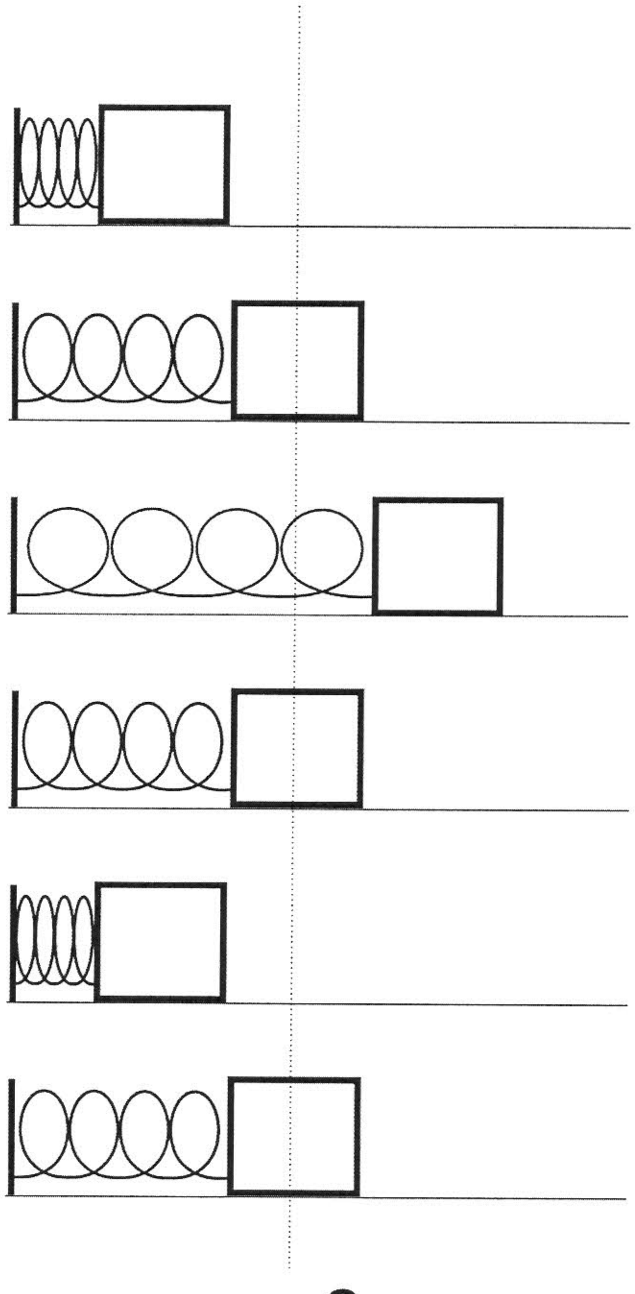
Oscillation – Set 1

Each row in the table below represents a snapshot of a mass attached to a spring. Assume that the mass starts from rest in the first row. In the second row, it is passing through x=0. In the third row, it has reached its maximum extension. In the fifth row, it has reached its maximum compression. In the cells below, mark an arrow indicating the direction of the associated force, acceleration, velocity, and position vectors for each row. If the magnitude is zero, put a zero in the cell.

F	a	v	x
→	→	0	←
0	0	→	0
←	←	0	→
0	0	←	0
→	→	0	←
0	0	→	0



x=0

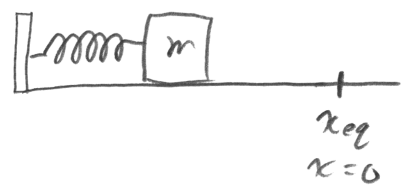
Oscillation – Set 1

Consider a mass attached to a spring allowed to move on a frictionless surface. Analyze the system as follows:

1) Draw a Free Body Diagram with a coordinate system. Let the positive direction be the direction of the initial displacement.

2) Write Newton's Second Law for the system. Substitute the Hook's Law in for F. Be careful with the minus signs.

3) Solve the resulting expression for the acceleration. Replace a with $\dfrac{d^2x}{dt^2}$.



$$F_s = ma$$

$$-kx = ma \Rightarrow a = -\frac{k}{m}x$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

If $x(t) = A\cos(\omega t + \varphi)$ is the position of a Simple Harmonic Oscillator, derive expressions for the velocity and acceleration of a Simple Harmonic Oscillator.

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left[A\cos(\omega t + \phi)\right]$$

$$\Rightarrow \boxed{v(t) = -\omega A \sin(\omega t + \phi)}$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left[-\omega A \sin(\omega t + \phi)\right]$$

$$\Rightarrow \boxed{a(t) = -\omega^2 A\cos(\omega t + \phi)}$$

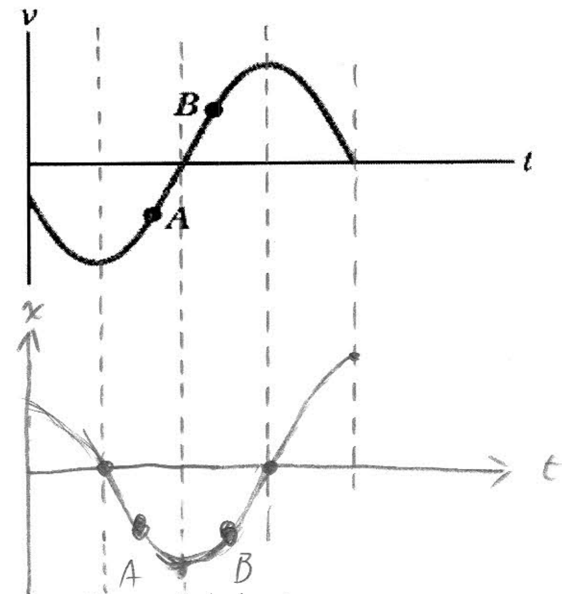
Oscillation – Set 1

If $x(t)=A\cos(\omega t+\varphi)$ is the position of a Simple Harmonic Oscillator, write expressions for the velocity and acceleration of a Simple Harmonic Oscillator.

$v(t) = -\omega A \sin(\omega t + \phi)$

$a(t) = -\omega^2 A \cos(\omega t + \phi)$

The figure to the right is the velocity of a SHO. Sketch the position versus time plot



Is the particle stationary, moving towards -x, or moving towards +x when the particle is at:

Point A: moving towards -x

Point B: moving towards +x

Is x=0, x>0, or x<0 when the particle is at:

Point A: $x < 0$

Point B: $x < 0$

Is the particle's speed (the magnitude of it's velocity) increasing, decreasing, or constant when the particle is at:

Point A: Decreasing

Point B: Increasing

Oscillation – Set 2

The acceleration of a particle in Simple Harmonic Motion is plotted in the figure below.

1) Which point(s) represent the particle's acceleration when it is at $x = -x_{max}$?

Point 2

2) Which point(s) represent the particle's acceleration when it is at $x = +x_{max}$?

Point 6

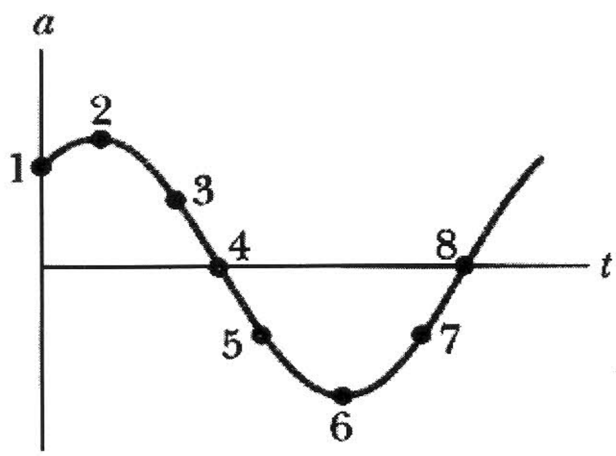
2) At point 4, is the velocity of the particle positive, negative, or zero?

Positive (just after a period of positive accel)

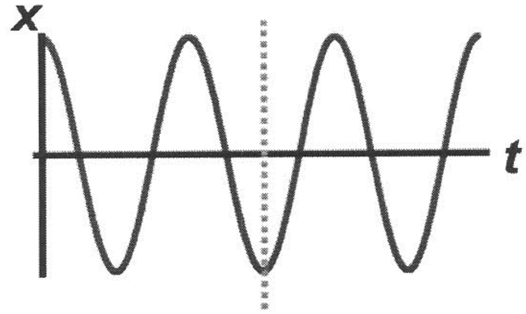
3) At point 5, what is the particle's position?

A) $x = 0$
B) $x = -x_{max}$
C) $x = +x_{max}$
D) $\boxed{0 < x < +x_{max}}$
E) $-x_{max} < x < 0$

$x(t) = A\cos(\omega t + \phi)$
$v(t) = -\omega A \sin(\omega t + \phi)$
$a(t) = -\omega^2 A \cos(\omega t + \phi)$



Below is a position versus time graph of a mass on a spring. What can you say about the velocity, net force, and acceleration at the time indicated by the dotted line?



Velocity: Positive, Negative, or zero: Zero

Force: Positive, Negative, or zero: Positive

Acceleration: Positive, Negative, or zero: Positive

If the amplitude of a simple harmonic oscillator is doubled, the maximum speed of the oscillator:

A) doubles

$V_{max} = \omega A$,

B) halves

C) stays the same

If the amplitude of a simple harmonic oscillator is doubled, the period of the oscillations:

A) doubles

Period is independent of Amplitude.

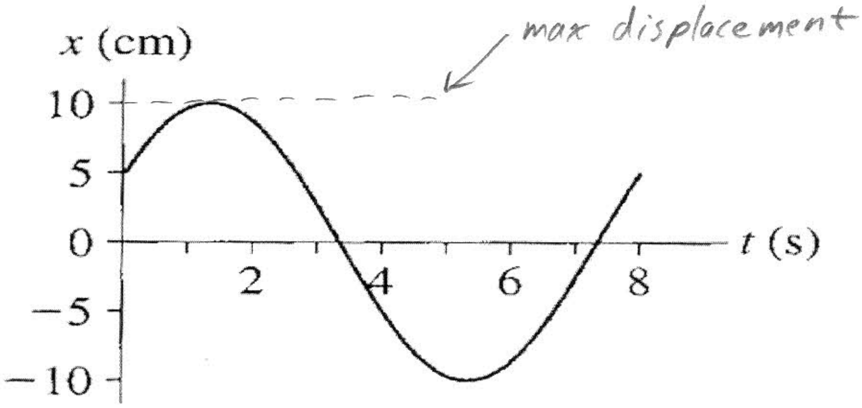
B) halves

C) stays the same

The figure below is a position versus time graph of a particle in simple harmonic motion. Assume that its position as a function of time is given by

$$x(t) = A\cos(\omega t + \varphi)$$

where A, ω and φ are constants.



a) What is the maximum displacement (amplitude) of the particle?

10cm

b) Which constant in the above equation gives the maximum displacement, or amplitude, of the oscillations? *(HINT: What's the maximum possible value of cosine?)*

max value of cosine is +1.

$x_{max} = A \cdot 1 = \boxed{A}$

c) What is the value *x(0)* (ie. *x* when *t* = 0)?

at $t=0$, $x = 5$cm. so: $x(0) = 5$

d) Given your answer to part c, solve $x(0) = A\cos(\omega t + \varphi)$ for φ (the phase constant) when t = 0?

$x(0) = A\cos(\omega t + \phi)$

$5 = 10\cos(0 + \phi) \Rightarrow \cos\phi = \frac{1}{2}$

$\boxed{\phi = \cos^{-1}\left(\frac{1}{2}\right)}$

e) What is the period (T) of the oscillations?

Period is the time required for one cycle.
According to the graph, $\underline{T = 8\,\text{sec.}}$

f) What are the units of ω? (*HINT: What are the units of the input to the cosine function?)*

Cosine takes an angle, (let's use radians)
so $(\omega t + \phi)$ must have units of radians.
$\Rightarrow$, $\boxed{\omega \text{ must be rad/sec}}$

g) What is the mathematical relationship between ω and T?

$T\left(\frac{\text{sec}}{\text{cycle}}\right)$ and there are $2\pi\left(\frac{\text{rad}}{\text{cycle}}\right)$ so $\boxed{T = \frac{2\pi}{\omega}}$

$\omega\left(\frac{\text{rad}}{\text{sec}}\right)$ $T = \dfrac{2\pi\left(\cancel{\text{rad}}/\text{cycle}\right)}{\omega\left(\cancel{\text{rad}}/\text{sec}\right)} = \dfrac{2\pi}{\omega}\left(\frac{\text{sec}}{\text{cycle}}\right)$

h) Good! Now calculate the numerical value of ω.

$$\omega = \frac{2\pi}{8} = \boxed{\frac{\pi}{4}\,\text{rad/s}}$$

d) What is the maximum velocity of the particle?
(HINT: What's the maximum possible value of sine?)

$v = -\omega A \sin(\omega t + \phi)$

when $\sin = -1$, $v = \omega A = \frac{\pi}{4} \cdot 10 = \boxed{\frac{5}{2}\pi}$

e) What is the maximum acceleration of the particle?
(HINT: What's the maximum possible value of cosine?)

$a = -\omega^2 A \cos(\omega t + \phi)$

when $\cos = -1$, $a = \omega^2 A = \frac{\pi^2}{16} \cdot 10 = \boxed{\frac{5}{8}\pi^2}$

Oscillation – Set 1

You are given the position and velocity of a simple harmonic oscillator (SHO) at some time t:

$$x(t)=x_0 \text{ and } v(t)=v_0 .$$

Starting with the equations for position and velocity:

$$x(t)=A\cos(\omega t+\varphi) \;,\quad v(t)=-\omega A\sin(\omega t+\varphi)$$

a) find an expression for the amplitude, A, of a **S**imple **H**armonic **O**scillator in terms of x_0 and v_0.

① Square both equations:

$x_0^2 = A^2\cos^2(\omega t+\phi)$, $v_0^2 = \omega^2 A^2 \sin^2(\omega t+\phi)$

② Divide v eq. by ω^2:

$v_0^2/\omega^2 = A^2 \sin^2(\omega t+\phi)$

③ Add them together

$x_0^2 + \frac{v_0^2}{\omega^2} = A^2\cos^2(\omega t+\phi) + A^2\sin^2(\omega t+\phi) \Rightarrow x_0^2 + \frac{v_0^2}{\omega^2} = A^2(\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi))$

$\Rightarrow \boxed{A = \left[x_0^2 + \frac{v_0^2}{\omega^2}\right]^{1/2}}$

b) find an expression for the phase angle, ϕ, of a **S**imple **H**armonic **O**scillator in terms of x_0 and v_0.

Divide v_0 by x_0:

$\frac{v_0}{x_0} = \frac{-\omega \cancel{A}\sin(\omega t+\phi)}{\cancel{A}\cos(\omega t+\phi)} \Rightarrow -\frac{v_0}{\omega x_0} = \tan(\omega t+\phi)$

$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right) - \omega t}$