SAMPLE TEST 6
PHYS 111, FALL 2011, SECTION 1

Name: ______________________________

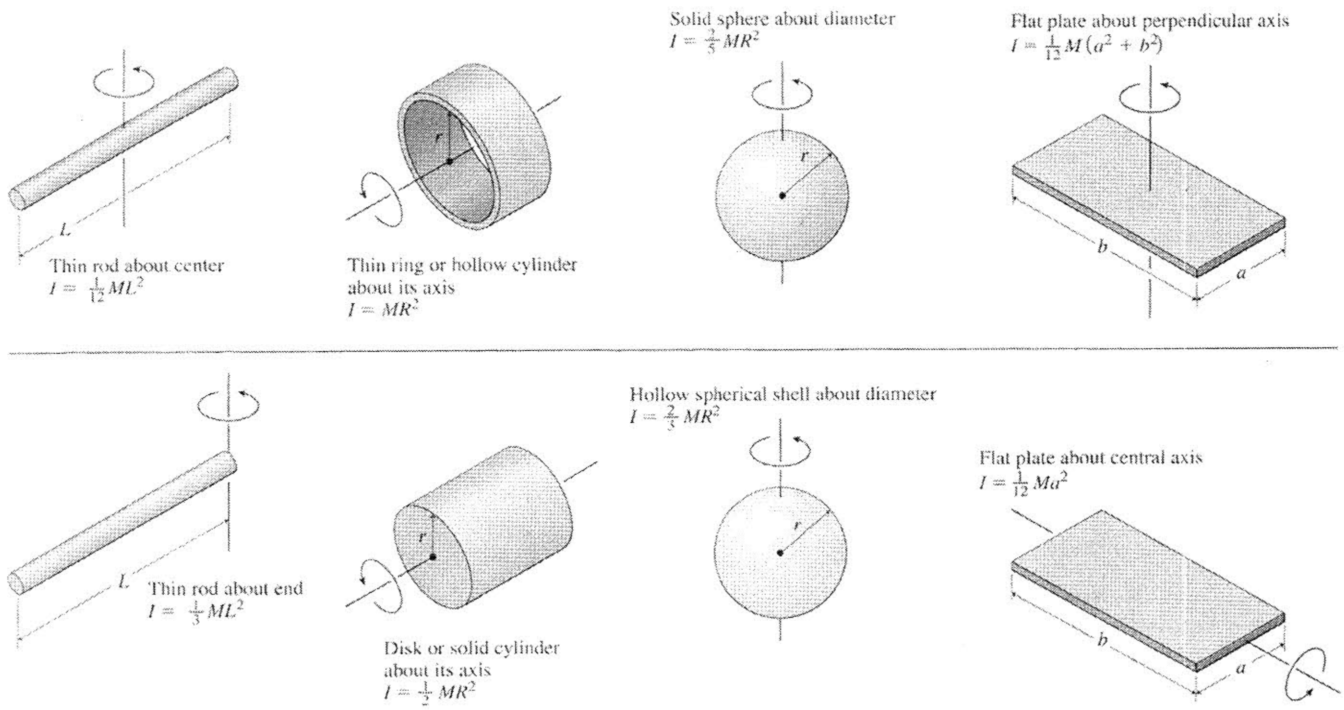
By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

A few potentially useful equations

Moment of Inertia, discrete definition	$I=\sum m_i r_i^2$
Moment of Inertia, integral definition	$I=\int r^2\,dm$
Parallel Axis Theorem	$I=I_{cm}+Md^2$
Superposition	$I_{Total}=\sum I_i$

TABLE 10.2 Rotational Inertias



SAMPLE TEST 6

PHYS 111, FALL 2011, SECTION 1

1) Derivations

a) (10pts) Given a differential equation of the form $\frac{d^2 x(t)}{dt^2} = -\omega^2 x(t)$, write the general solution for $x(t)$, $v(t)$, and $a(t)$ in terms of the angular frequency ω, the amplitude A, and the phase angle φ.

$x(t) = A\cos(\omega t + \phi)$

$v(t) = -\omega A \sin(\omega t + \phi)$

$a(t) = -\omega^2 A \cos(\omega t + \phi)$

b) (10pts) Given the boundary conditions $x(t_0) = x_0$ and $v(t_0) = v_0$, derive an expression for the phase angle φ and the amplitude A in terms of x_0, v_0, and ω.

Find ϕ - Divide v/x

$$x_0 = A\cos(\omega t_0 + \phi)$$
$$v_0 = -\omega A \sin(\omega t_0 + \phi)$$

$$\Rightarrow \frac{v_0}{x_0} = \frac{-\omega \cancel{A} \sin(\omega t + \phi)}{\cancel{A} \cos(\omega t + \phi)}$$

$$\Rightarrow \frac{-v_0}{x_0 \omega} = \tan(\omega t + \phi) \Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{-v_0}{x_0\omega}\right) - \omega t}$$

Find A - Square both eq. divide v by ω^2 and add

$$x_0^2 = A^2 \cos^2(\omega t + \phi), \quad v_0^2/\omega^2 = A^2 \sin^2(\omega t + \phi)$$

$$\left(x_0^2 + v_0^2/\omega^2\right) = A^2\left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right)$$

$$\Rightarrow \boxed{A = \left(x_0^2 + \frac{v_0^2}{\omega^2}\right)^{1/2}}$$

SAMPLE TEST 6
PHYS 111, FALL 2011, SECTION 1

2) Multiple Choice

2.1) A mass attached to a spring oscillates with a period T. If the amplitude of the oscillation is doubled, the period will be:

A) T
B) 1.5 T
C) $2T$
D) ½ T
E) $4T$

$\omega = \sqrt{\frac{k}{m}}$, $T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

independent of $\underline{A}$

2.2) An object of mass m, oscillating on the end of a spring with spring constant k has amplitude A. Its maximum speed is:

A) $A\sqrt{\frac{k}{m}}$
B) $A^2\frac{k}{m}$
C) $A\sqrt{\frac{m}{k}}$
D) $A\frac{m}{k}$

$v = -\omega A \sin(\omega t + \phi)$

$v_{max} = \omega A$, $\omega = \sqrt{k/m}$

2.3) In simple harmonic motion, the magnitude of the acceleration is greatest when:

A) the displacement is zero
B) the displacement is maximum ← greatest restoring Force
C) the speed is maximum
D) the force is zero
E) the speed is between zero and its maximum

SAMPLE TEST 6

PHYS 111, FALL 2011, SECTION 1

2.4) The displacement of an object oscillating on a spring is given by $x(t) = A\cos(\omega t + \phi)$.
If the initial displacement is zero and the initial velocity is in the negative x direction, then the phase constant φ is:

A) 0 radians
B) π/2 radians
C) π radians
D) 3π /2 radians
E) 2 π radians

$0 = A\cos(\phi) \Rightarrow \phi = \boxed{\pi/2}, \ 3\pi/2$

$v = -\omega A \sin(\phi)$

To keep minus sign

2.5) A simple pendulum of length L and mass M has frequency f. To increase its frequency to 2f:

A) increase its length to 4L
B) increase its length to 2L
C) decrease its length to L/2
D) decrease its length to L/4
E) decrease its mass to < M/4

$\omega = \sqrt{g/l}$

$\omega_2 = 2\omega_1$

$\frac{\omega_1}{\omega_2} = \left(\frac{l_2}{\cancel{g}} \cdot \frac{\cancel{g}}{l_1}\right)^{1/2}$

$\frac{\cancel{\omega_1}}{2\cancel{\omega_1}} = \left[\frac{l_2}{l_1}\right]^{1/2} \Rightarrow \frac{1}{4} = \frac{l_2}{l_1} \Rightarrow \boxed{l_1 = 4 l_2}$

$\Rightarrow \boxed{l_2 = \tfrac{1}{4} l_1}$ *

A block with a mass of m = 2.00 kg is attached to a spring with a spring constant k = 100 N/m. When t = 1.00 s, the position and velocity of the block are x(1s) = 0.129 m and v(1s) = 3.415 m/s.

a) Find the angular frequency, ω, of the oscillator.

b) Find the phase constant.

c) Find the amplitude

d) What was the position of the block at t = 0.00 s?

Step 1 – Use NSL to find the oscillator frequency.

FBD

NSL

$$\sum F = ma$$

$$F_s = m\frac{d^2x}{dt^2}, \quad F_s = -kx$$

$$\Rightarrow -kx = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x} \Rightarrow$$ Simple Harmonic Oscillator!

Now, to get the oscillator frequency, we can plug our general solution into the above equation.

General solution: $x(t) = A\cos(\omega t + \phi)$

$$\Rightarrow \frac{d^2}{dt^2}\left[A\cos(\omega t + \phi)\right] = -\frac{k}{m}A\cos(\omega t + \phi)$$

$$\Rightarrow -\omega^2 \cancel{A\cos(\omega t + \phi)} = -\frac{k}{m}\cancel{A\cos(\omega t + \phi)}$$

$$\Rightarrow -\omega^2 = -\frac{k}{m} \Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}} \Rightarrow \omega = \sqrt{50} = 7.07$$

Oscillation Set 1, P4 continued

Step ② – Use Physics to find the initial conditions

In this case, they're given in the problem statement.

at $t = 1.0s$, $\boxed{x(1) = 0.129m}$, $\boxed{v(1) = 3.415\ m/s}$

Step ③ – Use the SHO general solution to solve the rest of the problem.

General Solution:

$$x(t) = A\cos(\omega t + \phi)$$
$$v(t) = -\omega A\sin(\omega t + \phi)$$

Apply our initial conditions

$$x(1) = A\cos(\omega(1) + \phi)$$
$$v(1) = -\omega A\sin(\omega(1) + \phi)$$

Now we can divide to eliminate A and solve for ϕ

$$\frac{v(1)}{x(1)} = \frac{-\omega A\sin(\omega+\phi)}{A\cos(\omega+\phi)} \Rightarrow \frac{v(1)}{x(1)} = -\omega\tan(\omega+\phi)$$

$$\Rightarrow \tan(\omega+\phi) = -\frac{v(1)}{\omega x(1)}$$

$$\Rightarrow \omega + \phi = \tan^{-1}\left(\frac{-v(1)}{\omega x(1)}\right)$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{-v(1)}{\omega x(1)}\right) - \omega}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{-3.415}{(7.07)(0.129)}\right) - 7.07 = \boxed{-8.38 \text{ radians}}$$

Oscillations Set 1, P4 continued

Plug our answer for ϕ back into the $x(1)$ eq. and solve for A:

$$x(1) = A\cos(\omega + \phi)$$

$$\Rightarrow \boxed{A = \frac{x(1)}{\cos(\omega + \phi)}}$$

$$\Rightarrow \boxed{A = \frac{0.129}{\cos(7.07 - 8.38)} = 0.5\,m}$$

Finally, Put it all together to find $x(0)$

$$x(0) = 0.5\cos(\omega \cdot (0) - 8.38)$$

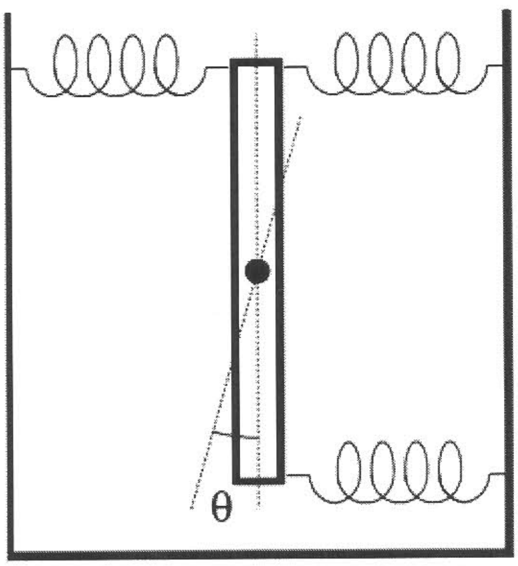
$$\boxed{x(0) = -0.25\,m}$$

SAMPLE TEST 6
PHYS 111, FALL 2011, SECTION 1

In the system shown below each of the three springs have a spring constant of 50 N/m and the bar is mounted on a frictionless pivot at its midpoint. The period of small oscillations is found to be 2.0 s.

The moment of inertia for the beam is $I=\frac{1}{12}ml^2$.

What is the mass of the bar?



Given	want
k, T	m

* Assuming that the pivot is at $\frac{l}{2}$ from the end and the springs are at the ends.

I'll use energy to get ω and then solve for m.

$$E_T = \underbrace{\tfrac{1}{2}I\omega^2}_{\text{rotating bar}} + \underbrace{3\left[\tfrac{1}{2}kx^2\right]}_{\text{3 Identical springs at equal displacements}}$$

continued ↓

Sample Test 6 continued:

Find ω by taking a derivative

$$\frac{dE_T}{dt} = I\omega\frac{d\omega}{dt} + 3kx\frac{dx}{dt}$$

$$\Rightarrow 0 = I\omega\alpha + 3kxv$$

Let's go rotation: $x = \frac{l}{2}\theta$, $v = \frac{l}{2}\omega$

$$\Rightarrow 0 = I\cancel{\omega}\alpha + 3k\frac{l}{2}\theta\frac{l}{2}\cancel{\omega}$$

$$\Rightarrow \alpha = -\frac{3kl^2}{4I}\theta \Rightarrow \alpha = -\frac{3k\cancel{l^2}}{4\cdot\frac{1}{\cancel{12}\,3}m\cancel{l^2}}\theta$$

$$\Rightarrow \alpha = -\left[\frac{9k}{m}\right]\theta \Rightarrow \omega = \left[\frac{9k}{m}\right]^{1/2}$$

and $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\left[\frac{m}{9k}\right]^{1/2}$

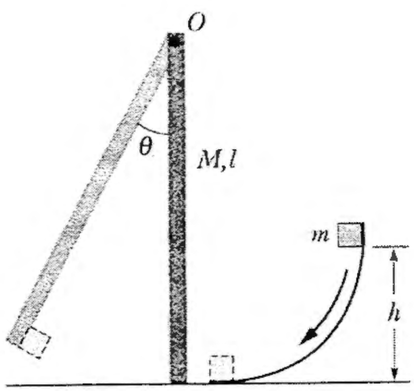
$$\Rightarrow \boxed{m = \frac{T^2}{4\pi^2}9k}$$

SAMPLE TEST 6

PHYS 111, FALL 2011, SECTION 1

A particle of mass m slides down a frictionless surface, collides with a uniform vertical rod of mass M and length l, and sticks. Let $m = M$ and Treat the mass m as a point mass at the end of the rod.

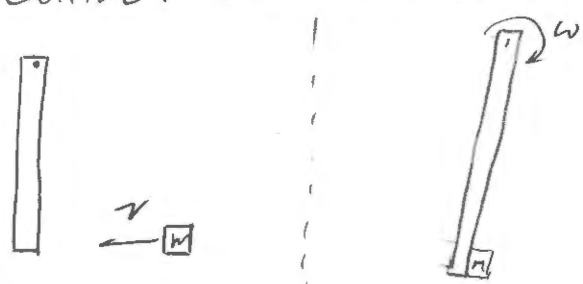
a. What is the amplitude of the resulting oscillator after the collision assuming the rod was initially at rest?
b. What is the angular frequency of the resulting oscillator?



① Drop the block:

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

② Collide: Conserve L



$$mlv = I\omega$$

$$\Rightarrow \omega = \frac{mlv}{I}$$

③ Swing up to max θ:

$$\frac{1}{2}I\omega^2 = 2mg\,l_{cm}(1-\cos\theta)$$

$$\frac{1}{2}\cancel{I}\frac{m^2l^2v^2}{I^{\cancel{2}}} = 2mg\,l_{cm}(1-\cos\theta)$$

$$\frac{1}{2}\frac{\cancel{m^2}\cancel{l^2}2gh}{\frac{4}{3}\cancel{m}\cancel{l^2}} = 2\cancel{mg}\frac{3}{4}l(1-\cos\theta)$$

$$1-\cos\theta = \frac{h}{2l} \Rightarrow \boxed{\theta_{max} = \cos^{-1}\left(1-\frac{h}{2l}\right)}$$

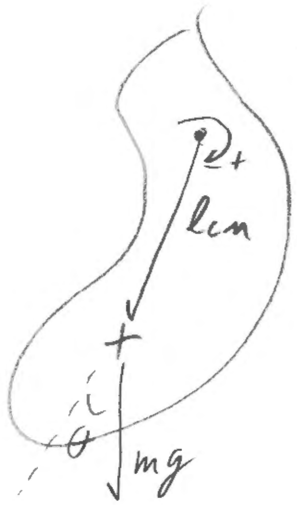
Rod ↓ block ↓

$$I = \frac{1}{3}ml^2 + ml^2$$

$$\Rightarrow I = \frac{4}{3}ml^2$$

$$l_{cm} = \frac{\frac{l}{2}m + lm}{2m} = \frac{3}{4}l$$

b) In general for a pendulum...



$$T = I\alpha$$

$$-mgl_{cm}\sin\theta = I\alpha$$

Small oscillations: $\sin\theta \cong \theta$

$$-mgl_{cm}\theta = I\alpha = \alpha = \frac{-mgl_{cm}}{I}\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgl_{cm}}{I}\theta$$

$$\Rightarrow \omega = \left[\frac{mgl_{cm}}{I}\right]^{1/2}$$ In general

For <u>our</u> pendulum:

$$I = \tfrac{4}{3}ml^2, \quad l_{cm} = \tfrac{3}{4}l$$

so $$\omega = \left[\frac{mg\frac{3}{4}l}{\frac{4}{3}ml^2}\right]^{1/2} \Rightarrow \boxed{\omega = \frac{3}{4}\sqrt{\frac{g}{l}}}$$