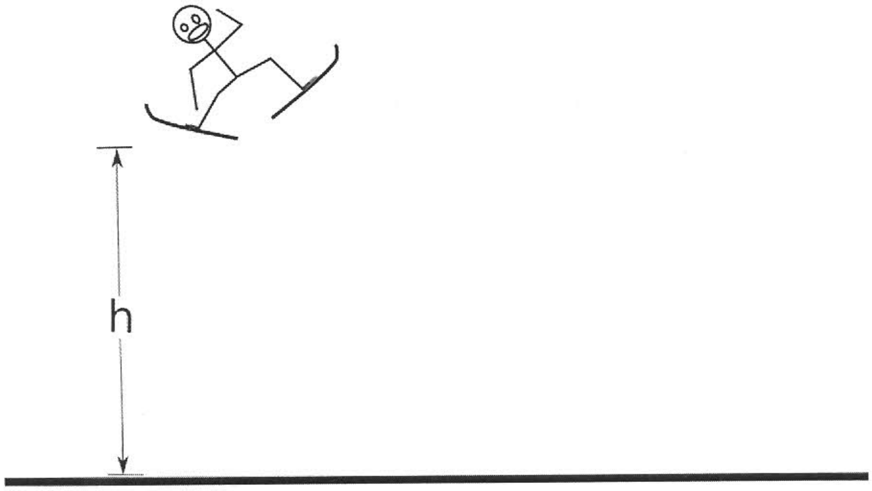
$$= mg\sin\theta \int_0^L dx = mgL\sin\theta = \underline{mgh}$$

$$\Rightarrow \boxed{W_{net} = W_g + W_{FR} = mgh}$$

$$W_{net} = \Delta K \Rightarrow \cancel{m}gh = \tfrac{1}{2}\cancel{m}v^2 \Rightarrow \boxed{v = \sqrt{2gh}}$$

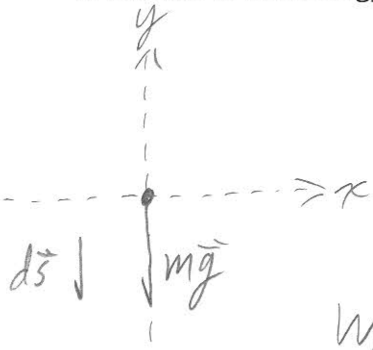
Energy Problems – Set 2

Stick skier won't stop talking about this awesome new video game that he got last week. His buddy gets so annoyed that he throws him off the mountain.



a. Calculate the net work done on stick skier and compare it to the net work from the previous question.

b. Use the **W**ork **E**nergy **T**heorem to find an expression for the final speed of the skier.



$$m\vec{g} = -mg\hat{j}$$

$$d\vec{s} = -dy\hat{j}$$

$$W_g = \int \vec{F}_g \cdot d\vec{s} = \int (-mg\hat{j})\cdot(-dy\hat{j}) = mg\int_0^h dy$$

$\boxed{W_g = mgh}$ och! Same as before!

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}v^2 \Rightarrow \boxed{v = \sqrt{2gh}}$$

has to move!

Energy Problems – Set 2

In order to do work on an object

$W = \int \vec{F} \cdot d\vec{s}$

a) it is necessary that friction be present.
b) it is necessary that friction not be present.
c) the object must move.
d) the applied force must be greater than the reaction force of the object.

A person drops one rock from rest and it falls 100 meters down a cliff before hitting the ground. The person then throws an identical rock downward.

The amount of kinetic energy gained by the rock dropped from rest is _______ that gained by the second rock (only consider the gain *after* the throw).

a) greater than
b) equal to
c) less than

Work done is mgh

The change in speed (the difference it's initial speed and it's speed just before hitting the ground)of the rock dropped from rest is _______ that of the second rock.

a) greater than
b) equal to
c) less than

The change in velocity (because it's squared) is greatest when $V_I = 0$

Three cars (car F, car G and car H) are moving with the same velocity, and slam on the brakes. The most massive car is F and the least massive is H.

Assuming all three have identical tires, for which car does friction do the most work?

a) Car F
b) Car G
c) Car H
d) Each car does the same friction work.

ΔK is greatest for most massive car.

Assuming all three have identical tires, which car travels the longest distance to skid to a stop?

a) Car F
b) Car G
c) Car H
d) They all travel the same distance.

$W = \Delta K$
$F_f d = \frac{1}{2} m V^2$

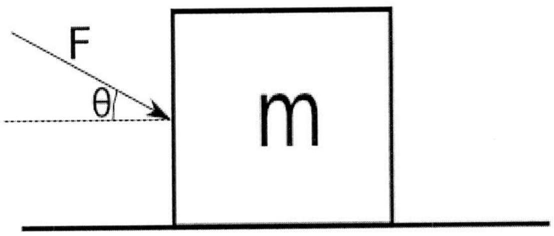
$\mu_s \cancel{m} g d = \frac{1}{2} \cancel{m} V^2 \rightarrow$ not dependant on m

Energy Problems – Set 2

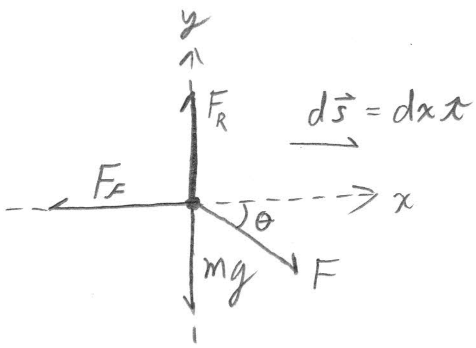
A box with mass *m*, initially at rest, is pushed a distance *d* along a surface with a force *F* making and angle θ with the horizontal. The coefficient of friction between the box and the surface is μ_k.

a) Find an expression for the final velocity of the box, V_f, using Work-Energy techniques.

b) Find an expression for final velocity of the box using Newton's Second Law and kinematics and show that the answer is the same.



a)



Find Frictional Force

NSL

$$\Sigma F_y = ma_y$$

$$y: \; F_R - mg - F\sin\theta = 0$$

$$\Rightarrow F_R = mg + F\sin\theta$$

$$\text{So: } F_f = \mu_s F_R \Rightarrow \boxed{F_f = \mu_s(mg + F\sin\theta)}$$

Calculate work

$$W_{F_R} = 0, \quad \vec{F_R} \perp d\vec{s}$$

$$W_g = 0, \quad \vec{g} \perp d\vec{s}$$

$$W_{F_f} = \int_0^d \vec{F_f} \cdot d\vec{s} = \int_0^d -\mu_s(mg + F\sin\theta)\hat{x} \cdot dx\,\hat{x} = -\mu_s(mg + F\sin\theta)d$$

$$W_F = \int_0^d (F\cos\theta\,\hat{x} - F\sin\theta\,\hat{y})(dx\,\hat{x}) = Fd\cos\theta$$

continued ↓

E2, P1 – continued

Apply WET

$$W_{net} = \Delta K$$

$$W_{F_R} + W_g + W_{F_f} + W_F = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m v_0^2$$

$$0 + 0 - \mu_s(mg + F\sin\theta)d + Fd\cos\theta = \tfrac{1}{2} m v^2$$

$$v^2 = \frac{2d}{m}\left[F\cos\theta - \mu_s mg - \mu_s F\sin\theta\right]$$

b) Use NSL and kinematics

NSL

$$x: F\cos\theta - \mu_s F_R = m a_x \quad ①$$

$$y: F_R - F\sin\theta - mg = 0$$

$$\Rightarrow F_R = F\sin\theta + mg \quad ②$$

Subst ② → ①:

$$F\cos\theta - \mu_s(F\sin\theta + mg) = ma$$

$$\Rightarrow a = \frac{1}{m}\left[F\cos\theta - \mu_s F\sin\theta - \mu_s mg\right] \quad ③$$

Kinematics

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$d = 0 + 0 + \tfrac{1}{2} a t^2 \quad ④$$

$$v = v_0 + at$$

$$v = 0 + at \Rightarrow t = \frac{v}{a} \quad ⑤$$

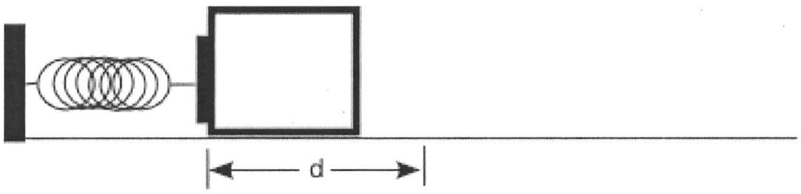
subst ⑤ → ④: $d = \tfrac{1}{2} \cancel{a} \dfrac{v^2}{a^{\cancel{2}}} \Rightarrow v^2 = 2da \quad ⑥$

subst ③ → ⑥: $\boxed{v^2 = \dfrac{2d}{m}\left[F\cos\theta - \mu_s F\sin\theta - \mu_s mg\right]}$

Springs exert a force that opposes being stretched or compressed. Hook's law states that the magnitude of the force exerted by the spring is not constant, but is proportional to the amount of compression/extension and in the opposite direction. Mathematically, it is written:

$$\vec{F}_s = -k\vec{x}$$

The spring constant, *k*, represents the strength of the spring and *x* is the displacement of the spring from it's equilibrium position (the position where it's not exerting any force). The negative sign indicates that the force opposes the displacement *x*.

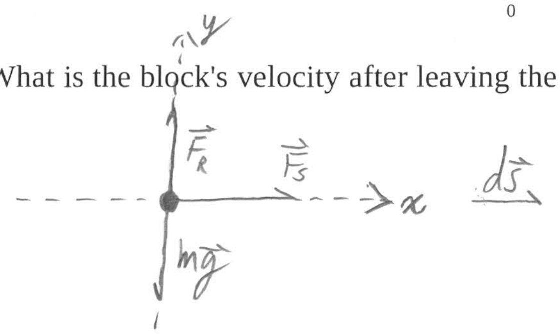


a) A block of mass m is pushed against a spring of spring constant *k* and the spring is compressed a distance *d*. Calculate the work done by the spring after it is released.

HINT: solve the following integral $W_s = \int_0^d \vec{F}_s \cdot (dx\,\hat{i})$

b) What is the block's velocity after leaving the spring?

a)



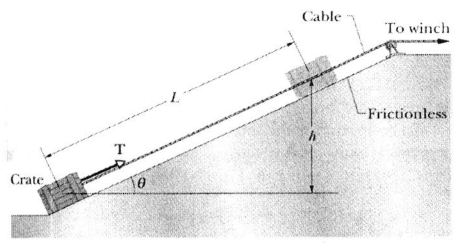
$d\vec{s} = dx\,\hat{i}$

$\vec{x} = -x\,\hat{i} \Rightarrow$ spring compressed in the $-\hat{i}$ direction

$$W_s = \int_0^d \vec{F}_s \cdot (dx\,\hat{i}) = \int_0^d -k(-x)\hat{i} \cdot (dx\,\hat{i}) = k(\hat{i}\cdot\hat{i})\int_0^d x\,dx = \tfrac{1}{2}kx^2$$

$$\boxed{W_s = \tfrac{1}{2}kd^2}$$

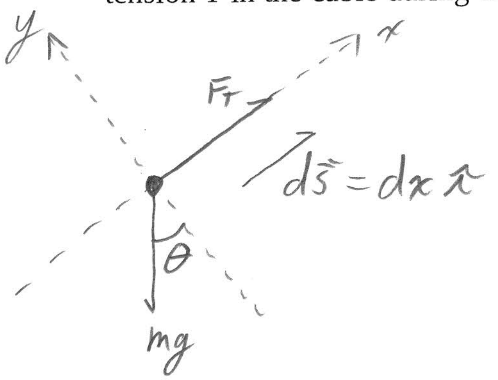
Energy Problems – Set 2



An initially stationary crate of mass m is pulled a distance L up a frictionless ramp to a height h where it stops.

a) Find an expression for the work W_g done on the crate by gravity during the lift in terms of m, h, and g.

b) Find an expression for the work W_T done on the crate by the tension T in the cable during the lift in terms of m, h and g.



a) $W_g = \int_0^L (mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j})(dx\,\hat{i})$

$= mg\sin\theta \int_0^L dx$

$W_g = mgL\sin\theta$, $\quad \underline{L\sin\theta = h}$

$\boxed{W_g = mgh}$

b) The problem statement tells us that $\Delta K = 0$.

The tension in the rope isn't constant. In fact we have no idea what the box <u>does</u> between start and finish.

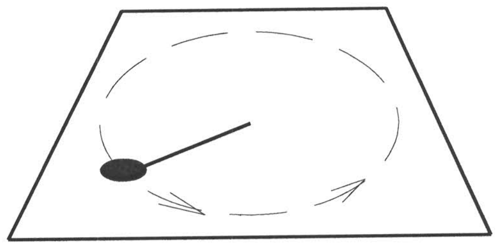
We <u>do</u> know that $W_{net} = \Delta K$ and $\Delta K = 0$

so: $W_{net} = 0$

$\Rightarrow W_g + W_T = 0 \Rightarrow mgh + W_T = 0 \Rightarrow \boxed{W_T = -mgh}$

Energy Problems – Set 2

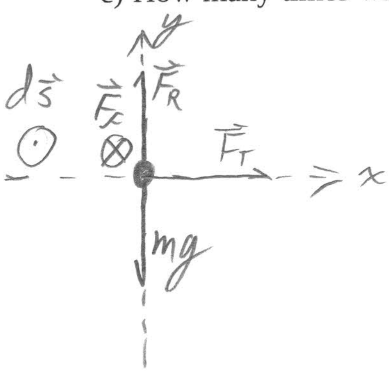
A particle of mass m moves in a horizontal circle of radius R on a rough table. It is attached to a string fixed at the center of the circle. The coefficient of friction between the mass and the table is μ_k.



a) Calculate the net work done on the puck after one revolution.

b) The initial velocity of the puck is v_0. After completing one revolution, the velocity of the puck is $\frac{1}{2}v_0$. Find an expression for μ_k in terms of v_0, π, R and g.

c) How many times will the particle go around? (You should get a number)



$W_{F_R} = 0, \quad \vec{F}_R \perp d\vec{s}$

$W_g = 0, \quad \vec{F}_g \perp d\vec{s}$

$W_{F_T} = 0, \quad \vec{F}_T \perp d\vec{s}$

NSL

$F_R - mg = 0$

$F_R = mg$

$\Rightarrow \boxed{F_f = \mu_k mg}$

$$W_{F_f} = \int_0^{2\pi R} (-\mu_k mg \hat{x})(ds\hat{x}) = -\mu_k mg 2\pi R$$

$$\boxed{W_{net1} = -\mu_k mg 2\pi R}$$

b) WET

$$W_{net} = \Delta K \Rightarrow -\mu_k mg 2\pi R = \tfrac{1}{2}mV^2 - \tfrac{1}{2}mV_0^2$$

$$-\mu_k \cancel{m} g 2\pi R = \tfrac{1}{2}\cancel{m}\left(\tfrac{1}{2}V_0\right)^2 - \tfrac{1}{2}\cancel{m}V_0^2$$

$$= \tfrac{1}{8}V_0^2 - \tfrac{1}{2}V_0^2$$

$$-\mu_k g 2\pi R = -\tfrac{3}{8}V_0^2$$

Continued ↓

EP2, P5 - continued

$$\boxed{+\mu_k = +\frac{3}{16}\frac{V_0^2}{\pi g R}}$$

c) Particle starts with kinetic energy $\frac{1}{2}mV_0^2$
It loses $\mu_k mg 2\pi R$ per revolution.
When the particle stops, its kinetic energy is zero.

Let n = # of revolutions:

WET

$$W_{net} = \Delta K$$

$$nW_{net1} = \cancel{\tfrac{1}{2}mV^2}^{0} - \tfrac{1}{2}mV_0^2 \,, \quad W_{net1} = \text{work in 1 rev.}$$

$$\Rightarrow -n\mu_k mg 2\pi R = -\tfrac{1}{2}mV_0^2$$

$$\Rightarrow n = \frac{1}{4}\frac{V_0^2}{\mu_k g \pi R}$$

subst in μ_k from part b

$$n = \frac{1}{\cancel{4}}\frac{\cancel{16}^{4}}{3}\frac{\cancel{\pi g R}}{\cancel{V_0^2}}\frac{\cancel{V_0^2}}{\cancel{g}\cancel{\pi}\cancel{R}} \Rightarrow \boxed{n = \frac{4}{3}} = \boxed{1.33 \text{ revcluions}}$$