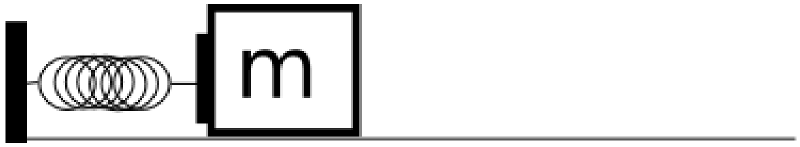
Oscillation – Set 2

Name: ______________________ Problems Solved __/ 9

A block of mass *m* is attached to a spring with spring constant *k* and allowed to move on a frictionless surface.



Step 1: Find the *Oscillator Frequency*

a) Draw a Free Body Diagram for this system.

b) Let the equilibrium position of the spring be x=0, write **Newton's Second Law**, and solve it for the acceleration of the system. Rewrite a as $\frac{d^2x}{dt^2}$.

c) Substitute the solution to the **Simple Harmonic Oscillator** equation into your differential equation.

d) Solve the resulting equation for ω.

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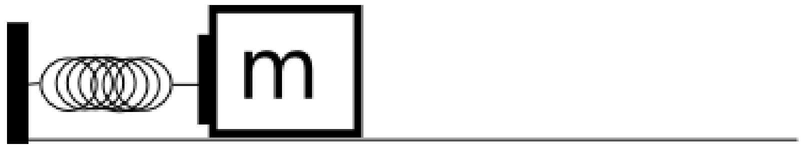

At $t=0$, the block is at $x=0$ (the spring's equilibrium point), and is moving to the right with a velocity $v=v_0$.

Step 2: Find the *Amplitude* and the *Phase angle* given a set of initial conditions.

a) Substitute the initial t, v, and x into your equations for position and velocity.
b) Solve the resulting system of equations for the phase angle φ.
c) Determine with of the two possible phase angles is correct.

d) Substitute the initial t, v, and x into your equations for position and velocity.
e) Solve the resulting system of equations for the amplitude A.

What happens to A and φ as v_0 is increased?

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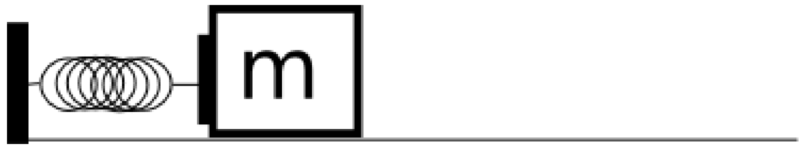
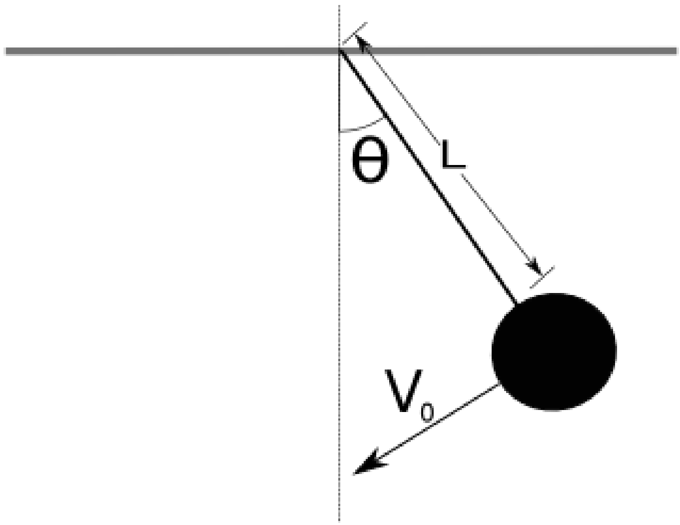


At $t=0$, the block is compressed a distance *d* and released from rest.

a) Find the amplitude *A* and the phase angle φ.
b) What happens to *A* as *d* is increased?
c) What happens to φ as *d* is increased?
d) Why is φ different than it was in the last problem?

Below is a simple pendulum consisting of a massless rod of length *L* with a point mass if mass *m* attached to the end.

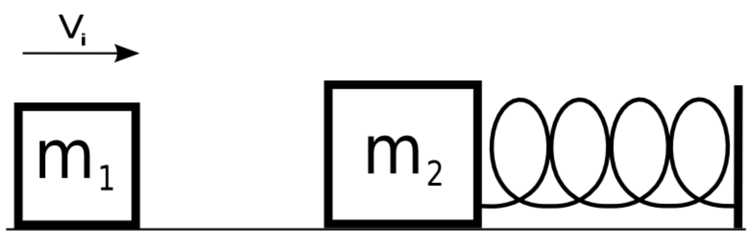
a) Find the frequency of small oscillations of the pendulum.
b) At t=0, the pendulum makes an angle θ_0 with the vertical and the point mass has a velocity V_0. What is the phase angle of the oscillator?



A meter stick with a mass M is suspended from one end and allowed to swing like a pendulum.

a) What is the **period** of small oscillations?
b) What length *L* does a simple pendulum (a point mass attached to a massless rod) need in order to have the same period?

A block with a mass of m_1 = 10 kg is moving to the right with a velocity V_i. It collides and sticks to a block with a mass of m_2 = 15 kg. The second mass is attached to a spring with spring constant k=3 N/m. Before the collision, the spring is at rest in it's equilibrium position.

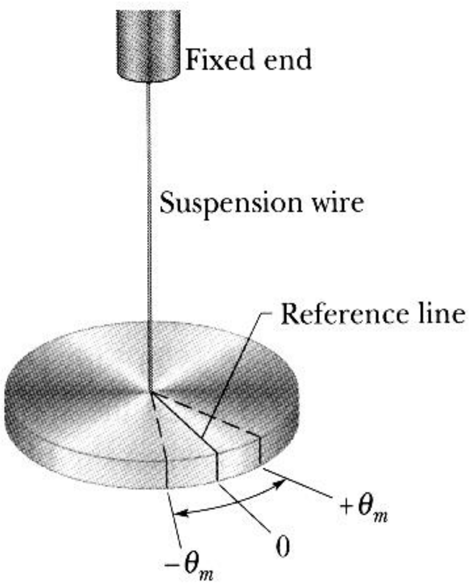


a) What is the angular frequency of the resulting oscillator after the collision?
b) Assuming that the moment of collision is t=0, find the phase constant of the oscillator.
c) If the resulting amplitude of the oscillator is A = 3 m, what was the initial velocity of m_1?

The device in the picture below is known as a torsion pendulum. It is a flat disk attached to a length of stiff wire. When the wire is twisted, it responds by providing a torque on the disk, much the same way a spring provides a force when it is stretched. The torque provided by the wire is $T = -\kappa\theta$, where κ (greek letter kappa) is the torsion constant and theta is the angular displacement from equilibrium.

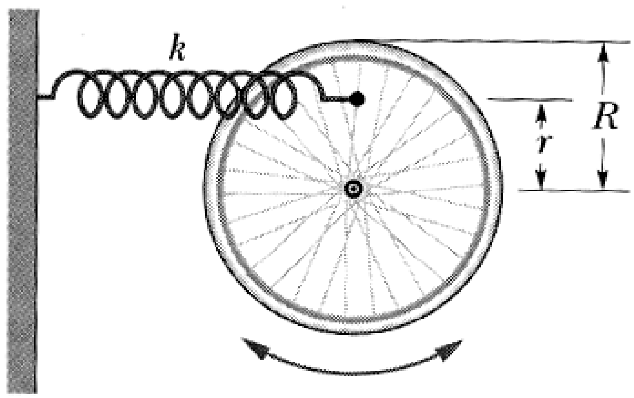
a) The moment of Inertia of the disk is $I = \frac{1}{2}MR^2$. Using the rotational version of Newton's Second Law, find the oscillator frequency of the torsion pendulum.

b) If a solid bar of length L, $I = \frac{1}{12}ML^2$, were suspended from the wire, what would the oscillator frequency be?



4) A wheel is free to rotate about a fixed axle. A spring with a spring constant *k* is attached to one of its spokes at a distacne *r* from the axle, as shown in the picture. Assume that the wheel is a hoop of mass *m* and radius *R* (the spokes have negligible mass).

a) Using **Newton's Second Law**, find the angular frequency of small oscillations in terms of *m, R, r* and the spring constant *k*.
b) What is the angular frequency if r = R.
c) What is the angular frequency if r = 0.



A solid cylinder of mass *M* is attached to a horizontal spring with force constant *k*. The cylinder can roll without slipping along the horizontal plane. When the system is displaced from the equilibrium position, it executes simple harmonic motion. Derive an expression for the period of the oscillations in terms of *M, k, I* and *R*.