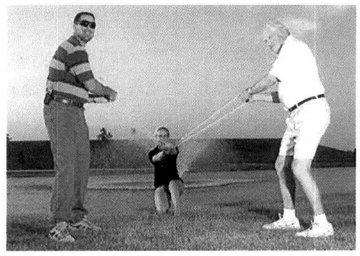
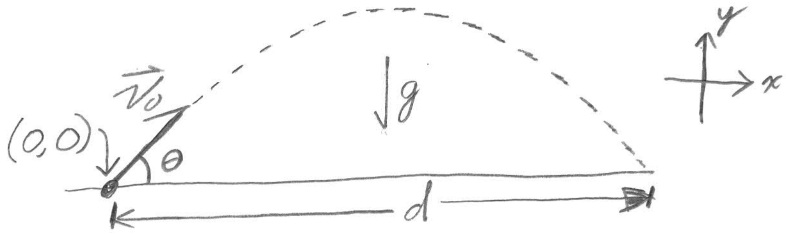
Kinematics



You and your buddies have created a water balloon slingshot. Using a radar gun, you measure that the velocity of the water balloon as it leaves the slingshot. Because you're taking physics, your friends want you to work out how far the balloon will go for a given launch angle.

Find an equation for the horizontal distance the balloon will travel in terms of the magnitude of it's initial velocity and the launch angle with respect to the horizontal.



Given	Want
v_0	d
θ	

Kinematics equations:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \vec{v} = \vec{v}_0 + \vec{a}t$$

Treat x and y axis independently:

x: $x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$ $\qquad v_x = v_{0x} + a_x t$

① $d = 0 + |\vec{v}_0|\cos\theta\, t + 0$ $\qquad$ ② $\boxed{v_x = |\vec{v}_0|\cos\theta} + 0$

$\boxed{\text{need } t}$ $\qquad$ Velocity in x is constant – no accel.

y: $y = y_0 + v_{0y}t + \tfrac{1}{2}at^2$ $\qquad v_y = v_{0y} + a_y t$

③ $0 = 0 + |\vec{v}_0|\sin\theta\, t - \tfrac{1}{2}gt^2$ $\qquad$ ④ $v_y = |\vec{v}_0|\sin\theta - gt$

continued ↓

Water Balloon continued

3 equations, (1, 3, 4) and 3 unknowns.

in Equation 3, only $\underline{t}$ is unknown, so we can use it to eliminate $\underline{t}$ from 1 and 4

From 3: $|\vec{v}_0| \sin\theta \cancel{t} = \frac{1}{2} g t^{\cancel{2}} \Rightarrow t = \frac{2|\vec{v}_0|\sin\theta}{g}$ 5

Plug into 1:

$$d = |\vec{v}_0| \cos\theta \cdot \frac{2|\vec{v}_0|\sin\theta}{g}$$

math aside:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$\boxed{d = \frac{|\vec{v}_0|^2}{g} 2\sin\theta\cos\theta} \Rightarrow \boxed{d = \frac{|\vec{v}_0|^2}{g}\sin(2\theta)}$$ 6

Although the problem statement doesn't request it, let's solve for v_y

$$v_y = |\vec{v}_0|\sin\theta - gt$$

$$= |\vec{v}_0|\sin\theta - \cancel{g} \cdot \frac{2|\vec{v}_0|\sin\theta}{\cancel{g}} = -|\vec{v}_0|\sin\theta$$

$$\boxed{\underline{v_y} = -|\vec{v}_0|\sin\theta = -\underline{v_{oy}} \quad \text{nifty!}}$$

continued ↓

What is the balloon's maximum height?

Maximize a function by taking the derivative and setting it to zero.

But that's the velocity!

So... max height when $v_y = 0$

From eq. 4

$$v_y = |\vec{v}_0| \sin\theta - gt, \quad \text{let } v_y = 0$$

$$0 = |\vec{v}_0| \sin\theta - g t_{apex}$$

$$\Rightarrow t_{apex} = \frac{|\vec{v}_0| \sin\theta}{g} \Rightarrow \text{Time at max height}$$

Compare to eq 5: $t_{apex} = \frac{1}{2} t$

So: $y_{max} = \cancel{y_0}^{0} + v_{0y} t - \frac{1}{2} g t^2$

$$y_{max} = |\vec{v}_0| \sin\theta \frac{|\vec{v}_0| \sin\theta}{g} - \frac{1}{2} \cancel{g} \frac{(|\vec{v}_0| \sin\theta)^2}{g^{\cancel{2}}}$$

$$\boxed{y_{max} = \frac{1}{2} \frac{|\vec{v}_0|^2 \sin^2\theta}{g}}$$

continued ↓

One last bit: What's x at the apex?

$$x_{apex} = \cancel{x_0}^{\,0} + v_{0x} t_{apex} + \cancel{\tfrac{1}{2} a_x t^2}^{\,0}$$

$$x_{apex} = |\vec{v}_0| \cos\theta \frac{|\vec{v}_0| \sin\theta}{g} = \frac{|\vec{v}_0|^2}{g} \sin\theta\cos\theta$$

$$x_{apex} = \frac{|\vec{v}_0|^2}{g} \frac{1}{2} \sin(2\theta)$$

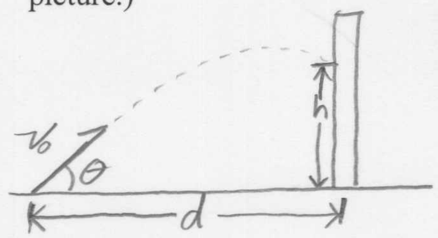
compare to eq. 6 we see that

$$\boxed{x_{apex} = \tfrac{1}{2} d \quad \text{groovy!}}$$

1D Kinematics, Part 2

2. You throw a ball toward a wall with a speed of 25.0 m/s and at an angle of 40.0º above the horizontal. The wall is 22.0 m from the release point of the ball.

   (a) How far above the release point does the ball hit the wall? (Be sure to draw a picture.)



Given
$v_0 = 25\ m/s$
$\theta = 40°$
$d = 22\ m$

Want
h

Part a

x

$x_f = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

Solve for t

$\rightarrow d = 0 + v_0 \cos\theta t + 0$

$t = \dfrac{d}{v_0 \cos\theta}$

y

$y_f = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$h = 0 + v_0 \sin\theta t - \frac{1}{2}gt^2$

$\xrightarrow{\text{Plug into y eq}} h = \cancel{v_0} \sin\theta \dfrac{d}{\cancel{v_0}\cos\theta} - \frac{1}{2}g\dfrac{d^2}{v_0^2\cos^2\theta}$

   (b) What are the horizontal and vertical components of its velocity as it hits the wall?

$\boxed{h = d\tan\theta - \frac{1}{2}\dfrac{gd^2}{v_0^2\cos^2\theta}}$ *

$h = 22 \cdot \tan(40) - \frac{1}{2}\dfrac{(9.8)(22)^2}{(25)^2\cos^2(40)}$

$\boxed{h = 12.0\ m}$

Part B - Find $\vec{v}_f$, Final velocity

x

$v_{fx} = v_{0x} + a_x t$

y

$v_{fy} = v_{0y} + a_y t$

   (c) When it hits, has it passed the highest point on its trajectory? How do you know?

$\boxed{v_{fx} = v_0 \cos\theta}$

$v_{fx} = (25)\cos(40)$

$= \boxed{19.2}\ m/s$

get from part a

$\rightarrow v_{fy} = v_0 \sin\theta - gt$

$\boxed{v_{fy} = v_0 \sin\theta - \dfrac{gd}{v_0\cos\theta}}$

$v_{fy} = 25\sin 40 - \dfrac{(9.8)(22)}{(25)\cos(40)}$

Part C Not past apex because v_{yF} is positive

$\boxed{v_{fy} = 4.8\ m/s}$

SAMPLE TEST 1

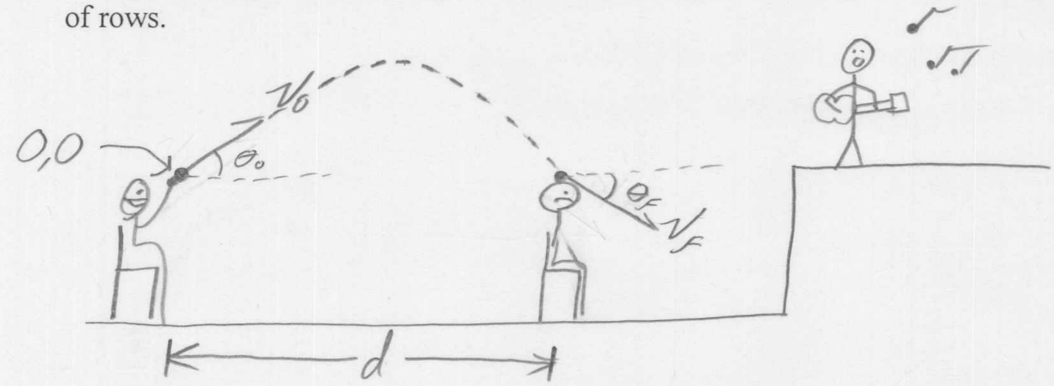
PHYS 111 FALL 2008

4 20pts) You are sitting in the middle of a field listening to a concert when a water balloon hits you from behind at a speed of 14 m/s coming in at an angle of -60° as measured off of the x axis. If the rows are separated by 1.5 m, how many rows behind you are the vandals sitting?

a. (5pts) Draw a sketch of the situation showing ALL relevant variables and define the coordinate system.

b. (13pts) Using the kinematics equations, derive an expression for the number of rows in terms of GIVEN variables defined in part a.

c. (2pts) Using your expression from part b and numbers given in the problem statement, calculate the number of rows.



GIVEN

$|\vec{V}_F| = 14\ m/s$

$\theta = -60°$

$r = 1.5\ m/row$

want

d in rows

$x_F = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$

$d = 0 + V_{0x}t + 0$

$y_F = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$0 = 0 + V_{0y}t - \frac{1}{2}gt^2$

hmm... use velocity eqs

$V_{Fx} = V_{0x} + at$

$|\vec{V}_F|\cos\theta_F = V_{0x} + 0$

$d = |\vec{V}_F|\cos\theta_F\, t$

$V_{Fy} = V_{0y} + at$

$|\vec{V}_F|\sin\theta = V_{0y} - gt$

$V_{0y} = |\vec{V}_F|\sin\theta + gt$

$0 = |\vec{V}_F|\sin\theta + gt - \frac{1}{2}gt$

$0 = |V_F|\sin\theta + \frac{1}{2}gt$

Continued ↓

Solve for t

$$t = -\frac{2|\vec{V}_F| \sin\theta}{g}$$

$$d = -\frac{2|\vec{V}_F|^2 \sin\theta \cos\theta}{g}$$

don't freak out!
Remember $\theta = -60$ so $\sin\theta$
will be negative and it
will cancel the negative

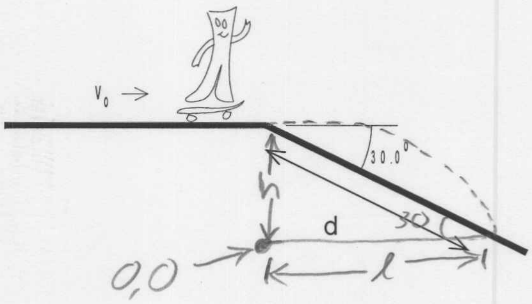
$$d = -\frac{2(14)^2 \sin(-60)\cos(-60)}{9.8}$$

$$= 17.3\,m \cdot \frac{1}{1.5}\ \text{rows}/m = \boxed{11.5\ \text{rows}}$$

1D Kinematics, Part 2

3. Gumby has just purchased a new skateboard; but, unfortunately, he does not know how to stop. Traveling at 8.0 m/s, he reaches the top of a hill sloping down at 30.0°.

   How far down the hill does Gumby crash (i.e., find the distance d)?



Given

$V_{0x} = 8.0\ m/s$

$V_{0y} = 0$

$\theta = 30^\circ$

want

d

x

$x_f = x_0 + V_{0x}t + \frac{1}{2}at^2$

$l = 0 + V_{0x}t + 0$

y

$y_f = y_0 + V_{0y}t + \frac{1}{2}at^2$

$0 = h + 0 - \frac{1}{2}gt^2$

Look at picture:
write l and h in terms of d and θ

$l = d\cos\theta, \quad h = d\sin\theta$

$d\cos\theta = V_{0x}t$

$t = \frac{d\cos\theta}{V_{0x}}$

$d\sin\theta = \frac{1}{2}gt^2$

$\cancel{d}\sin\theta = \frac{1}{2}g\frac{d^{\cancel{2}}\cos^2\theta}{V_{0x}^2}$

Solve for d

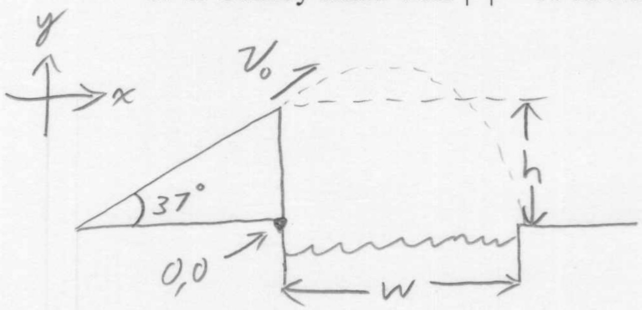
$\boxed{d = \frac{2V_{0x}^2\sin\theta}{g\cos^2\theta}} \Rightarrow d = \frac{2(8)^2\sin(30)}{(9.8)\cos^2(30)} =$

$\boxed{d = 8.7\ m}$

4. Having recovered from an earlier crash, Gumby is ready to try new and more exciting stunts on his skateboard. After some prodding from the Blockheads, he decides to jump across a river. Gumby knows that the far bank is 3.0 m below the top of the ramp. The ramp is inclined at 37.0° above the x-axis, and he is moving at 15 m/s when he leaves it.

a. How wide of a river can Gumby jump if he puts the ramp on the edge of the riverbank?

b. If Gumby lands with $|\vec{v}| > 16$ m/s his leg will break. Does Gumby need crutches?



$h = 3.0\,m$

$V_0 = 15\,m/s$

$\theta = 37.0°$

Want: w, $|\vec{V}_f|$

x

$x = x_0 + V_{0x}t + \cancel{\tfrac{1}{2}at^2}^{\,0}$

$w = V_0\cos\theta\, t$

y

$y = y_0 + V_{0y}t + \tfrac{1}{2}at^2$

$0 = h + V_0\sin\theta\, t - \tfrac{1}{2}gt^2$

$$t = \frac{1}{g}\left[V_0\sin\theta \oplus \left(V_0^2\sin^2\theta + 2gh\right)^{1/2}\right]$$

$$* \boxed{w = \frac{V_0\cos\theta}{g}\left[V_0\sin\theta + \left(V_0^2\sin^2\theta + 2gh\right)^{1/2}\right]} *$$

$$= \frac{15\cos(37)}{9.8}\left[15\sin 37 + \left(15^2\sin^2 37 + (2)(9.8)(3)\right)^{1/2}\right]$$

$$= \underline{25\,m}$$

(b) Given: $W = 25m$

What is $|\vec{V}_F|$

x

$W = V_0 \cos\theta \, t \Rightarrow t = \frac{W}{V_0 \cos\theta}$

$$\boxed{V_{xF} = V_0 \cos\theta}$$

$V_{xF} = 15 \cos(37)$

$V_{xF} = 12.0 \ m/s$

y

$0 = h + V_0 \sin\theta \, t - \frac{1}{2} g t^2$

$V_{yF} = V_0 \sin\theta - gt$

$$\boxed{V_{yF} = V_0 \sin\theta - g \frac{W}{V_0 \cos\theta}}$$

$V_{yF} = 15 \cdot \sin 37 - \frac{(9.8)(25)}{15 \cos 37}$

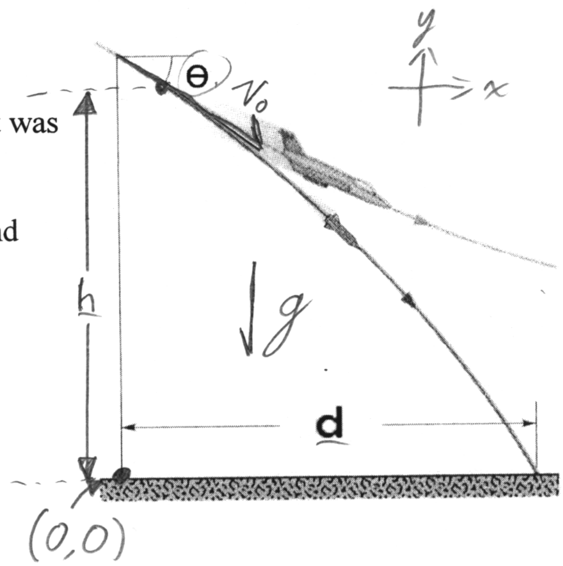
$V_{yF} = -11.4 \ m/s$

$V_F = (12^2 + 11.4^2)^{1/2} = 16.5 \ m/s$

Gumby needs cruthes

An airplane is flying at a speed of V_0 and diving at an angle of θ degrees below the horizontal when it drops a radar decoy. The decoy strikes the ground a horizontal distance of d from where it was released.

Find an expression for the height of the airplane above the ground when the decoy was released.



Given	Want
$\lvert\vec{V}_0\rvert$	h
θ	
d	

x

$$x = x_0 + V_{0x}t + \tfrac{1}{2}a_x t^2$$

① $d = 0 + \lvert\vec{V}_0\rvert \cos\theta \, t + 0$

$\Rightarrow t = \dfrac{d}{\lvert\vec{V}_0\rvert \cos\theta}$ ③

y

$$y = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2$$

② $0 = h - \lvert\vec{V}_0\rvert \sin\theta \, t - \tfrac{1}{2} g t^2$

Initial y Velocity is negative

Gravity is negative

Subst ③ into ② and solve for h:

$$\Rightarrow h = \cancel{\lvert\vec{V}_0\rvert} \sin\theta \frac{d}{\cancel{\lvert\vec{V}_0\rvert}\cos\theta} + \tfrac{1}{2} g \left(\frac{d}{\lvert\vec{V}_0\rvert\cos\theta}\right)^2$$

$$\Rightarrow \boxed{h = d\tan\theta + d^2 \frac{g}{2(\lvert\vec{V}_0\rvert\cos\theta)^2}}$$