Rotation – Set 3

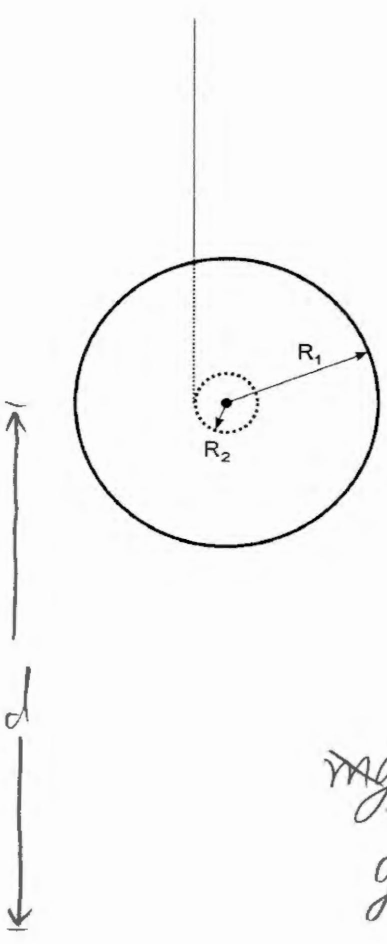
Name: ______________________ **Problems Solved __/ 2**

The picture below represents the side view of a yo-yo. The inner dashed circle represents the axle that the string is wound around. The top of the string is held stationary and the yo-yo is allowed to fall, unwinding the string as it descends.

The moment of inertia of the yo-yo is: $I_{cm} = \frac{1}{2} M R_1^2$

Use Work/Energy techniques to answer the following questions.

a) If the yo-yo starts from rest, what is its angular velocity after a length of string, *d*, is unwound?
b) What is the translational velocity of the center of the yo-yo



Given	Want
I	ω, v
M	
d	

$U_I = mgd, \quad U_F = 0$

$K_I = 0, \quad K_F = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$v = R_2 \omega$

$$\cancel{m} g d = \frac{1}{2} \cancel{m} v^2 + \frac{1}{2}\left(\frac{1}{2} \cancel{M} R_1^2\right)\omega^2$$

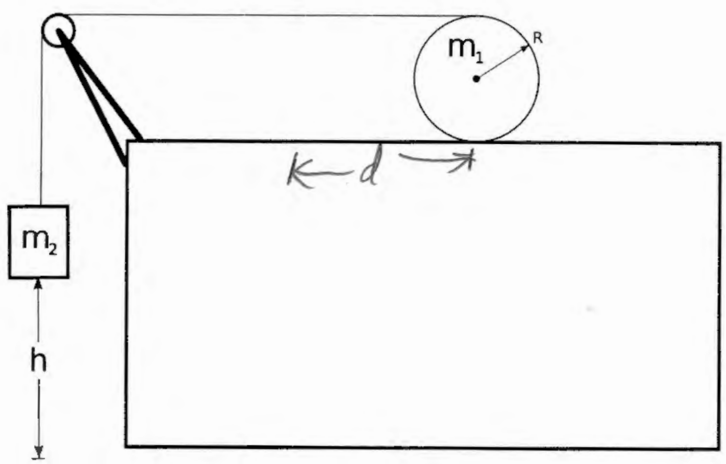
$$g d = \frac{1}{2} R_2^2 \omega^2 + \frac{1}{4} R_1^2 \omega^2$$

$$\Rightarrow \omega = \left[\frac{g d}{\frac{1}{2} R_2^2 + \frac{1}{4} R_1^2}\right]^{1/2}$$

$$\Rightarrow v = R_2 \left[\frac{g d}{\frac{1}{4} R_2^2 + R_1^2}\right]^{1/2}$$

In the picture below, a cylinder with mass m_1, and radius R is sitting on a table. One end of a string is wound around the cylinder. The other end of the string passes over a massless and frictionless pulley and is attached to a hanging weight of mass m_2.

What is the angular velocity of m_1 after m_2 has fallen a distance h?



Given: m_1, m_2, h, R

Want: ω

$U_I = m_2 g h, \quad U_F = 0$

$K_I = 0, \quad K_F = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I \omega^2$

Because the string unwinds from the cylinder as it rolls, m_2 falls twice as far as m_1 rolls

So: $\underline{h = 2d} \Rightarrow \underline{v_2 = 2v_1}$

Then: $v_1 = R\omega, \quad v_2 = 2R\omega$

$\Rightarrow m_2 g h = \frac{1}{2} m_2 4R^2\omega^2 + \frac{1}{2} m_1 R^2 \omega^2 + \frac{1}{2}\left(\frac{1}{2} m_1 R^2\right)\omega^2$

$\Rightarrow \boxed{\omega = \left[\frac{gh}{2m_2 + \frac{3}{4}m_1}\right]^{1/2}}$