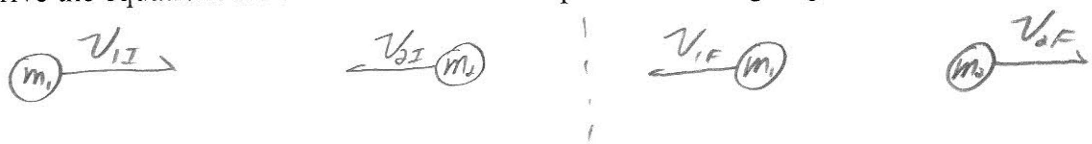
Sample Test 4
Phys 111 Spring 2010

Name: ______________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

Show your work on all of the problems – your approach to the problem is as important as (if not MORE) important than) your final answer.

1) Derive the equations for the final velocities of particles undergoing an elastic collision in 1 dimension.



conserve momentum: $m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{2F}$ ①

conserve energy : $\frac{1}{2} m_1 v_{1I}^2 + \frac{1}{2} m_2 v_{2I}^2 = \frac{1}{2} m_1 v_{1F}^2 + \frac{1}{2} m_2 v_{2F}^2$ ②

Get all m_1 on right and all m_2 on left

$m_1(v_{1I} - v_{1F}) = m_2(v_{2F} - v_{2I})$ ③

$m_1(v_{1I}^2 - v_{1F}^2) = m_2(v_{2F}^2 - v_{2I}^2)$ ④

Divide $\frac{④}{③}$: $\frac{\cancel{m_1}(v_{1I}^2 - v_{1F}^2)}{\cancel{m_1}(v_{1I} - v_{1F})} = \frac{\cancel{m_2}(v_{2F}^2 - v_{2I}^2)}{\cancel{m_2}(v_{2F} - v_{2I})}$

Remember:
$(a^2 - b^2) = (a+b)(a-b)$: $\frac{\cancel{(v_{1I} - v_{1F})}(v_{1I} + v_{1F})}{\cancel{(v_{1I} - v_{1F})}} = \frac{\cancel{(v_{2F} - v_{2I})}(v_{2F} + v_{2I})}{\cancel{(v_{2F} - v_{2I})}}$

$v_{1I} + v_{1F} = v_{2I} + v_{2F}$ ⑤

Solve ⑤ for v_{2F}: $v_{2F} = v_{1I} + v_{1F} - v_{2I}$ ⑥

continued
↓

Sample Test 4, P1 continued

Put ⑥ → ①

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 (v_{1I} + v_{1F} - v_{2I})$$

$$m_1 v_{1I} + m_2 v_{2I} = m_1 v_{1F} + m_2 v_{1I} + m_2 v_{1F} - m_2 v_{2I}$$

$$v_{1F}(m_1 + m_2) = (m_1 - m_2) v_{1I} + 2 m_2 v_{2I}$$

$$\boxed{v_{1F} = \frac{m_1 - m_2}{m_1 + m_2} v_{1I} + \frac{2 m_2}{m_1 + m_2} v_{2I}}$$

And swap subscripts for v_{2F}

$$\boxed{v_{2F} = \frac{m_2 - m_1}{m_1 + m_2} v_{2I} + \frac{2 m_1}{m_1 + m_2} v_{1I}}$$

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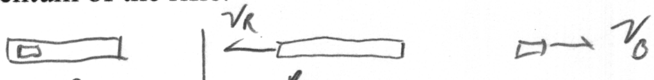
2.1 When you fire a rifle, the bullet flies out and the rifle recoils. The magnitude of the momentum of the bullet is _____ the momentum of the rifle.



a. greater than
b. the same as
c. less than

P_I | P_F

$0 = P_r + P_B \Rightarrow \boxed{P_r = -P_B} \Rightarrow \boxed{v_R = -\frac{m_B}{m_R} v_B}$

2.2 When you fire a rifle, the bullet flies out and the rifle recoils. Assuming that the bullet weighs less than the rifle, the Kinetic Energy of the bullet is _____ the Kinetic Energy of the rifle.

a. greater than
b. the same as
c. less than

$\frac{K_R}{K_B} = \frac{\frac{1}{2} m_R v_R^2}{\frac{1}{2} m_B v_B^2} = \frac{m_R}{m_B} \frac{1}{v_B^2} \cdot \frac{m_B^2}{m_R^2} v_B^2 \Rightarrow \frac{K_R}{K_B} = \frac{m_B}{m_R}$

Two cars with masses m_1 and m_2 undergo a perfectly elastic collision. Assume that the initial velocity of car 2, V_{2I}, is zero and that $m_1 > m_2$.

2.3) The post collision velocity of car 1, V_{1F}, will be:

a. Positive
b. Negative
c. Zero
d. No way to know.

$V_{1F} = \frac{m_1 - m_2}{m_1 + m_2} V_{1I}$ ← POS... IF $m_1 > m_2$

2.4) The post collision velocity of car 1, V_{1F}, will be _____ the post collision velocity of car 2, V_{2F}.

a. greater than
b. equal to
c. less than
d. No way to know.

$\frac{V_{1F}}{V_{2F}} = \left[\frac{m_1 - m_2}{\cancel{m_1 + m_2}} \cancel{V_{1I}}\right] \cdot \left[\frac{\cancel{m_1 + m_2}}{2m_1} \frac{1}{\cancel{V_{1I}}}\right] = \frac{m_1 - m_2}{2m_1} < 1$

2.5) According to Newton's Second Law for systems of particles, if the sum of the external forces is zero:

a. The position of the center of mass must be constant
b. The velocity of the center of mass must be zero
c. The momentum of the system must be constant.
d. None of the above statements is true.

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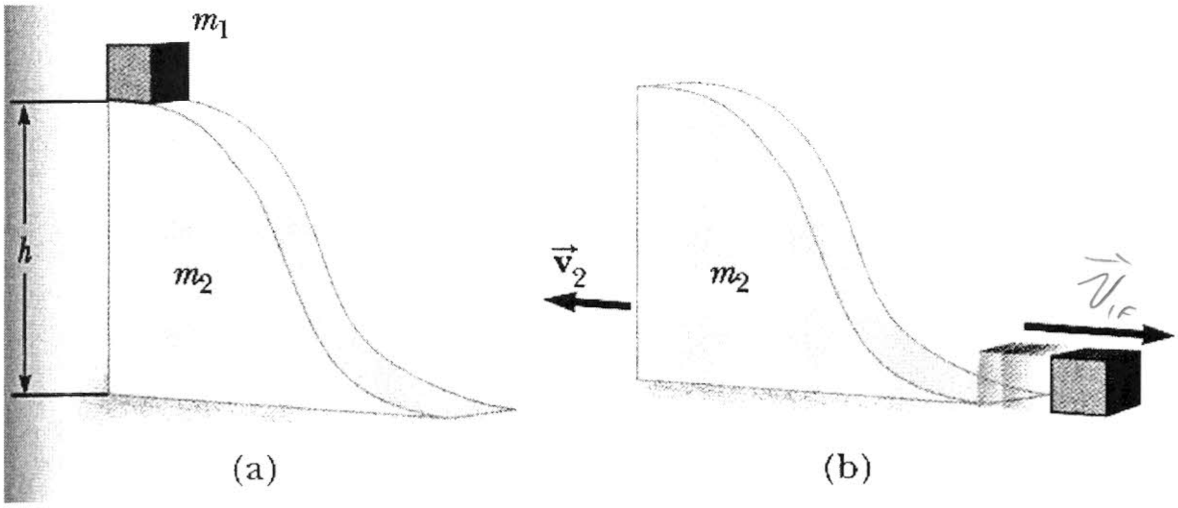
4. A small block of mass m_1 is released from rest from the top of a curve-shaped frictionless wedge of mass m_2 that sits on a frictionless horizontal surface. After leaving the wedge, m_1 has a velocity v_1.

Find an expression for the height of the ramp, h, in terms of m_1, m_2, and v_1.



Because there is no friction, $\sum \vec{F}_{ext} = 0$ and we can conserve momentum:

$$m_1 \cancel{v_{1I}}^{0} + m_2 \cancel{v_{2I}}^{0} = m_1 v_{1F} + m_2 v_{2F} \quad (1)$$

We can also conserve Energy:

$$U_I + \cancel{K_I}^{0} + \cancel{W_{NCF}}^{0} = \cancel{U_F}^{0} + K_F$$

$$m_1 g h + 0 + 0 = 0 + \tfrac{1}{2} m_1 v_{1F}^2 + \tfrac{1}{2} m_2 v_{2F}^2 \quad (2)$$

I will solve (1) for v_{2F} and sub it into (2):

From (1): $v_{2F} = -\frac{m_1}{m_2} v_{1F}$

into (2): $\cancel{m_1} g h = \tfrac{1}{2} \cancel{m_1} v_{1F}^2 + \tfrac{1}{2} m_2 \frac{m_1^2}{m_2^2} v_{1F}^2$

$$\Rightarrow \boxed{h = \frac{v_{1F}^2}{2g}\left[1 + \frac{m_1}{m_2}\right]}$$

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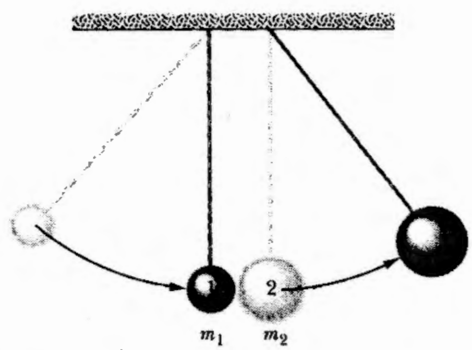
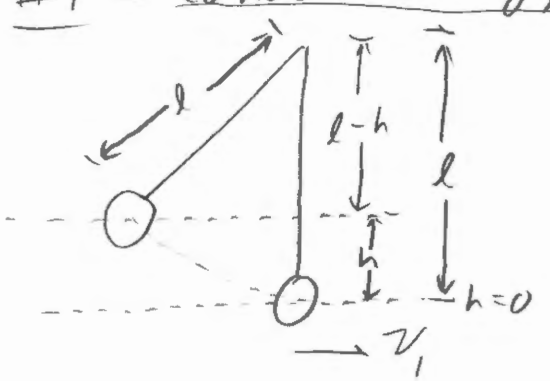
Two metal spheres of mass m_1 and m_2, are suspended by cords so that they just touch when the cords are vertical. m_1 is pulled back so that its cord makes an angle θ_1 with the vertical, is released from rest, and allowed to collide **elastically** with m_2.

Find the maximum angle, θ_2, that the cord attached to m_2 makes with the vertical.

Given	Want
m_1	θ_2
m_2	
θ_1	



#1 – Conserve energy



$U_I = mgh$

$= m_1 g(l - l\cos\theta)$

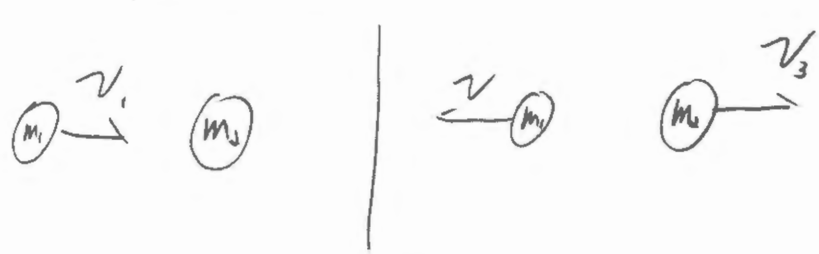
$K_I = 0$

$U_F = 0$

$K_F = \frac{1}{2} m_1 v_1^2$

$$\boxed{\cancel{m_1} g(l - l\cos\theta_1) = \frac{1}{2}\cancel{m_1} v_1^2}$$

#2 – Elastic collision



$$v_{2F} = \frac{m_2 - m_1}{m_2 + m_1} \cancel{v_{2I}}^{\,0} + \frac{2m_1}{m_1 + m_2} v_{1I} \Rightarrow \boxed{v_3 = \frac{2m_1}{m_1 + m_2} v_1}$$

#3 - Conserve energy

Same as #1, But backwards...

$U_I = 0$ $\qquad K_I = \frac{1}{2} m_2 V_3^2$

$U_F = m_2 g(l - l\cos\theta_2)$ $\qquad K_F = 0$

$$\frac{1}{2} \cancel{m_2} V_3^2 = \cancel{m_2} g(l - l\cos\theta_2)$$

Put it together

$$2g(l - l\cos\theta_1) = \left(\frac{m_1+m_2}{2m_1}\right)^2 V_3^2$$

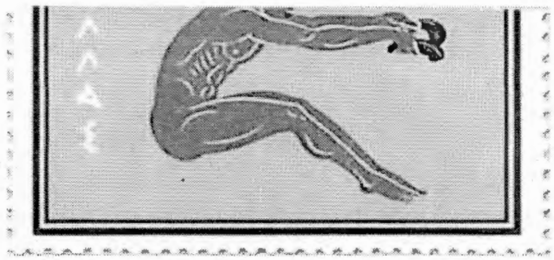
$$\cancel{2g}(\cancel{l} - \cancel{l}\cos\theta_1) = \left(\frac{m_1+m_2}{2m_1}\right)^2 \cancel{2g}(\cancel{l} - \cancel{l}\cos\theta_2)$$

$$1 - \cos\theta_1 = \left(\frac{m_1+m_2}{2m_1}\right)^2 (1 - \cos\theta_2)$$

$$\boxed{\cos\theta_2 = 1 - \left(\frac{2m_1}{m_1+m_2}\right)^2 (1 - \cos\theta_1)}$$

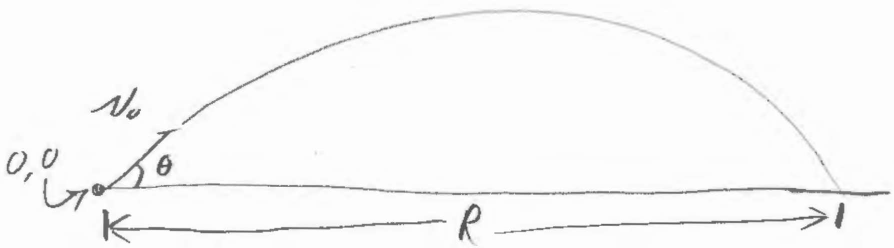
hmm... ick.

lengthen their jumps. The weights were swung in front of them just before liftoff (as pictured) and then thrown behind them during the flight.



Let the mass of the athlete be M_A, the mass of the halteres be M_H. The athlete leaves the ground (with or without the *halteres)* with an initial velocity V_0 at an angle θ with the horizontal. At the peak of his trajectory, he throws the weights backward so that their velocity with respect to the ground is zero. How much distance is added to the athlete's jump by throwing the weights?

#1 - Trajectory to get R_{cm} (Kinematics)



$x: \quad x = \cancel{x_0}^{0} + V_{0x}t + \cancel{\tfrac{1}{2}a_x t^2}^{0}$

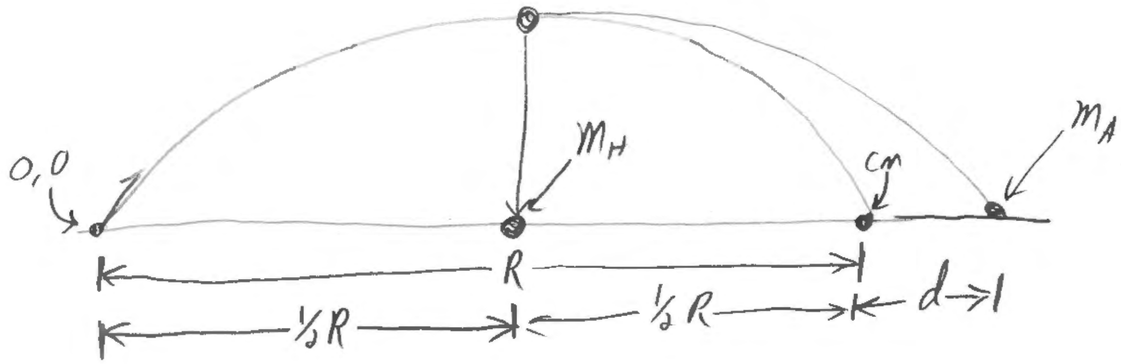
$R = V_0 \cos\theta t$

$y: \quad y = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2$

$0 = 0 + V_0 \sin\theta \cancel{t} - \tfrac{1}{2}g t^{\cancel{2}} \Rightarrow t = \tfrac{2}{g} V_0 \sin\theta$

$\Rightarrow \boxed{R = \tfrac{2}{g} V_0^2 \sin\theta \cos\theta}$

#2 - Center of mass calculation



Now, CM lands at R, m_H lands at $\frac{1}{2}R$, and m_A lands at $R+d$.

So: $$R = \frac{m_H \frac{1}{2}R + m_A(R+d)}{m_H + m_A} \Rightarrow m_H R + \cancel{m_A R} = m_H \frac{R}{2} + \cancel{m_A R} + m_A d$$

$$\Rightarrow \frac{1}{2} m_H R = m_A d$$

$$\Rightarrow d = \frac{m_H}{2m_A} R$$

$$\Rightarrow \boxed{d = \frac{m_H}{m_A} \cdot \frac{1}{g} v_0^2 \sin\theta \cos\theta}$$