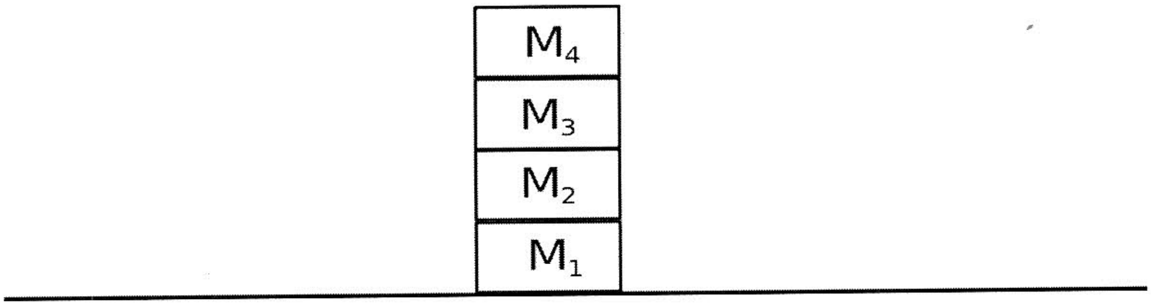
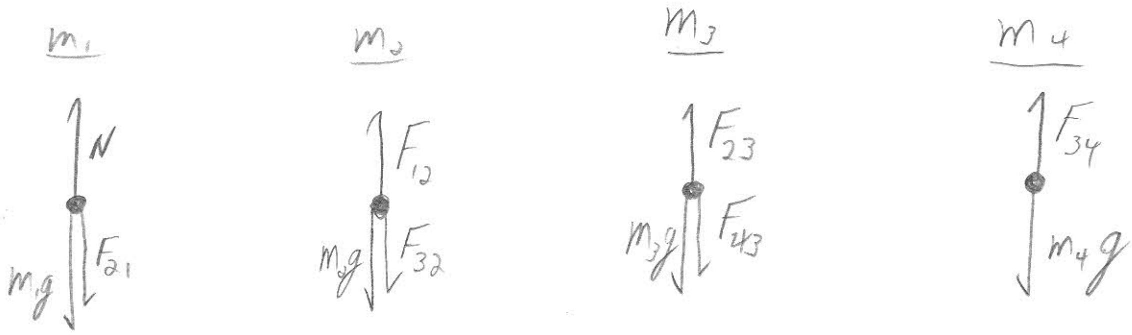
Force Problems – Set 2

In the picture below, four blocks are stacked on top of each other. Nothing is in motion.



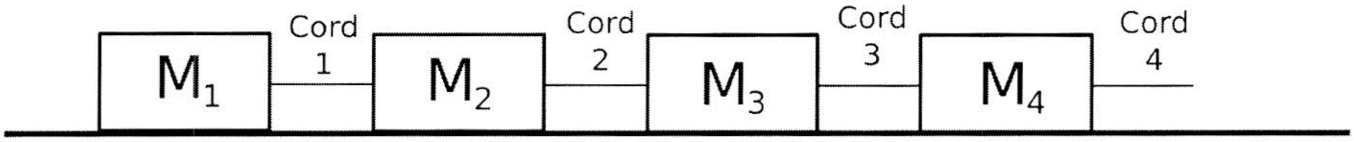
a) Draw a free body diagrams for each block.



b) Rank the reaction forces from the free body diagrams above in order of magnitude, greatest first. If any are equal, place an equals sign between them.

$N > F_{21} = F_{12} > F_{32} = F_{23} > F_{43} = F_{34}$

The picture below shows four blocks connected by chords. The system is accelerated to the right by pulling on cord 4.



a) Rank the blocks according to their acceleration, greatest first.

all accelerations are equal.

The system is mechanically linked.
All of the blocks must move together

b) Rank the cords according to their tension, greatest first.

$T_4 > T_3 > T_2 > T_1$

~~Cor~~

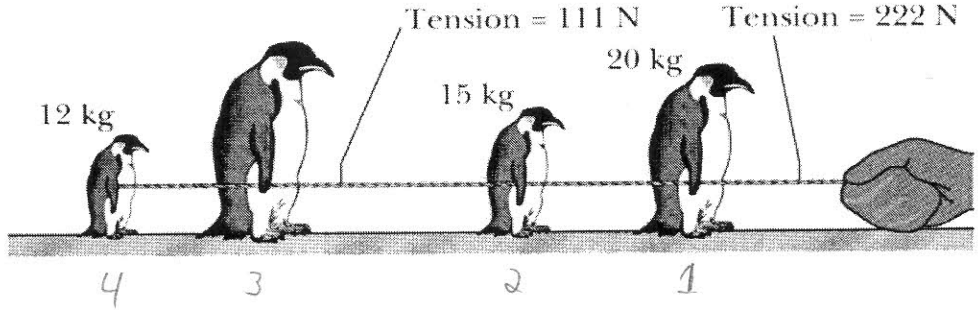
c) Which is greater: The total mass accelerated by cord 4 or the total mass accelerated by cord 1?

Cord 4 accelerates the entire system.
Cord 1 accelerates only block 1.

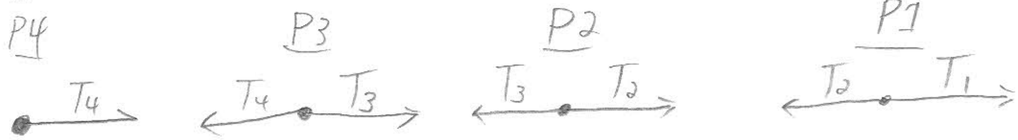
So, cord 4.

Force Problems – Set 2

Four penguins are being playfully pulled along a very slippery (frictionless) ice by a zoo keeper. The masses of the first, second, and third penguins and two cord tensions are shown below. Find the mass of the third penguin.



a) Draw four free body diagrams, one for each penguin. Keep very careful track of the tension forces. State the given information.



Given

$m_4 = 12\ kg,\ m_2 = 15\ kg,\ m_1 = 20\ kg,\ T_3 = 111\ N,\ T_1 = 222\ N$

b) Write Newton's Second Law for each free body diagram.

P4: $T_4 = m_4 a$

P2: $T_2 - T_3 = m_2 a$

P3: $T_3 - T_4 = m_3 a$

P1: $T_1 - T_2 = m_1 a$

givens are circled
unknowns are underlined.

4 equations, 4 unknown variables.

c) Solve the system of equations above to isolate the mass of the ~~fourth~~ third penguin.

combine P4 and P3, solve for T_3

$T_3 - m_4 a = m_3 a \Rightarrow \boxed{T_3 = (m_3 + m_4) a}$

solve P2 for T_2 and subst into P1

$T_2 = T_3 + m_2 a \Rightarrow T_1 - T_3 - m_2 a = m_1 a$

solve for $T_1 - T_3$; $\boxed{T_1 - T_3 = (m_2 + m_1) a}$

Divide the two equations and solve for m_3

$$\frac{T_3}{T_1 - T_3} = \frac{(m_3 + m_4)\cancel{a}}{(m_1 + m_2)\cancel{a}} \Rightarrow \boxed{m_3 = \frac{T_3}{T_1 - T_3}(m_1 + m_2) - m_4}$$

$\boxed{m_3 = 23\ kg}$

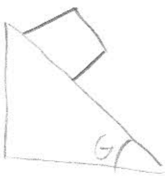
Force Problems – Set 2

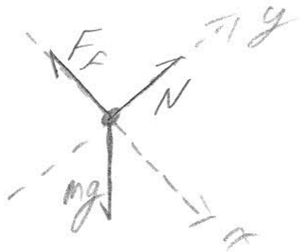
For the following situations, draw free-body diagrams to indicate all forces acting on the object(s) in question. *Indicate relative magnitudes of forces by drawing long, short, or equal-length vectors.*

a) A block is being pushed across a rough horizontal table at a constant velocity (there's friction).

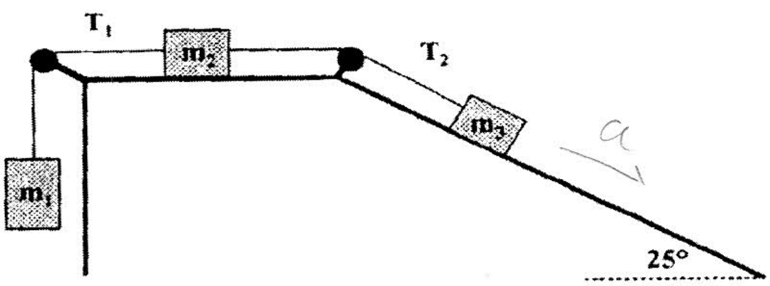
$F_p = F_f$, const $v \Rightarrow a = 0$

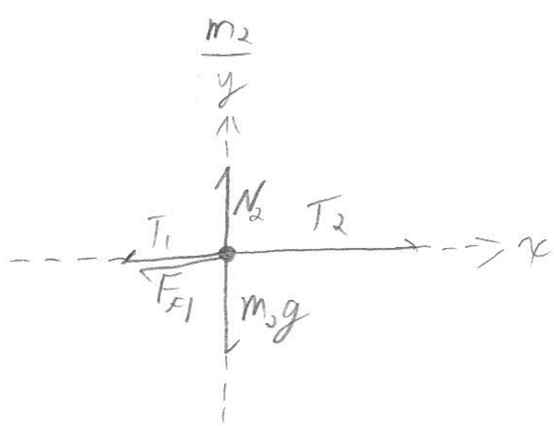
b) A block is resting on a rough incline plane without sliding.

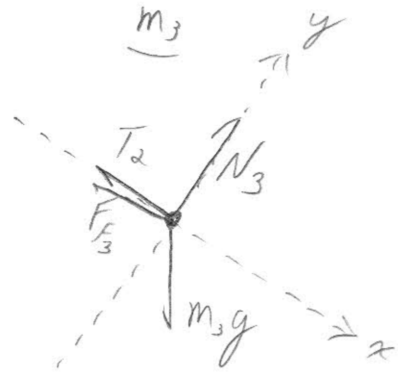




c) Three blocks are attached by strings as in the diagram. All surfaces are rough. Draw free body diagrams of each block. Block 3 is BIG, system is accelerating.







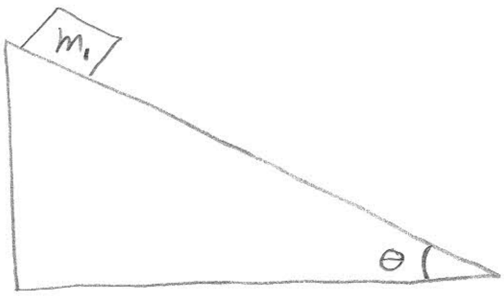
Force Problems – Set 2

A block rests on an incline plane that makes an angle θ with the horizontal. The coefficient of static friction between the block and the plane is μ_s and the coefficient of kinetic friction is μ_k.

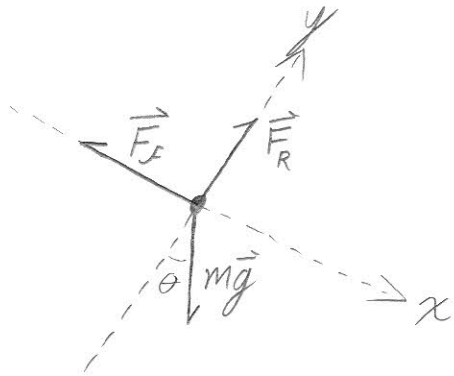
a) Find an expression for the maximum angle of the incline before the block slips.

HINT: The equation for the force of static friction ($F = \mu_s N$) represents the ***maximum*** *force that friction can provide.*

b) Assuming that the block is in motion, find an expression for the ~~velocity~~ acceleration of the block in terms of θ and μ_k.



FBD



a) The block isn't moving $\Rightarrow a_x = a_y = 0$. When the force in the positive x exceeds the max frictional force, the block slips.

$\boxed{\text{Write NSL}}$

x

$$\sum F_x = m a_x$$

$\boxed{\text{no slipping}}$

$$mg\sin\theta - F_f = 0$$

$$\boxed{mg\sin\theta = F_f} \quad (1)$$

y

$$\sum F_y = m a_y$$

$$F_R - mg\cos\theta = 0$$

$$\Rightarrow \boxed{F_R = mg\cos\theta} \quad (2)$$

continued

Force problems - Set 2, P1 continued

When eq 1 is true, the block will not slip.

We know that $F_f \leq \mu_s F_R$

So, as long as:

$$\boxed{mg\sin\theta \leq \mu_s F_R} \quad (3)$$

The block won't slip

Now, we need to plug in 2 into 3 to eliminate the unknown F_R.

$$\Rightarrow \cancel{mg}\sin\theta \leq \mu_s \cancel{mg}\cos\theta$$

$$\Rightarrow \sin\theta \leq \mu_s\cos\theta$$

$$\Rightarrow \tan\theta \leq \mu_s$$

$$\Rightarrow \boxed{\theta \leq \tan^{-1}(\mu_s)}$$

If θ exceeds this value, the block slips.

continued ↓

Force Problems - Set 2, P1 continued

b) This one we set up normally because the force from kinetic friction is constant.

$\boxed{\text{Write NSL}}$

$\underline{x}$

$\sum F_x = ma_x$

$mg\sin\theta - F_f = ma_x$

(1) $\boxed{mg\sin\theta - \mu_k F_R = ma_x}$

$\underline{y}$

$\sum F_y = ma_y$

$F_R - mg\cos\theta = 0$

$\Rightarrow \boxed{F_R = mg\cos\theta}$ (2)

- Combine (1) and (2) to eliminate F_R

$\Rightarrow \cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a_x$

$\boxed{a_x = g(\sin\theta - \mu_k\cos\theta)}$

Force Problems – Set 2

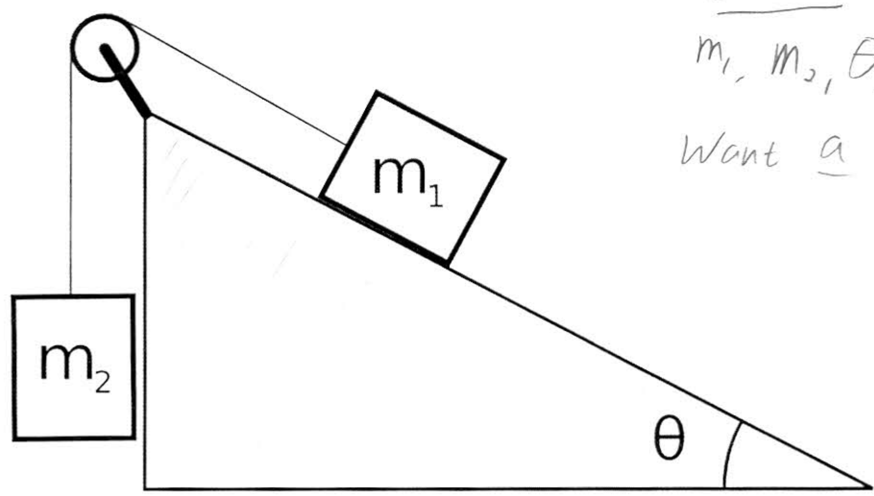
In the picture below, the coefficient of kinetic friction between the ramp and m1 is $\cancel{\mu_s}$ μ_k. A rope connecting m1 and m2 passes over a massless frictionless pulley.

Calculate the acceleration of the system.

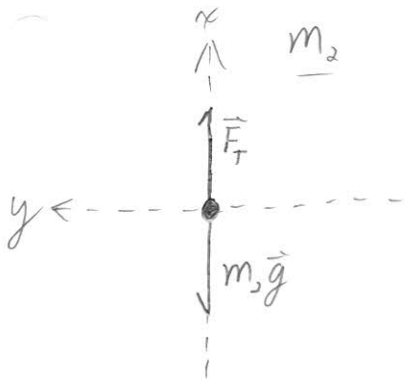
<u>Given</u>

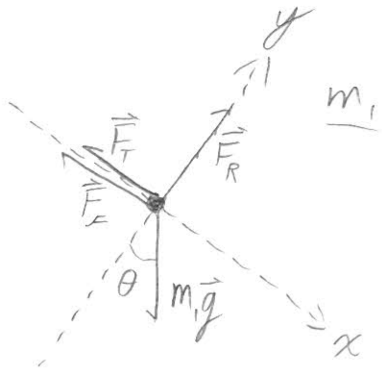
m_1, m_2, θ, μ_k

Want <u>a</u>



<u>Draw FBDs</u>





The coordinate systems here are aligned so that the positive x-axes are in the direction of the acceleration of the block.

continued

↓

Force Problems - Set 2 P4 continued

Write $\boxed{NSL}$

m_1

x

$\sum F_x = ma_x$

$m_1 g \sin\theta - F_T - F_f = m_1 a_{1x}$

and $F_f = \mu_k F_R$ so...

$\Rightarrow \boxed{m_1 g \sin\theta - F_T - \mu_k F_R = m_1 a_{1x}}$ 1

y

$\sum F_y = ma_y$

$\boxed{F_R - m_1 g \cos\theta = 0}$ 2

m_2

x

$\sum F_x = ma_x$

$\boxed{F_T - m_2 g = m_2 a_{2x}}$ 3

y

$\sum F_y = ma_y$

$0 = 0$

$a_{1x} = a_{2x}$ because we aligned the coordinate systems and they are connected by a rope.

Solve 2 for F_R and subst $\rightarrow$ eq 1:

$\Rightarrow m_1 g \sin\theta - F_T - \mu_k m_1 g \cos\theta = m_1 a_{1x}$ 4

continued

$\downarrow$

Force Problems - Set 2 P4 continued

Solve 3 For F_T and subst into 4

From 3: $F_T = m_2 g + m_2 a_{1x}$ ← replaced a_{2x} with a_{1x}

into 4: $\boxed{m_1 g \sin\theta - m_2 g - m_2 a_{1x} - \mu_K m_1 g \cos\theta = m_1 a_{1x}}$ 5

Eq 5 has no unknown variables. Isolate a_{1x} (which is the same as a_{2x})

From 5

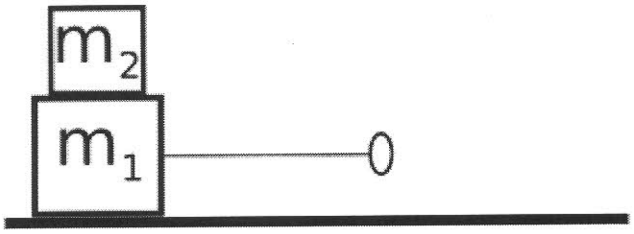
$$m_1 a_{1x} + m_2 a_{1x} = m_1 g \sin\theta - \mu_K m_1 g \cos\theta - m_2 g$$

$$\Rightarrow (m_1 + m_2) a_{1x} = g\left[m_1(\sin\theta - \mu_K \cos\theta) - m_2\right]$$

$$\Rightarrow \boxed{a_{1x} = g\left[\frac{m_1(\sin\theta - \mu_K\cos\theta) - m_2}{m_1 + m_2}\right]}$$

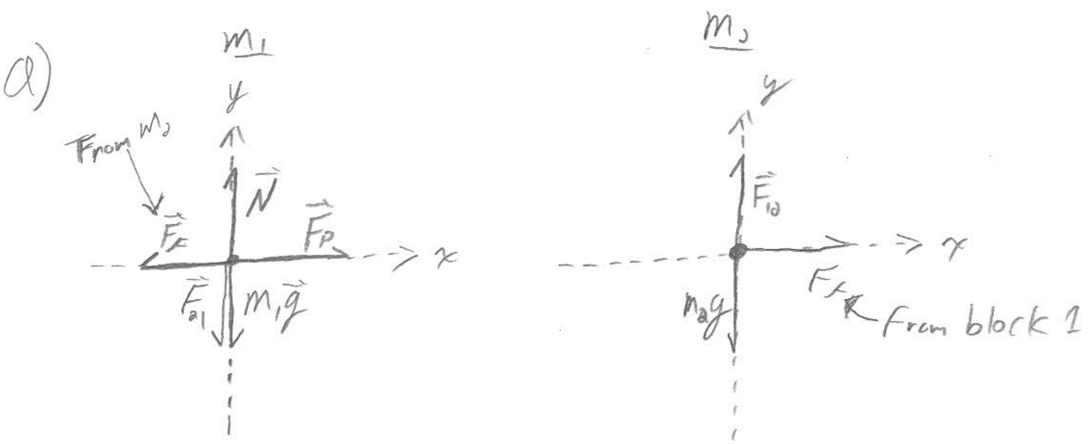
Force Problems – Set 2

Two blocks with masses m_1 and m_2 are stacked up as shown in the picture below. A rope with a handle is attached to m_1 as shown. There is no friction between m_1 and the table. The coefficient of static friction between m_2 and m_1 is μ_s.



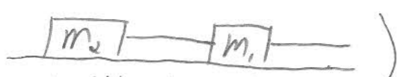
a) Draw free body diagrams for m_1 and m_2.
b) Find an expression for the maximum force that can be applied to to the rope on m_1 without m_2 slipping.
c) Find an expression for the reaction force of the floor acting on m_1 in terms of g, m_1, and m_2.

a)



The frictonal force from the m_1 m_2 interface is what accelerates block 2, but the presence of block 2 must cause opposition to block 1's acceleration.

(Imagine this scenario:



with no friction.)

continued

↓

Force problems set 2, (P7) continued.

b) What's given? Given: m_1, m_2, μ_s, g That's all!

want: $F_{P\,max}$, For block 2 to not slip

Write NSL For each block

m_1

x: $\boxed{F_P - F_f = m_1 a}$ (1)

y: $\boxed{N - F_{21} - m_1 g = 0}$ (2)

m_2

x: $\boxed{F_f = m_2 a}$ (3)

y: $\boxed{F_{12} - m_2 g = 0}$ (4)

In this case, the "normal" force in F_f is F_{21} or F_{12} since this is the surface where the contact is.

combine (1) and (3) to eliminate unknown a:

From (3): $\boxed{a = \frac{F_f}{m_2}}$ (5)

Subst: (5) → (1) $F_P - F_f = m_1 \cdot \frac{F_f}{m_2}$

$\Rightarrow \boxed{F_P = F_f\left(1 + \frac{m_1}{m_2}\right)}$ (6)

But we know that: $\boxed{F_f \leq \mu_s F_{12}}$

and From (4) we have: $F_{12} = m_2 g \Rightarrow \boxed{F_f \leq \mu_s m_2 g}$ (7)

continued
↓

Force Problems set 2, P7 continued

Now subst. 7 → 6:

$$F_p \le \mu_s m_2 g \left(1 + \frac{m_1}{m_2}\right)$$

$$\Rightarrow \boxed{F_p \le \mu_s g\,(m_1 + m_2)}$$

As long as this is true, tle block will not slip.

c) From 2: $N = F_{21} + m_1 g$ From 4: $F_{12} = m_2 g$

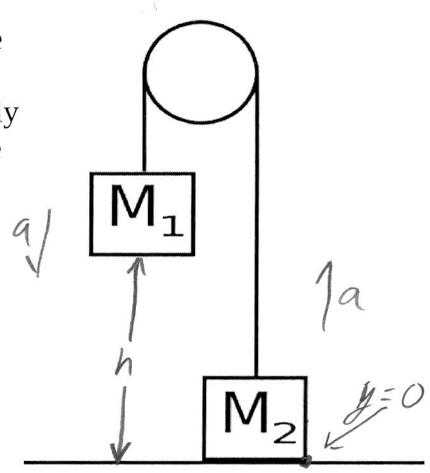
$$\Rightarrow N = m_2 g + m_1 g$$

$$\Rightarrow \boxed{N = (m_1 + m_2) g}$$

So, the floor holds the combined weight of both blocks.

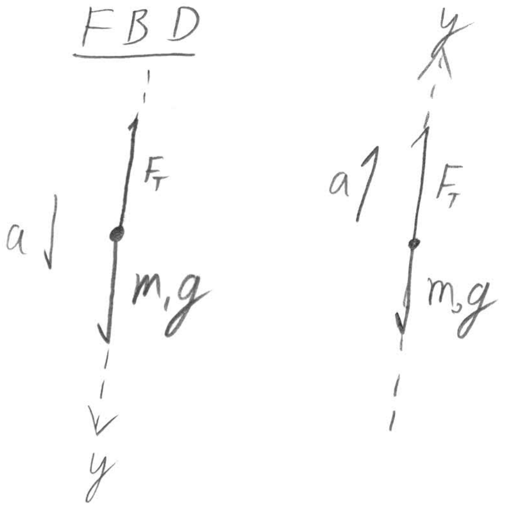
One end of a rope is connected to a mass M_1=10kg. The rope passes over a massless frictionless pulley and the other end is connected to a mass M_2 = 5kg. M_2 is initially resting on the ground and M_1 is suspended 3m above the ground. The system is initially at rest.

If M_1 is released and allowed to hit the ground, what is the maximum height that M_2 will reach?



*HINT: When M_1 hits the ground, M_2 will still have an upward velocity. The rope will go slack and M_2 will **continue** upward until its velocity is zero.*

FBD



NSL

① $m_1 g - F_T = m_1 a$

② $F_T - m_2 g = m_2 a$

From ②: $F_T = m_2 a + m_2 g$

Into ①: $m_1 g - m_2 a - m_2 g = m_1 a$

$$(m_1 - m_2) g = (m_1 + m_2) a$$

③ $$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

continued ↓

Atwood — continued

we have acceleration, now we'll do kinematics.

M_2 will accelerate through a distance h with a from the force analysis. Then, it will essentially be in "free fall" with some initial upward velocity V_1.

Find V_1

$$y = y_0 + V_0 t + \tfrac{1}{2} a t^2$$

$$h = 0 + 0 + \tfrac{1}{2} a t^2$$

$$t = \left(\frac{2h}{a}\right)^{1/2}$$

$$V = V_0 + at$$

$$V_1 = 0 + at$$

$$V_1 = a\left(\frac{2h}{a}\right)^{1/2} = (2ha)^{1/2}$$

$$\boxed{V_1 = (2ha)^{1/2}} \quad 4$$

Now find h_{max}

$$y = y_0 + V_0 t + \tfrac{1}{2} a t^2$$

$$V = V_0 + at$$

$$0 = V_1 - gt$$

$$t = \frac{V_1}{g}$$

$$h_{max} = h + V_1 t - \tfrac{1}{2} g t^2$$

$$h_{max} = h + \frac{V_1^2}{g} - \frac{1}{2}\frac{V_1^2}{g}$$

$$\boxed{h_{max} = h + \tfrac{1}{2}\frac{V_1^2}{g}} \quad 5$$

continued

↓

Atwood continued.

combine 3, 4, and 5

$$h_{max} = h + \frac{\cancel{2}ha}{\cancel{2}g}$$

$$h_{max} = h + \frac{h}{\cancel{g}} \frac{m_1 - m_2}{m_1 + m_2} \cancel{g}$$

$$h_{max} = h\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) = h \frac{m_1 + \cancel{m_2} + m_1 - \cancel{m_2}}{m_1 + m_2}$$

$$\boxed{h_{max} = \frac{2m_1}{m_1 + m_2} h}$$

So... If $m_2 < m_1$, $h_{max} > h$.

$$h_{max} = \frac{(2)(10)}{10 + 5}(3) = \frac{20}{15} 3 = 4_m$$