# Physics 112 Sample Test 1

*By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.*

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

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$$k = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9\,\mathrm{N}\frac{\mathrm{m}^2}{\mathrm{C}^2}$$

$$\varepsilon_0 = 8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$

$$|q| = 1.6\times10^{-19}\,\mathrm{C}\ \text{ for electron and proton}$$

$$m_{electron} = 9.11\times10^{-31}\,\mathrm{kg}$$

$$m_{proton} = 1.67\times10^{-27}\,\mathrm{kg}$$

$$v = v_{0x} + a_x t$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$x = x_0 + \tfrac{1}{2}(v_x + v_{0x})t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x = x_0 + v_x t - \tfrac{1}{2}a_x t^2$$

$$\int \frac{x\,dx}{\left(x^2+a^2\right)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

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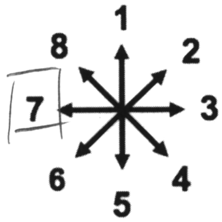
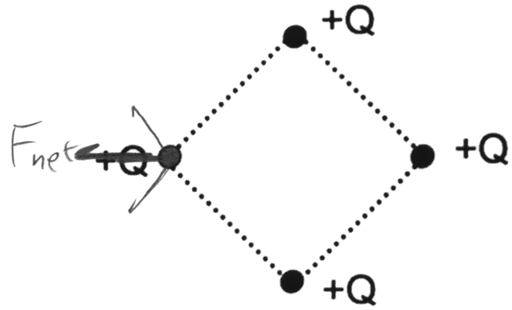
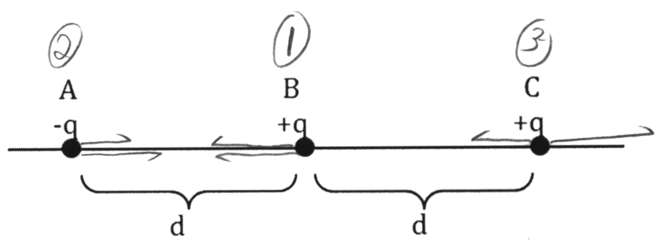
1.1) Four equal charges +Q are fixed to the corners of a square, as shown below. Select the direction of the net force on the charge on the left (represented by a gray circle).




1.2) Consider the electric charges shown below. List the charges in order of the magnitude of the force they experience, starting with the smallest. C A B

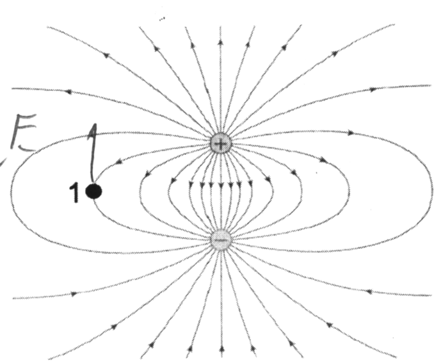
a) B, A, C
b) A, B, C
c) C, B, A
d) C, A, B
e) None of the above




1.3) The arrows represent electric field lines. What is the direction of the electric force on a ***negative*** test charge at point 1?

a) up
b) down
c) left
d) right
e) the net force is zero

$F = qE$



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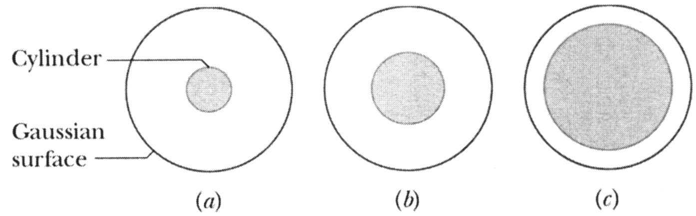
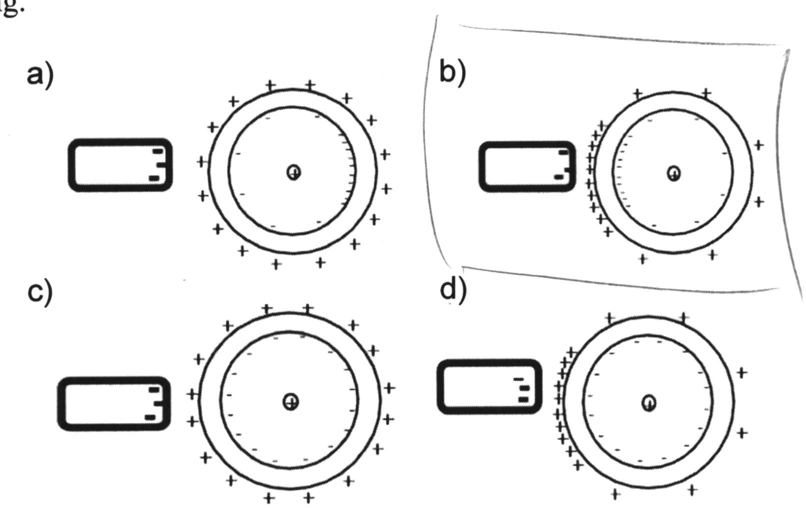
1.4) The following figure shows cross sections of three blue cylinders, each with charge Q uniformly distributed throughout its volume. Concentric with each cylinder is a red cylindrical Gaussian surface, all three have the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface.

a) $E_a>E_b>E_c$
b) $E_c>E_b>E_a$
c) $E_b>E_c>E_a$
d) ~~$E_b>E_a>E_c$~~
e) $\boxed{E_a=E_b=E_c}$




$$\Phi = \frac{Q_{enc}}{\epsilon_0}$$

1.5) A positive charge +Q is kept fixed at the center of a spherical neutral conducting shell. An object with negative charge –Q is brought near the outside of the sphere. Which of the following represents the charge distribution of this system? Remember to explain your reasoning.



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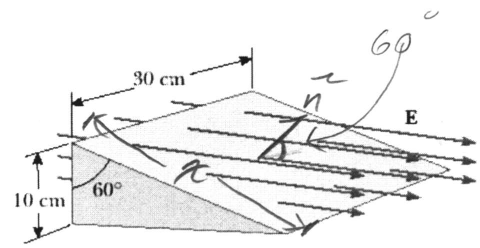
1.6) Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7800$ N/C. Calculate the electric flux through (i) the vertical rectangular surface, (ii) the slanted surface, and (iii) the entire surface of the box.




$x\cos 60 = 10$

$\Phi = \vec{E}\cdot\vec{A}$

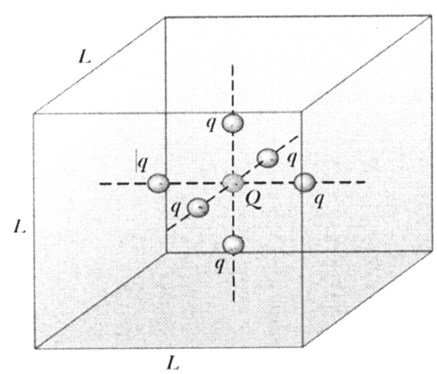
Flux through the back wall is equal and opposite the flux through the wedge front.

i) $-(7800)(10\times10^{-2})(30\times10^{-2}) = -2340\ \frac{N\cdot m^2}{C}$

ii) $2340\ \frac{N\cdot m^2}{C}$

iii) zero.

1.7) A particle with charge $Q = 5.0\times10^{-6}$ C is located at the center of a cube of edge $L = 0.10$ m. In addition, six other identical charged particles having $q = -1.0\times10^{-6}$ C are positioned symmetrically around $Q$, as shown. Determine the electric flux through one face of the cube.




Gauss's Law gives flux through the cube:

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \Phi = \frac{1}{\epsilon_0}\cdot(Q+6q)$$

$$= \frac{1}{8.8\times10^{-12}}\left(5\times10^{-6} - 6(1\times10^{-6})\right)$$

$$\Phi = -1.1\times10^{5}\ \frac{N\cdot m^2}{C}$$

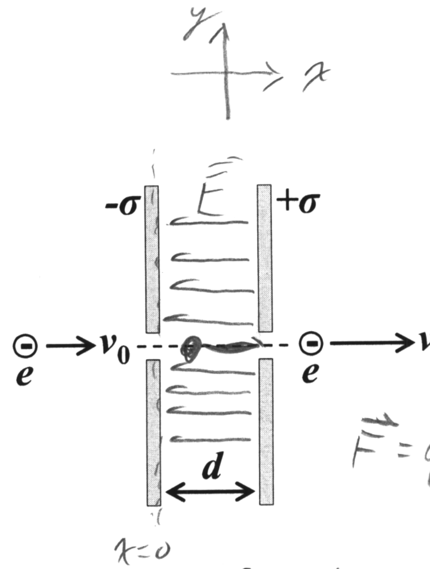
By symmetry, each face carries equal flux:

$$\boxed{\Phi_F = \frac{\Phi}{6} = -1.89\times10^{4}\ \frac{N\cdot m^2}{C}}$$

## Physics 112 Sample Test 1

2) A beam of electrons each with charge -*e* is accelerated by passing it through a region between two very large, charged, parallel, insulating plates separated by a distance *d*. Calculate the surface charge density σ on the plates if the electrons accelerate from $v_o$ to $v$.

As part of your answer, you should derive the electric field between the plates rather than simply using the result from lecture. (Ignore the holes in the plates when considering the electric field between the plates.)



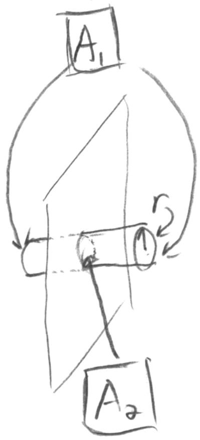

$\vec{F} = q\vec{E}$

Given
$-e, m, d$
$v_0, v$

want
$\sigma$

Find $\vec{E}$ between the plates

→ Field above one plate.




$$\Phi = \frac{Q_{enc}}{\epsilon_0}$$

$$EA_1 = \frac{\sigma A_2}{\epsilon_0},$$

$A_2$ = Area of plate enclosed
$A_1$ = Area of gaussian surface

$$E \cdot 2\cancel{\pi}\cancel{r^2} = \frac{\sigma \cancel{\pi}\cancel{r^2}}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}, \text{ For 1 plate}$$

Field between plates: $\boxed{\vec{E} = \frac{-\sigma}{\epsilon_0}\hat{\imath}}$ ①

Find electron velocity

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad v = v_0 + at$$

$$d = v_0 t + \tfrac{1}{2} a t^2 \qquad t = \frac{v - v_0}{a}$$

$$d = \frac{v_0(v - v_0)}{a} + \frac{1}{2} a \frac{(v - v_0)^2}{a^2} \Rightarrow \boxed{d = \frac{[v^2 - v_0^2]}{2a}}$$ ②

continued ↓

Sample Test 1, P2 continued

<u>Combine ①, ②, and NSL and solve for $\sigma$</u>

$$\vec{F} = m\vec{a} \Rightarrow -e\vec{E} = m\vec{a}$$

$$\Rightarrow -e\left(\frac{-\sigma}{\epsilon_0}\hat{\imath}\right) = m\vec{a}$$

$$\Rightarrow \boxed{\vec{a} = \frac{\sigma e}{m}\hat{\imath}}$$ – positive in x

So: $d = \dfrac{v^2 - v_0^2}{2}\dfrac{m}{\sigma e}$

$$\Rightarrow \boxed{\sigma = \frac{m(v^2 - v_0^2)}{2e}}$$

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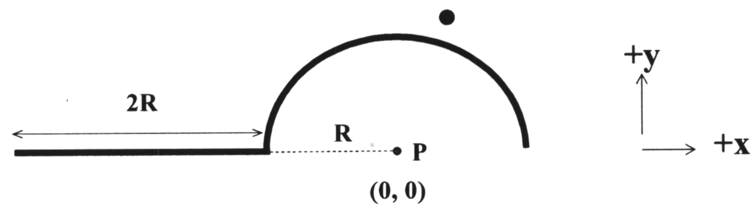
3) In this figure, the entire black entire black line (straight segment plus arc) has a total charge $Q$ and a uniform linear charge density $\lambda$.



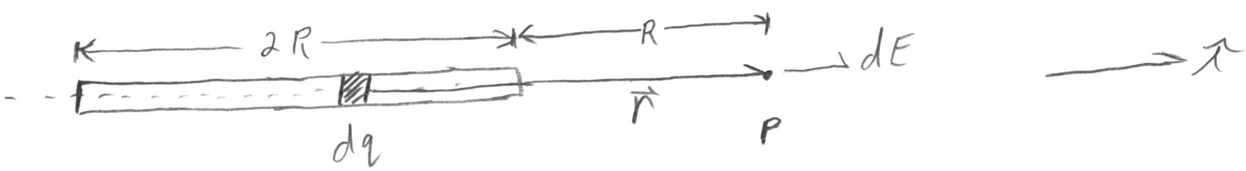

(a) Write $\lambda$ in terms of $Q$ and $R$.

(b) Find the net electric field vector at point $P$ in the figure. Write your answer in terms of $R$, $Q$, and constants (but not $\lambda$). (Pay attention to where the origin is located, and be careful when writing the bounds of your integral. Use appropriate + and – signs.)

a) $\lambda = \frac{Q}{L}$, $\quad L = 2R + \pi R = R(2+\pi)$

$$\boxed{\lambda = \frac{Q}{R(2+\pi)}}$$

b) Line charge




$$\vec{E} = \int d\vec{E} \Rightarrow \vec{E} = \int_{-3R}^{-R} \frac{k\,dq}{|\vec{r}|^2}\hat{r}, \quad dq = \lambda\,dx, \quad \vec{r} = x\hat{\imath}, \quad \hat{r} = \hat{\imath}$$

$$\Rightarrow \vec{E} = \int_{-3R}^{-R} \frac{k\lambda}{x^2}\,dx\,\hat{\imath} \Rightarrow \vec{E} = \frac{kQ}{R(2+\pi)} \int_{-3R}^{-R} \frac{1}{x^2}\,dx\,\hat{\imath}$$

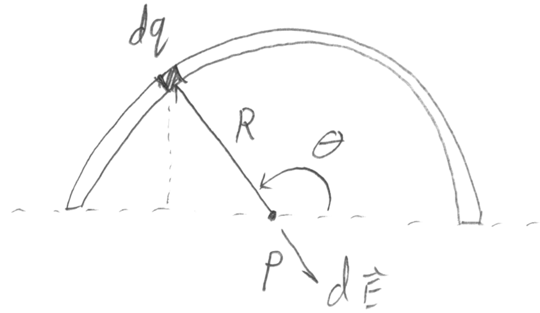
$$\Rightarrow \vec{E} = \frac{kQ}{R(2+\pi)} \left(-\frac{1}{x}\right)\Big|_{-3R}^{-R} \hat{\imath} \Rightarrow \vec{E} = \frac{kQ}{R(2+\pi)}\left(\frac{1}{R} - \frac{1}{3R}\right)$$

$$\Rightarrow \boxed{\vec{E}_1 = \frac{2kQ}{3R^2(2+\pi)}\hat{\imath}} \quad (1)$$

continued

Sample Test 1, p3 continued

Arc of charge




By symmetry, $\hat{\imath}$ cancels

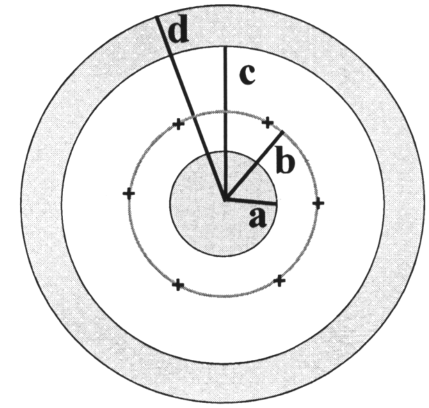
$$\vec{E} = \int d\vec{E} \Rightarrow \vec{E} = \int \frac{k\,dq}{|\vec{R}|^2}\hat{r}\ , \quad dq = \lambda\, ds = \lambda R\, d\theta, \quad \hat{r} = \cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}$$

$$\Rightarrow \vec{E} = \int_0^{\pi} \frac{k}{R^{\cancel{2}}}\lambda \cancel{R}\left(-\sin\theta\,\hat{\jmath}\right) d\theta = -\frac{kQ}{R^2(2+\pi)}\int_0^{\pi} \sin\theta\, d\theta\,\hat{\jmath}$$

$$\boxed{\vec{E}_2 = \frac{-2kQ}{R^2(2+\pi)}\hat{\jmath}}$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \boxed{\frac{kQ}{R^2(2+\pi)}\left[\frac{2}{3}\hat{\imath} - 2\hat{\jmath}\right]}$$

4) Three concentric spheres have charges placed on them. The inner sphere (radius = $a$) is a solid, uniformly charged insulator with total charge $-5q$. The sphere of radius $b$ is a very thin insulating shell with total charge $+2q$ spread uniformly over its surface. The outer sphere is a thick conducting shell (inner radius = $c$, outer radius = $d$) with a total charge $-7q$.




a) Determine the total charge on the inner and outer surfaces of the conducting sphere.

b) Determine the electric fields for $r<a$, $a<r<b$, $b<r<c$, $c<r<d$, and $r>d$ as measured from the center of the system. Write your answers in terms of $q$, the radii ($a$, $b$, $c$, $d$), and constants.

c) Accurately plot the electric field magnitudes vs. $r$ for all values of $r$. Label the field values at the beginning and ending of each section of your graph.

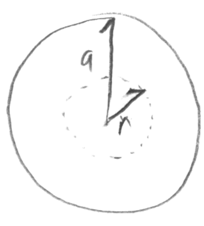
a) Inner surface encloses total charge: $Q_{T\,in} = -5q + 2q = -3q$

So, $\boxed{Q_{in} = 3q}$ to cancel the E field.

Must see $-10q$ outside outer surface so:

$Q_{out} + 3q = -10q \Rightarrow \boxed{Q_{out} = -13q}$

b) $r<a$




$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow EA = \frac{\rho V}{\epsilon_0}, \quad \rho = \frac{-5q}{4/3\pi a^3}$

$\Rightarrow E \cdot 4\pi r^{\cancel{2}} = \frac{1}{\epsilon_0}\frac{-5q}{\cancel{4/3\pi} a^3}\cancel{4/3\pi} r^{\cancel{3}}$

$\Rightarrow \boxed{\vec{E} = \frac{-5q}{4\pi\epsilon_0} \cdot \frac{r}{a^3}\hat{r}}$

continued ↓

Sample Test 1, p4 continued

<u>$a < r < b$</u>

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E \cdot A = \frac{-5q}{\epsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{-5q}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{-5q}{4\pi\epsilon_0 r^2}\hat{r}}$$

<u>$b < r < c$</u>

$$E \cdot 4\pi r^2 = \frac{-3q}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{-3q}{4\pi\epsilon_0 r^2}\hat{r}}$$

<u>$c < r < d$</u>

inside conductor, $\boxed{\vec{E} = 0}$

<u>$r > d$</u>

$$\boxed{\vec{E} = \frac{-10q}{4\pi\epsilon_0 r^2}\hat{r}}$$

c)