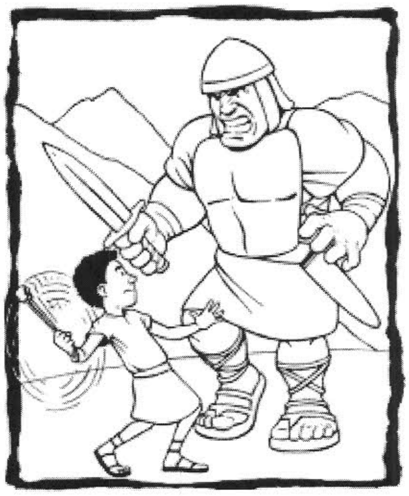
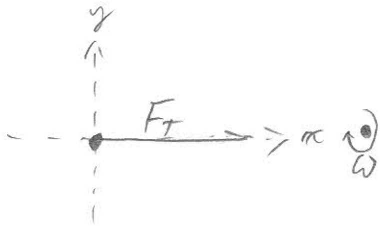
Force Problems – Set 3



David is fighting Goliath. To defeat him, David's rock must leave his 0.5 m sling at 20 m/s. Unfortunately, his sling was damaged and the maximum tension the rope can withstand is 500 N. What is the mass of largest rock that David can sling? Ignore gravity.

a) Draw a free body diagram of the rock (pretend it's David vs Goliath in space and Ignore Gravity). State the Given and Wanted information.



Given

$v = 20\ m/s$

$r = 0.5m$

$F_{T_{max}} = 500N$

Wanted

$m = ?$

b) QUICK! Without thinking about Newton's Second Law, write an expression for the acceleration of the rock. (HINT: It's in Uniform Circular Motion

$$\boxed{a = \frac{v^2}{r}}$$

c) Write Newton's Second Law for the rock based on your answer to parts a and b. Solve it for the mass of the rock.

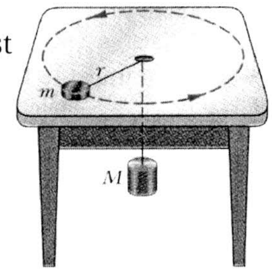
NSL

$$F_T = m\frac{v^2}{r} \Rightarrow \boxed{m = \frac{F_T r}{v^2}}$$

d) Calculate the mass of the rock.

$$m = \frac{(500N)(0.5m)}{(20\ m/s)^2} = \boxed{0.625\ kg}$$

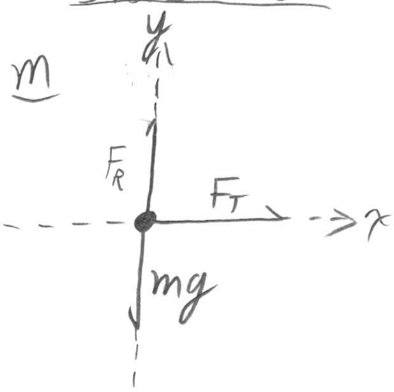
A mass m on a frictionless table is attached to a hanging mass M by a cord through a hole in the table. Find the speed with which m must move in order for M to stay at rest.



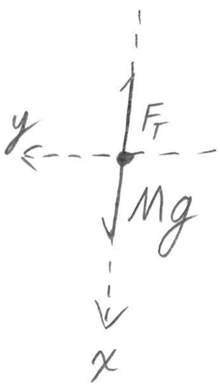
FBD

Side view

m



M



NSL

m

$x: \sum F_x = ma_x$

① $F_T = m\frac{v^2}{r} \Rightarrow$ UCM

$y: \sum F_y = ma_y$

② $F_R - mg = 0 \Rightarrow$ No motion in y

NSL

M

$x: \sum F_x = ma_x$

③ $Mg - F_T = 0$

$y: 0 = 0$

So, From ③: $F_T = Mg$

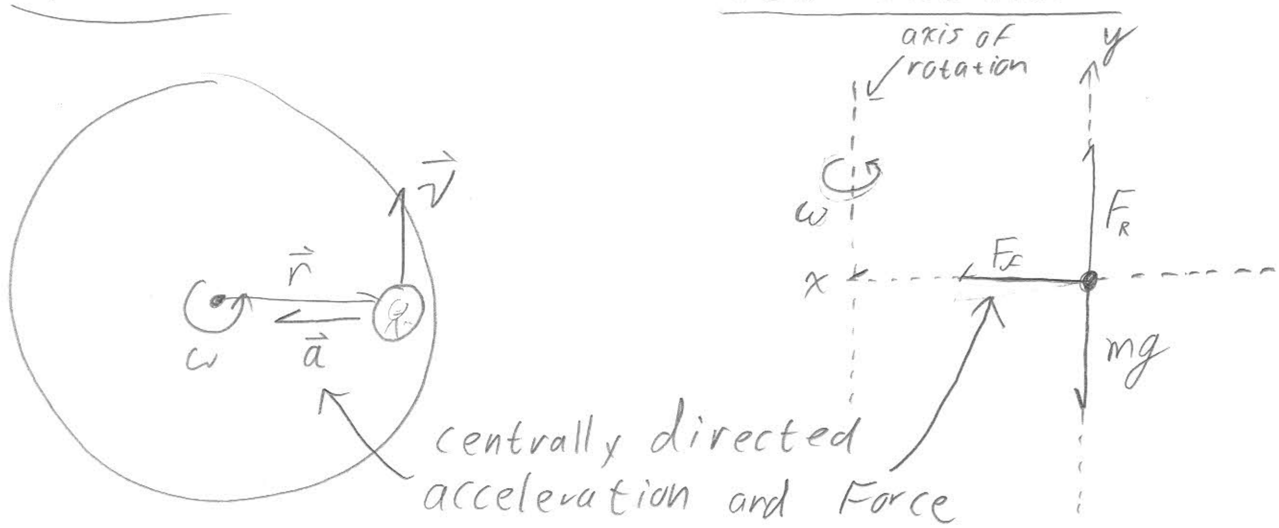
then into ①: $Mg = m\frac{v^2}{r}$

$\Rightarrow \boxed{v = \left[\frac{M}{m}gr\right]^{1/2}}$

A penny of mass *m* is placed on a turntable (remember those?) and the turntable begins spinning. The coefficient of static friction between the penny and the turntable is μ_s. What is the fastest angular velocity ω that the turntable can spin before the penny slips.

Top view

FBD – Side view



NSL

x

$$\Sigma F_x = ma_x$$

$$F_f = ma_x$$ ← centripetal acceleration

$$\boxed{\mu_s F_R \geq mr\omega^2} \quad (1)$$

↑

Because $\boxed{F_f \leq \mu_s F_R}$

Subst (2) → (1): $\mu_s \cancel{m} g \geq \cancel{m} r\omega^2$

$$\Rightarrow \boxed{\omega \leq \left(\frac{\mu_s g}{r}\right)^{1/2}}$$

y

$$\Sigma F_y = ma_y$$

$$F_R - mg = 0$$

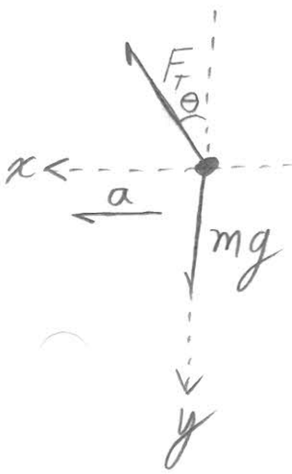
$$\boxed{F_R = mg} \quad (2)$$

Napoleon lands a flippin' sweet blow to the tether ball, which is attached to a rope of whose length is L = 0.60m sending it whipping around the pole with a period of of 0.75s.

a) Napoleon is jealous of your awesome physics skills and needs your help finding an expression for the angle, θ, that the rope makes with the pole.

b) ~~Calculate a numeric value for θ using your expression from part a and use it to calculate the tension in the rope.~~

FBD



Given

L = 0.6 m
P = 0.75 s

wanted

θ = ?

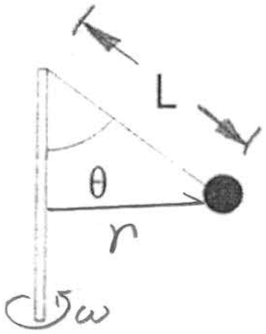
NSL

$x: \sum F_x = ma_x$

① $F_T \sin\theta = m\frac{v^2}{r}$

$y: \sum F_y = ma_y$

② $F_T \cos\theta - mg = 0$



$r = L \sin\theta$

Let's eliminate F_T Between ① and ② First and then deal with the period.

From ②: $F_T \cos\theta = mg$ ③

Divide $\frac{①}{③}$: $\frac{\cancel{F_T}\sin\theta}{\cancel{F_T}\cos\theta} = \frac{\cancel{m}\frac{v^2}{r}}{\cancel{m}g} \Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$ ④

Now we can relate v and r to the period.

$v = \frac{d}{t} \Rightarrow \boxed{v = \frac{2\pi r}{P}}$ Plug into ④

continued ↓

Napoleon - continued

From 4: $\tan\theta = \frac{4\pi^2 \cancel{r}}{P^2}\frac{1}{\cancel{Kg}}$

$$\tan\theta = \frac{4\pi^2 r}{g P^2} \quad 5$$

But $r = L\sin\theta$

So: $\tan\theta = \frac{4\pi^2}{g P^2} L \sin\theta$

$$\Rightarrow \frac{\cancel{\sin\theta}}{\cos\theta} = \frac{4\pi^2 L}{g P^2}\cancel{\sin\theta}$$

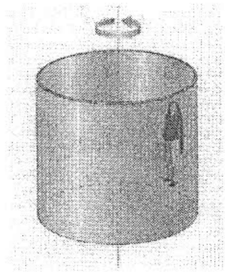
$$\Rightarrow \boxed{\cos\theta = \frac{g P^2}{4\pi^2 L}}$$

$$\theta = \cos^{-1}\left(\frac{gP^2}{4\pi^2 L}\right) = \cos^{-1}\left[\frac{(9.8)(0.75)^2}{(4)\pi^2(0.6)}\right]$$

$$\boxed{\theta = 76^\circ}$$

Force Problems – Set 3

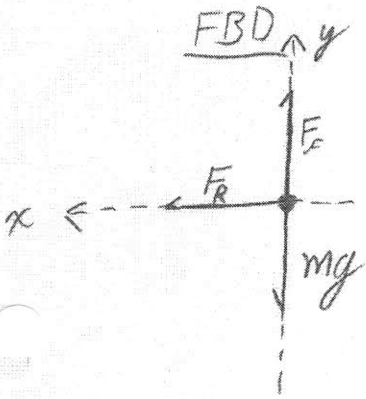
The Cyclone is an exciting state fair ride consisting of a large vertical cylinder that spins about its axis. Riders stand inside the cylinder with their backs against the wall. When the cylinder is spinning fast enough, the floor drops away and the rider "sticks" to the wall.



If the The coefficient of static friction between a wall and person is μ_s, and the radius of the cylinder is R, show that the maximum period of revolution is given by:

$$P=\left(\frac{4\pi^2 R\mu_s}{g}\right)^{\frac{1}{2}}$$

FBD



NSL

$$x: \sum F_x = ma_x$$

$$① \ F_R = mr\omega^2$$

$$y: \sum F_y = ma_y$$

$$F_f - mg = 0$$

$$② \ \mu_s F_R - mg = 0$$

Given

r, μ_s, g

Wanted

P

Subst ① → ② : $\mu_s \cancel{m} r\omega^2 = \cancel{m}g$ $\qquad \omega = \frac{2\pi}{P}$

$$\mu_s r \frac{4\pi^2}{P^2} = g$$

$$\boxed{P = \left(\frac{4\pi^2 \mu_s r}{g}\right)^{1/2}}$$