Electrostatics – Set 6

Name: ______________________ **Problems Solved __/ 8**

Conceptual questions: (Remember to explain your reasoning)

Any excess charge placed on a conductor will:

1) move to the interior of the conductor
2) distribute itself evenly within the conductor
3) move to the exterior surface of the conductor
4) None of the above

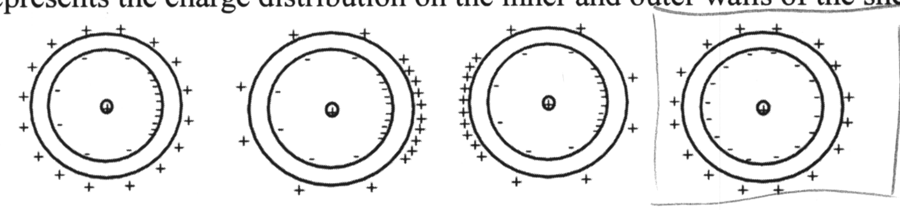
The Excess charges will move as far apart as possible and $\vec{E} = 0$ inside the conductor

Which of the following is a TRUE statement?

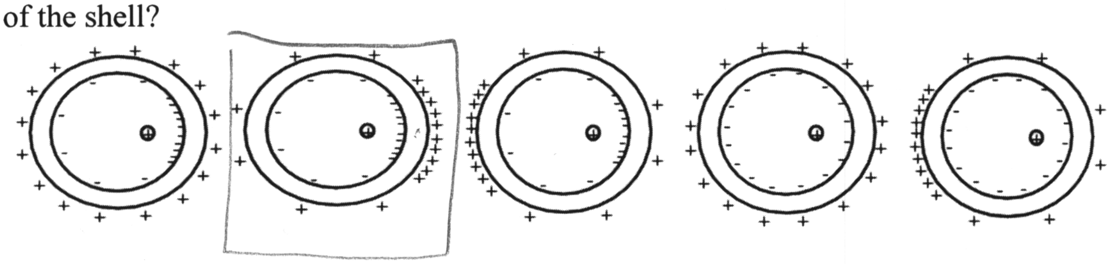
5) The electric field within a conductor in equilibrium is never zero
6) The electric field within a conductor in equilibrium is always zero
7) The electric field within a conductor in equilibrium is sometimes zero
8) None of the above

If $\vec{E}$ wasn't zero, the charges would move.

A positive charge is kept (fixed) at ***the center*** inside a fixed spherical ***neutral*** conducting shell. Which of the following represents the charge distribution on the inner and outer walls of the shell?



The positive charge is now moved and kept fixed ***off-center*** inside the fixed spherical neutral conducting shell. Which of the following represents the charge distribution on the inner and outer surfaces of the shell?



A 250 nC point charge is placed at the center of an uncharged spherical conducting shell 20 cm in radius.

(a) Find the surface charge density on the outer surface of the shell
(b) Find the electric field strength at the shell's outer surface.

a)

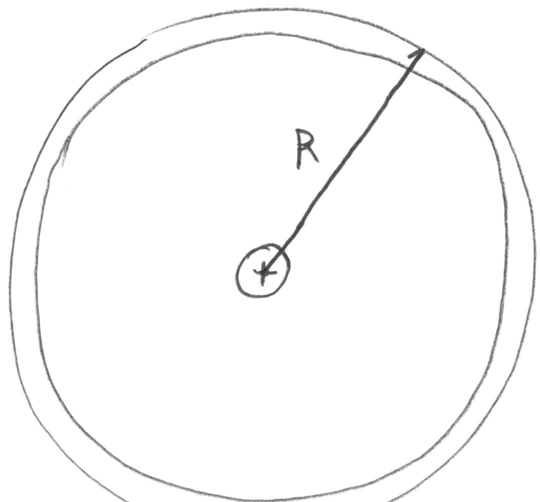
Given

$Q = 250\,nC$

$R = 20\,cm$

Want

$\sigma, |\vec{E}|$



The conductor will have $-Q$ on its inner surface and $+Q$ on its outer surface.

$$\boxed{\sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2}} \Rightarrow \sigma = \frac{250 \times 10^{-9}\,C}{4\pi (20 \times 10^{-2}\,m)^2}$$

$$\boxed{\sigma = 0.4\,\frac{nC}{m^2}}$$

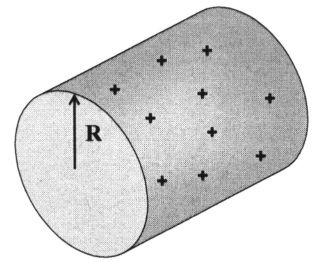
b) Place a Gaussian surface at the surface of the shell

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \vec{E} \cdot \vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow \boxed{|\vec{E}| = \frac{Q_{enc}}{A \cdot \epsilon_0}} \Rightarrow |\vec{E}| = \frac{Q_{enc}}{4\pi R^2 \epsilon_0} = \boxed{\frac{kQ}{R^2}}$$

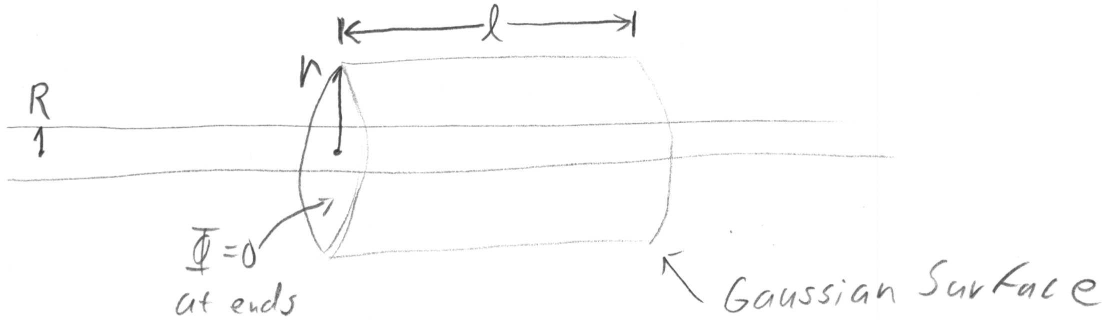
$$\Rightarrow |\vec{E}| = \frac{9 \times 10^9 \left(\frac{N \cdot m^2}{C^2}\right)(250 \times 10^{-9}\,C)}{(20 \times 10^{-2}\,m)^2} = \boxed{5.6 \times 10^4\,N/C}$$

Calculate the electric field both inside and outside of an infinitely long cylindrical shell (it's hollow) of radius R and surface charge density σ. Write your answers in terms of the constants k (=1/4πε$_o$) and σ. After finding the field outside the cylinder, compare your answer to that of an infinite line charge.



A Gaussian surface at $r < R$ would enclose no charge:

$$\Phi_{r<R} = \frac{Q_{enc}}{\epsilon_0}, \quad \text{But } Q_{enc} = 0 \Rightarrow \boxed{\Phi_{r<R} = 0}$$



$$\Phi_{r>R} = \frac{Q_{enc}}{\epsilon_0}; \quad Q_{enc} = \sigma A_1,$$

A_1 – Surface area of cylindrical shell Enclosed

$$A = 2\pi R l$$

$$\Phi_{r>R} = \vec{E} \cdot \vec{A}_2$$

A_2 – Surface area of the wall of the gaussian surface

$$\Phi_{r>R} = |\vec{E}| 2\pi r l$$

$\vec{E} \perp \vec{A}_2$ at all points

$$\Rightarrow |\vec{E}| \cancel{2\pi} r \cancel{l} = \frac{\sigma \cancel{2\pi} R \cancel{l}}{\epsilon_0} \Rightarrow \boxed{|\vec{E}| = \frac{\sigma R}{\epsilon_0 r} = \frac{k 4\pi R \sigma}{r}}$$

The result for a line is:

$$|\vec{E}| = \frac{2k\lambda}{r}$$, same $1/r$ dependence as our cylinder

* What's λ for the cylinder?

$$\lambda = \sigma \cdot \underbrace{2\pi R}_{\text{integrate around the circumference}}, \Rightarrow \sigma = \frac{\lambda}{2\pi R}$$

So: $$|\vec{E}| = \frac{k4\pi R\sigma}{r} = \frac{k \overset{2}{\cancel{4}} \cancel{\pi} \cancel{R}}{r} \frac{\lambda}{\cancel{2\pi R}}$$

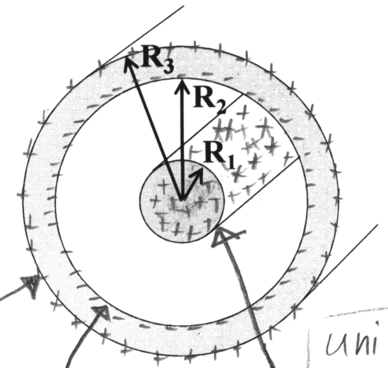
$$\boxed{|\vec{E}| = \frac{2k\lambda}{r}}$$ $\rightarrow$ same result as the line.

(1)

Electrostatics – Set 6

An infinitely long insulating cylinder is surrounded by a hollow conducting cylinder. The solid inner cylinder has a radius of R_1 and a charge per unit length of $+6\lambda$, with the charge uniformly distributed throughout its volume. The outer cylinder has an inner radius of R_2, an outer radius of R_3, and a net charge per unit length of -4λ.

(a) Show how the charge is distributed throughout the system.
(b) Find expressions for the electric field for all values of the distance r.
(c) Draw a graph of the electric field for all values of r.



radial

Positive charge on outer wall

negative charge on inner wall

uniform distribution on central cylinder

b) $\boxed{r<R_1}$ - Inside inner cylinder

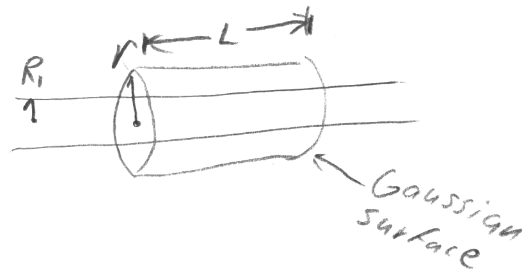
$$\Phi_1 = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E_1 A_r = \frac{\rho V_r}{\epsilon_0}, \quad V_r = \pi r^2 L$$

$$\boxed{\rho = \frac{Q}{V_{R_1}} = \frac{6\lambda \cancel{L}}{\pi R_1^2 \cancel{L}} \Rightarrow \rho = \frac{6\lambda}{\pi R_1^2}}$$

Find ρ

$$\Rightarrow E_1(\cancel{2}\pi \cancel{r} \cancel{L}) = \frac{1}{\epsilon_0}\cdot\frac{\overset{3}{\cancel{6}}\lambda}{\cancel{\pi} R_1^2}\cdot\cancel{\pi} r^{\cancel{2}}\cancel{L}$$



$$\Rightarrow \boxed{\vec{E}_1 = \frac{3\lambda}{\epsilon_0 \pi R_1^2} r\hat{r}}$$

$\ast\,\boxed{R_1 < r < R_2}$ → outside inner cylinder but inside inner conductor

$$\Phi_2 = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E_2 A_r = \frac{6\lambda L}{\epsilon_0} \Rightarrow E_2 \cancel{2}\pi r \cancel{L} = \frac{\overset{3}{\cancel{6}}\lambda \cancel{L}}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E}_2 = \frac{3\lambda}{\pi\epsilon_0 r}\hat{r}}$$

continued ↓

Electrostatics Set 6, P4 continued

$\boxed{R_3 > r > R_2}$ – inside conductor, $|\vec{E}_3| = 0$

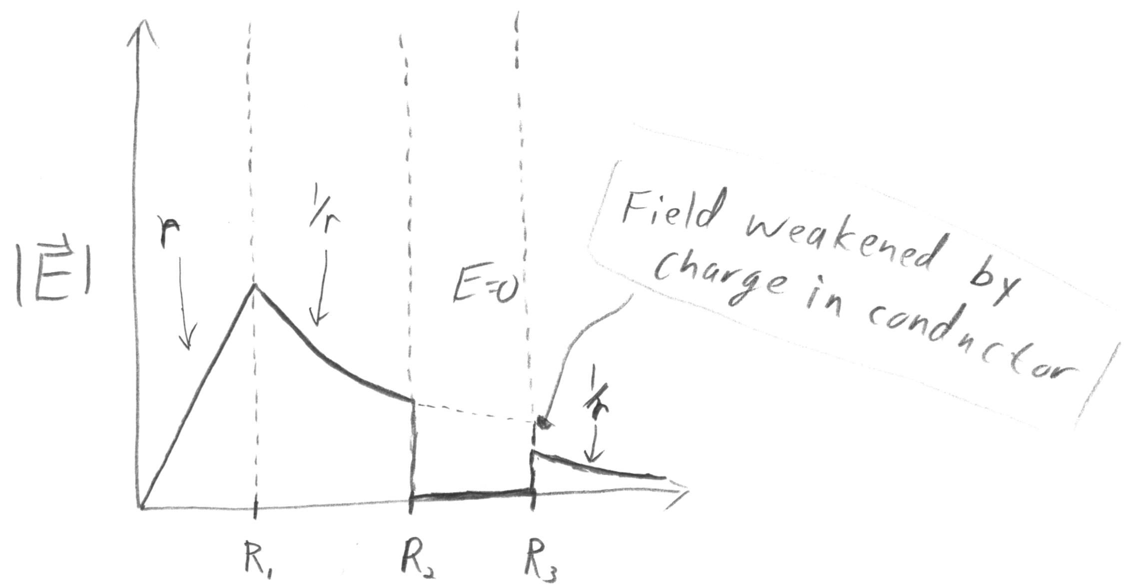
$\boxed{r > R_3}$ – outside of the conductor

$\Phi_4 = \frac{Q_{enc}}{\epsilon_0}$, $Q_{net} = (6\lambda - 4\lambda)L$

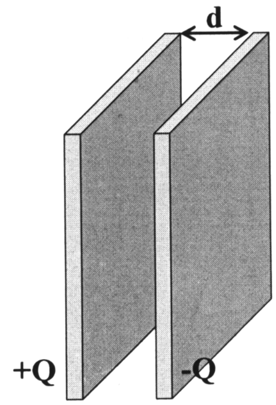
$E(4\pi r \cancel{L}) = (6\lambda - 4\lambda)\cancel{L} \frac{1}{\epsilon_0}$

$\Rightarrow \boxed{\vec{E}_4 = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$

c)



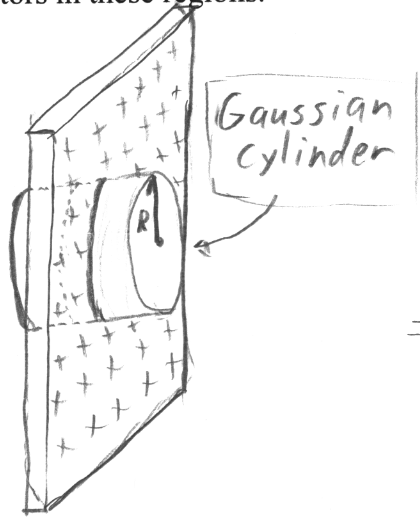
The arrangement of two electrodes shown in the figure is often called a "parallel-plate capacitor" and plays an important role in many electric circuits. One electrode has a charge $+Q$ while the other has a charge $-Q$. The plates are separated by a distance d. If the plates are large compared to their separation, you can assume them to be infinite in extent.



(a) Use Gauss's Law to find the field a distance d above an infinite sheet of charge.

(b) Using the result of (a), calculate the electric field to the left, to the right, and in between the plates. Draw arrows representing the electric field vectors in these regions.

a)



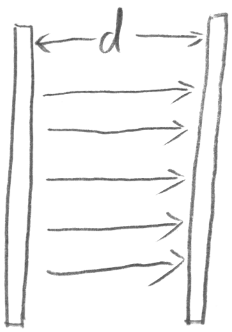
$$\Phi_E = \frac{Q_{enc}}{\epsilon_0}$$

$$EA = \frac{\sigma \pi R^2}{\epsilon_0}$$

$$\Rightarrow E\cdot 2\cancel{\pi}\cancel{R^2} = \frac{\sigma \cancel{\pi}\cancel{R^2}}{\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

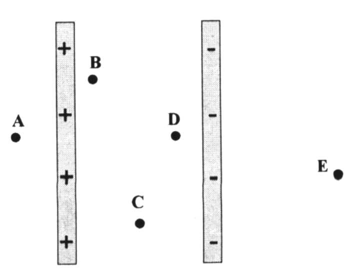
b)

$E=0$



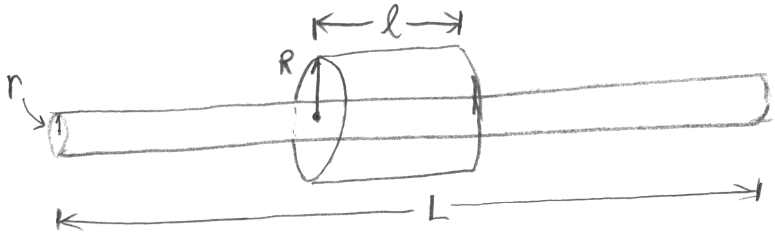
$E=0$ $\quad E = \frac{\sigma}{\epsilon_0}$ in between the plates.

(c) Rank in order, from largest to smallest, the forces F_A to F_E a proton would experience if placed at points A-E near this parallel plate capacitor.



$A = E < B = C = D$

Wolfson, Volume II, 2nd Edition, Problem 21.35



Given

$L = 50\,cm$

$r = 1.0\,cm$

$Q = 2.0\,\mu C$

$R_1 = 4.0\,mm$

$R_2 = 23\,m$

Want

E_{R1}

E_{R2}

In general:

$$\Phi = \frac{Q_{enc}}{\epsilon_0}, \quad Q_{enc} = \lambda l, \quad \lambda = \frac{Q}{L}$$

$$EA = \frac{Q}{\epsilon_0}\frac{l}{L} \Rightarrow E \cdot 2\pi R \cancel{l} = \frac{Q}{\epsilon_0}\frac{\cancel{l}}{L}$$

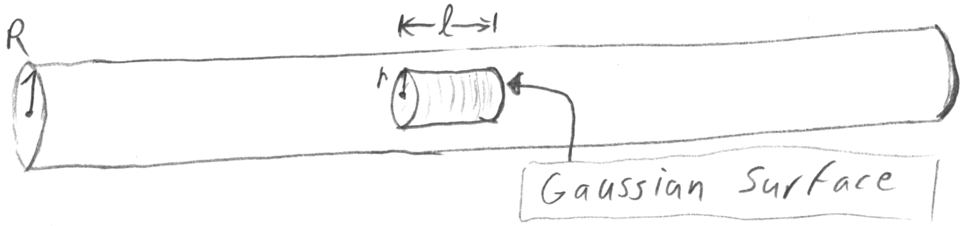
$$\Rightarrow \boxed{E = \frac{Q}{2\pi\epsilon_0 R L}}$$

a) $$E_{R1} = \frac{2\times10^{-6}\,C}{2\pi(8.8\times10^{-12}\,C^2/N\cdot m^2)(1.4\times10^{-2}\,m)(50\times10^{-2}\,m)} = \boxed{5.2\times10^{6}\,N/C}$$

b) $$E_{R2} = \frac{2\times10^{-6}\,C}{2\pi(8.8\times10^{-12}\,C^2/N\cdot m^2)(23.01\,m)(50\times10^{-2}\,m)} = \boxed{3.1\times10^{3}\,N/C}$$

Wolfson, Volume II, 2nd Edition, Problem 21.54

Given	Want
R	E_r
ρ	
r	

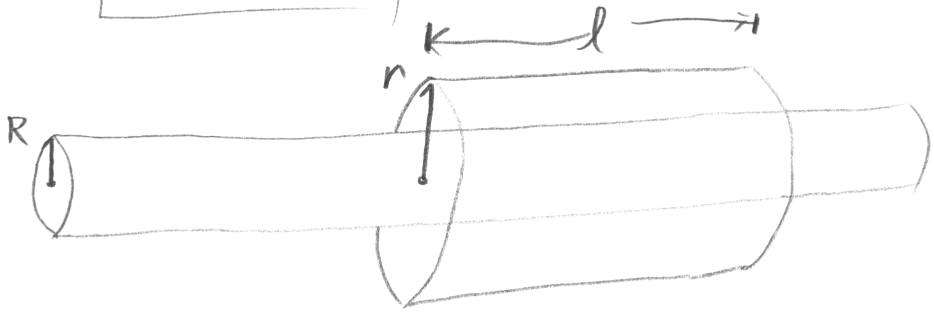


$r<R$:

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{\rho \cdot \pi r^2 l}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\cancel{\pi}\cancel{r}\cancel{l} = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{l}}{\epsilon_0}$$

$$\Rightarrow \boxed{E_{r<R} = \frac{\rho r}{2\epsilon_0}}$$



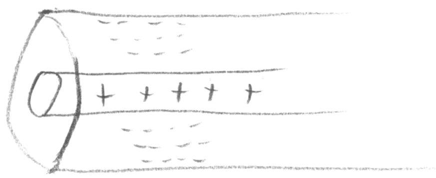
$r>R$:

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA_G = \frac{\rho V}{\epsilon_0}$$

$$\Rightarrow E\, 2\cancel{\pi} r \cancel{l} = \frac{\rho \cancel{\pi} R^2 \cancel{l}}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\rho R^2}{2 r \epsilon_0}}$$

Wolfson, Volume II, 2nd Edition, Problems 21.71, 72, and 73
(These three multiple choice questions go together. Please explain your answers or justify them with math.)

71) 

Inside field goes as $1/r$
Outside field is zero
since net charge is zero
inside conductor, field is
also zero.

$\boxed{\text{answer: a}}$

72) Excess charge on a conductor moves to the outer surface.

$\boxed{\text{Answer: a}}$

73) As stated in (71), it is the field of an infinite line charge and goes as $1/r$.

$\boxed{\text{Ansewer: b}}$