Electric Potential Energy & Electric Potential IV
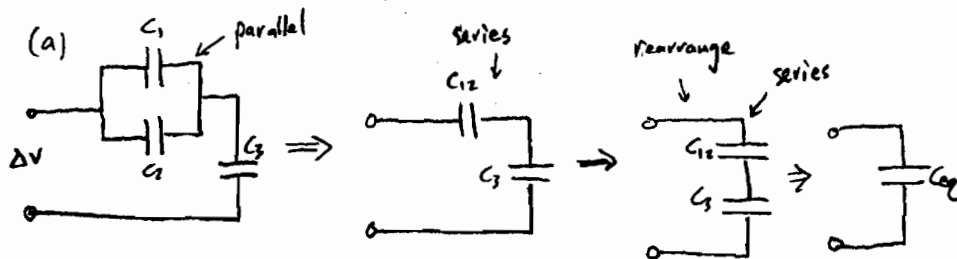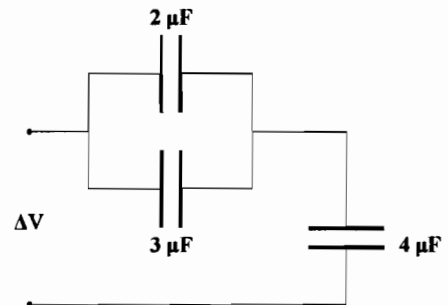
Name: _Key_   **# of problems you seriously attempted, groupwork + homework = _____**

1. (a) Find the equivalent capacitance of the network of three capacitors shown in the figure.

(b) If $\Delta V = 10$ V, find the charge on each capacitor and the voltage across each capacitor.

(c) Find the total electric potential energy stored in this system.

$$C_{12} = C_1 + C_2 = \underline{5\mu F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{C_3 + C_{12}}{C_3 C_{12}}$$

$$\Rightarrow C_{eq} = \frac{C_3 C_{12}}{C_3 + C_{12}} = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3} = \frac{4(2+3)\mu F}{(2+3+4)\mu F} = \boxed{2.22\mu F = C_{eq}}$$

(b) $\quad C_{eq} = \dfrac{Q_{tot}}{\Delta V} \Rightarrow Q_{tot} = C_{eq}\Delta V = (2.22\mu F)(10v) = \boxed{22.2\mu C}$

$$\boxed{= Q_{tot} = Q_3 = Q_{12}}$$

Since $C_{12} + C_{eq}$ are in series

$$C_{12} = \frac{Q_{12}}{\Delta V_{12}} \Rightarrow \Delta V_{12} = \frac{Q_{12}}{C_{12}} = \frac{22.2\mu C}{5\mu F} = \boxed{4.44v = \Delta V_1 = \Delta V_2}$$

$$\Delta V_{12} + \Delta V_3 = \Delta V \Rightarrow \Delta V_3 = \Delta V - \Delta V_{12} = 10v - 4.44v = \boxed{5.56v = \Delta V_3}$$

$$Q_2 = C_2 \Delta V_2 = (3\mu F)(4.44v) = \boxed{13.3\mu C}$$

$$Q_1 = C_1 \Delta V_1 = (2\mu F)(4.44v) = \boxed{8.9\mu C}$$

(c)
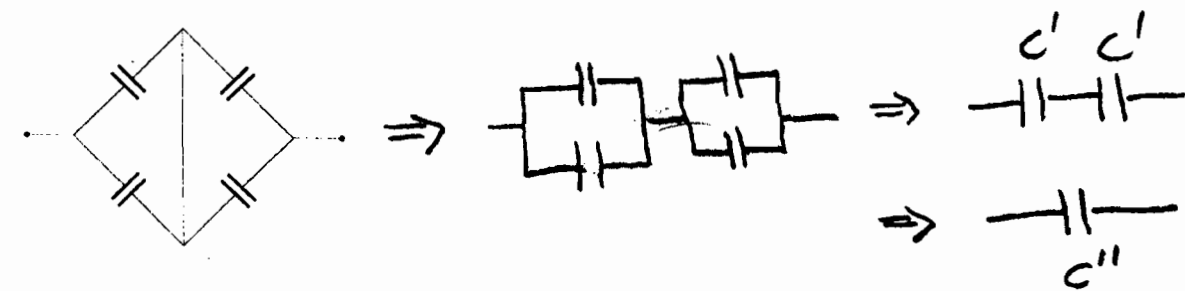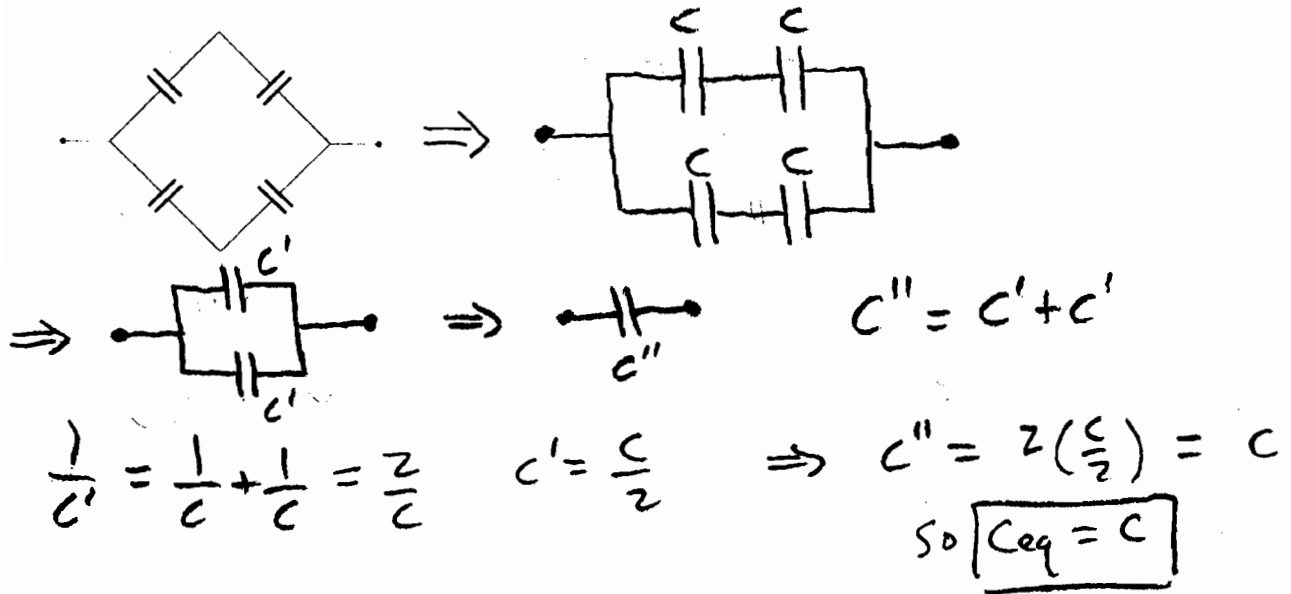
The total electric potential energy stored in the system is

$$U_{tot} = U_1 + U_2 + U_3$$

$$= \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + \frac{Q_3^2}{2C_3} = \frac{(8.9\times10^{-6})^2}{2(2\times10^{-6})} + \frac{(13.3\times10^{-6})^2}{2(3\times10^{-6})} + \frac{(22.2\times10^{-6})^2}{2(4\times10^{-6})}$$

$$\boxed{U_{tot} = 1.1\times10^{-4} J}$$

2. Find the equivalent capacitance of the following systems. All of the capacitors have a capacitance of C.

$$C'' = C' + C'$$

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \qquad C' = \frac{C}{2} \qquad \Rightarrow \qquad C'' = 2\left(\frac{C}{2}\right) = C$$

$$\text{So} \boxed{C_{eq} = C}$$

$$C' = 2C$$

$$C'' = \frac{1}{\frac{1}{C'} + \frac{1}{C'}} = \frac{1}{\frac{1}{2C} + \frac{1}{2C}} = C \qquad\qquad \text{So} \boxed{C_{eq} = C}$$
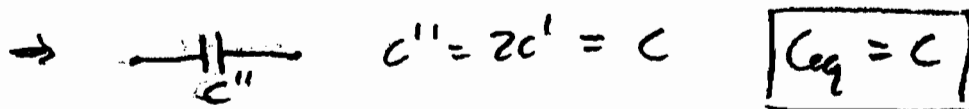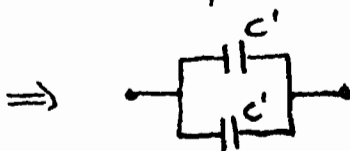
$$C_{eq} = 3c$$
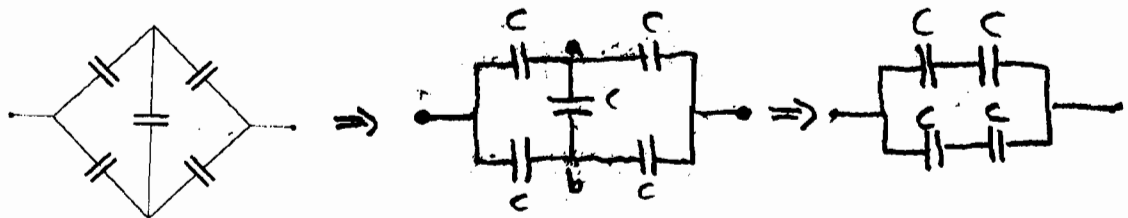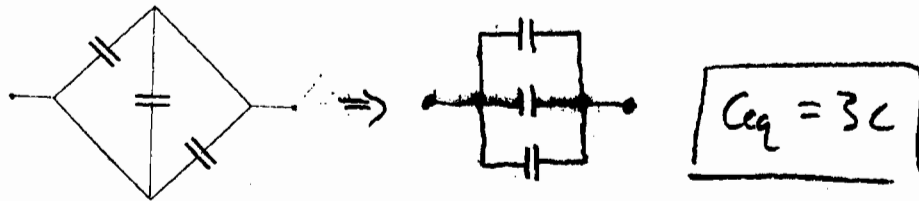
Tricky: Since all the capacitors are the <u>same</u>, the potential at point "a" must equal the potential at point "b". (Think <u>symmetry</u>.)

⟹ The center capacitor doesn't do anything! $\Delta V_{ab} = 0$

So you can ignore it completely.   ← This wouldn't necessarily be true if the caps were different.

⟹

$$\frac{1}{c'} = \frac{1}{c} + \frac{1}{c} \qquad c' = \frac{c}{2}$$

→

$$c'' = 2c' = c \qquad \boxed{C_{eq} = c}$$

3. In the figure below, each capacitance C_1 is 6.9 µF, and each capacitance C_2 is 4.6 µF.

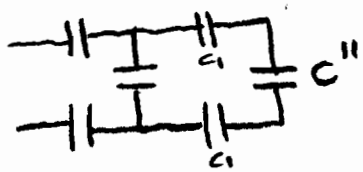   (a) Compute the equivalent capacitance of the network between points a and b.
   (b) Compute the charge on each of the three capacitors nearest a and b when $\Delta V_{ab} =$ 420 V.
   (c) With 420 V across a and b, compute ΔV_{cd}.
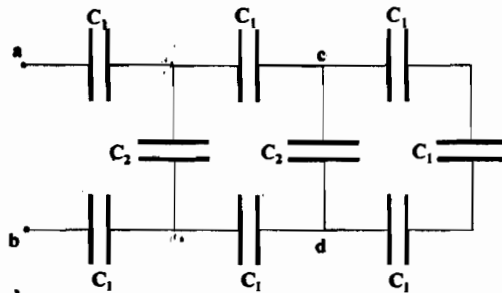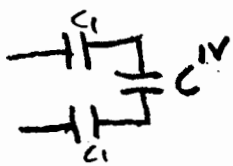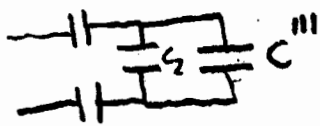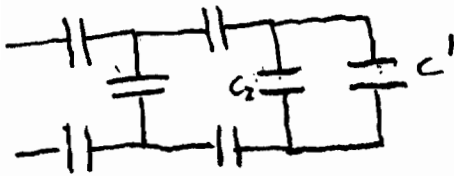
(a) $c' = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1}} = \dfrac{C_1}{3}$

$c'' = c' + C_2 = \dfrac{C_1}{3} + C_2 = \dfrac{6.9\,\mu F}{3} + 4.6\,\mu F = 6.9\,\mu F = c''$

$c''' = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{c''}} = \dfrac{1}{\frac{2}{C_1} + \frac{1}{c''}} = \dfrac{C_1 c''}{2c'' + C_1} = \dfrac{(6.9)(6.9)}{2(6.9) + (6.9)} = 2.3\,\mu F = c'''$
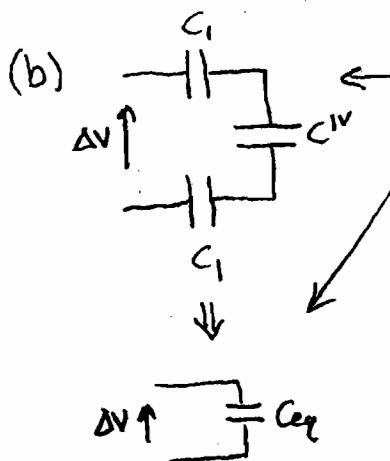
$c^{IV} = C_2 + c''' = 4.6 + 2.3 = 6.9\,\mu F$

$C_{eq} = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{c^{IV}}} = \dfrac{1}{\frac{2}{C_1} + \frac{1}{c^{IV}}} = \dfrac{C_1 c^{IV}}{2c^{IV} + C_1} = \dfrac{(6.9)(6.9)}{2(6.9) + 6.9}$
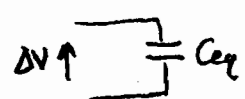
$\boxed{C_{eq} = 2.3\,\mu F}$

(b)

← The charge on each of these caps is the same, and it equals the charge on C_{eq}. (since they're all in series)

So $Q_{c_1} = C_{eq} \, \Delta V = (2.3 \mu F)(420V)$
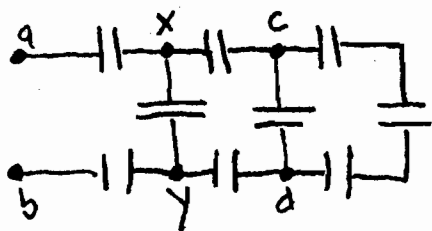
$$\boxed{Q_{c_1} = 9.7 \times 10^{-4} C}$$

Now find Q_{c_2}. Notice that $C_1 = C_2 = C^{IV} = 6.9 \mu F$. Since they are all the same, the potential difference across each cap is the same: $\Delta V_{c_1} = \Delta V_{c^{IV}} = \Delta V_{c_1}$. Also, $\Delta V_{c_1} + \Delta V_{c^{IV}} + \Delta V_{c_1} = \Delta V$
$\qquad\qquad\qquad\qquad \uparrow \qquad\qquad\qquad \uparrow \qquad\qquad\qquad\qquad\qquad\qquad = 420V$
$\qquad\qquad\qquad$ top $\qquad\qquad$ bottom

So $\Delta V_{c^{IV}} = \dfrac{\Delta V}{3} = \dfrac{420V}{3} = \underline{140V}$. Notice that $\Delta V_{c^{IV}} = \Delta V_{c_2}$ (the same wires connect them) $\Rightarrow Q_{c_2} = C_2 \Delta V_{c_2} = (4.6 \mu F)\left(\dfrac{420}{3} V\right)$

$$\boxed{Q_{c_2} = 6.44 \times 10^{-4} C}$$

(c) Utilize the circuit's symmetry here.

If $\Delta V_{xy} = \frac{1}{3} \Delta V_{ab}$, as we determined in part (b), then symmetry tells us that $\Delta V_{cd} = \frac{1}{3} \Delta V_{xy} = \frac{1}{9} \Delta V_{ab}$

$$\boxed{\Delta V_{cd} = 46.7V}$$

This only works because of symmetry. If the caps had been different along the top & bottom, the ratio would not have been a simple $\frac{1}{3} \cdot \frac{1}{3}$.

HOMEWORK

1. Find the charge, potential difference, and stored energy in every capacitor in this circuit. Let $V_{ab} = 60$ V, $C_1 = 4.0\mu F$, $C_2 = 7.0\mu F$, $C_3 = 5.0\mu F$, $C_4 = 6.0\mu F$.

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{7} + \frac{1}{5} = \frac{12}{35} \rightarrow C_{23} = \frac{35}{12} = \boxed{2.92 \mu F}$$

$$q_{23} = C_{23}\Delta V_{23} = C_{23}\Delta V_{ab} = (2.92)(60) = \boxed{175 \mu C = q_2 = q_3}$$

$$\Delta V_2 = \frac{q_2}{C_2} = \frac{175 \mu C}{7 \mu F} = \boxed{25.0 V} \quad \Delta V_3 = \frac{q_3}{C_3} = \frac{175 \mu C}{50 \mu F} = \boxed{35.0 V}$$

$$\Delta V_1 = \Delta V_{ab} = \boxed{60.0 V} \rightarrow q_1 = \Delta V_1 C_1 = (60)(4.0\mu F) = \boxed{240 \mu C}$$

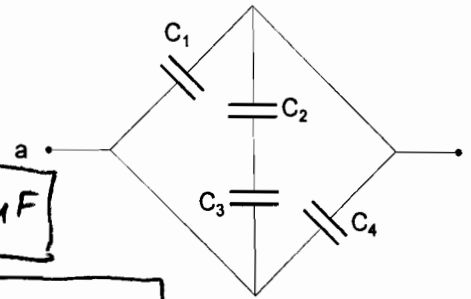$$\Delta V_4 = \Delta V_{ab} = \boxed{60.0 V} \rightarrow q_4 = \Delta V_4 C_4 = (60)(6.0\mu F) = \boxed{360 \mu C}$$

Energies:

$$U_1 = \frac{1}{2}C_1 \Delta V_1^2 = \boxed{7.2 mJ} \qquad U_3 = \frac{1}{2}C_3 \Delta V_3^2 = \boxed{3.06 mJ}$$
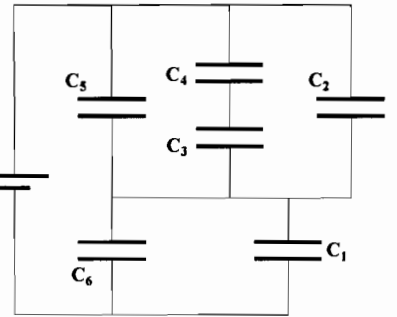
$$U_2 = \frac{1}{2}C_2 \Delta V_2^2 = \boxed{2.19 mJ} \qquad U_4 = \frac{1}{2}C_4 \Delta V_4^2 = \boxed{10.8 mJ}$$

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2. Find the equivalent capacitance of the following circuit.
$C_1 = 3.0\mu F$, $C_2 = 2.0\mu F$, $C_3 = 4.0\mu F$, $C_4 = 4.0\mu F$, $C_5 = 2.0\mu F$, $C_6 = 3.0\mu F$.

① $\dfrac{1}{C_{34}} = \dfrac{1}{C_3} + \dfrac{1}{C_4} = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$ $\Rightarrow$ $C_{34} = 2.0\mu F$

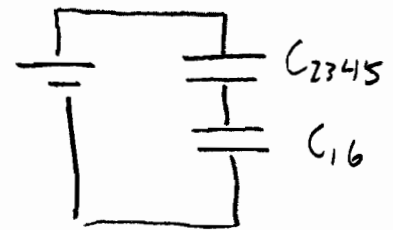② $C_{2345} = C_2 + C_{34} + C_5 = 2 + 2 + 2 = 6.0\mu F$

$C_{16} = C_1 + C_6 = 3 + 3 = 6.0\mu F$

③ $\dfrac{1}{C_{eq}} = \dfrac{1}{C_{2345}} + \dfrac{1}{C_{16}}$

$= \dfrac{1}{6} + \dfrac{1}{6} = \dfrac{1}{3}$
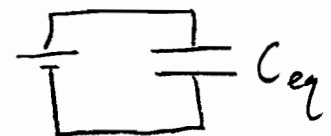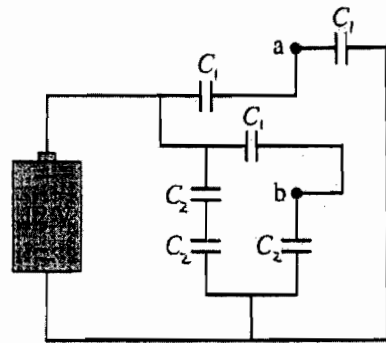
$\Rightarrow$ $\boxed{C_{eq} = 3.0\mu F}$
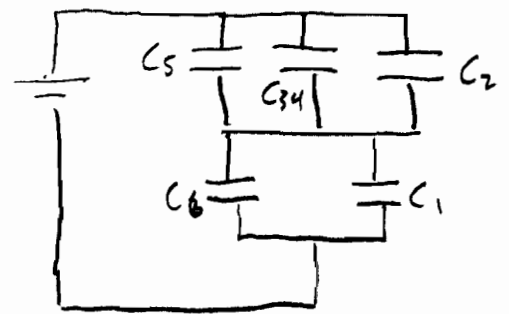
Electric Potential Energy & Electric Potential IV

3. Three capacitors with capacitance $C_1 = 10\ \mu F$ and three with capacitance $C_2 = 20\ \mu F$ are connected as shown in the figure.

(a) Find the equivalent capacitance of this system.

②

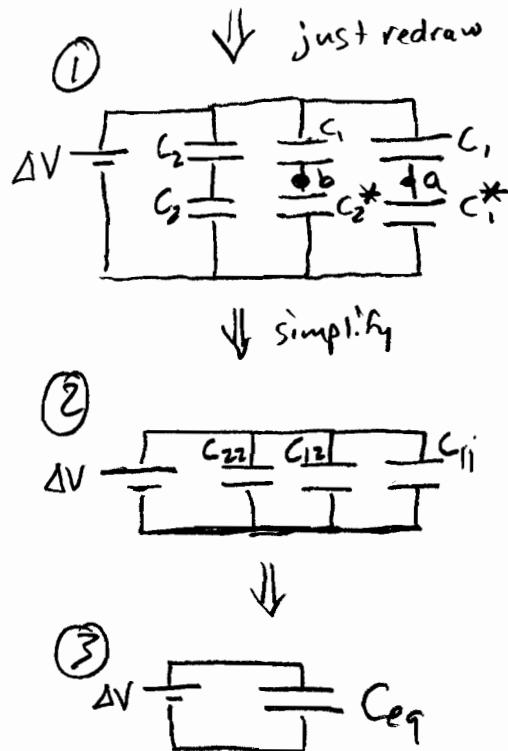$$\frac{1}{C_{22}} = \frac{1}{C_2} + \frac{1}{C_2} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} \implies C_{22} = 10\ \mu F$$

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20} \implies C_{12} = 6.67\ \mu F$$

$$\frac{1}{C_{11}} = \frac{1}{C_1} + \frac{1}{C_1} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5} \implies C_{11} = 5.0\ \mu F$$

③

$$C_{eq} = C_{11} + C_{12} + C_{22} = \boxed{21.67\ \mu F}$$

↓ just redraw

①

↓ simplify

②

⬇

③

(b) What is the potential difference between points a and b? Indicate which point is at a higher potential.

$$\Delta V_{C1^*} = \frac{q_{C1^*}}{C1^*} = \frac{60\ \mu C}{10\ \mu F} = {}^+6V \quad \text{"above" bottom wire}$$

(see ①)

$$q_{C1^*} = q_{11} = C_{11}\,\Delta V = (5\ \mu F)(12V) = 60\ \mu C$$

$$q_{C2^*} = q_{12} = C_{12}\,\Delta V = (6.67\ \mu F)(12V) = 80\ \mu C$$

$$\Delta V_{C2^*} = \frac{q_{C2^*}}{C2^*} = \frac{80\ \mu C}{20\ \mu F} = {}^+4V \quad \text{"above" bottom wire}$$

So, $\boxed{\text{point } a \text{ is at a higher potential}}$ and $\Delta V_{ab} = V_a - V_b = (6-4)V = \boxed{2V}$

4. Three capacitors having capacitances of 8.4 μF, 8.4 μF, and 4.2 μF are connected in series across a 36 V potential difference.

(a) What is the charge on the 4.2 μF capacitor?
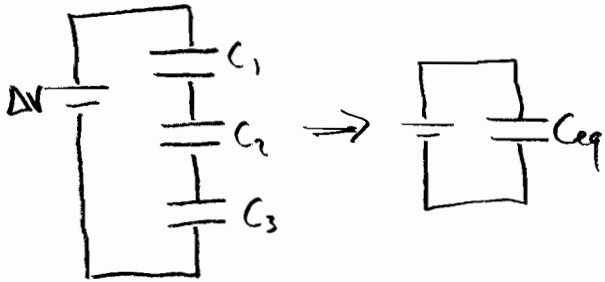
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{8.4} + \frac{1}{8.4} + \frac{1}{4.2} = \frac{1}{0.476}$$

$$\rightarrow C_{eq} = 2.1 \mu F$$

$$q_{eq} = C_{eq} \Delta V = \boxed{75.6 \mu C = q_1 = q_2 = q_3}$$

(b) What is the total energy stored in all three capacitors?

$$U_{eq} = \frac{1}{2} q_{eq} \Delta V = \frac{1}{2}(75.6 \mu C)(36V) = \boxed{1361 J} \quad \text{for whole system}$$

(c) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other, with the positively charged plates connected together. What is the voltage across each capacitor in the parallel combination?
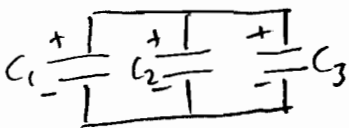
$$\Delta V' = \frac{q_{tot}}{C_{eq}} = \frac{q_1 + q_2 + q_3}{C_1 + C_2 + C_3} = \frac{226.8 \mu C}{21 \mu F} = \boxed{10.8V}$$

ΔV is now the same for all 3 (definition of parallel)

$$\Delta V_1' = \Delta V_2' = \Delta V_3' = \Delta V_{eq}'$$

$$* (\neq \Delta V \text{ original of battery}) *$$

(d) What is the total energy now stored in the capacitors?

$$U_{eq}' = \frac{1}{2} q_{tot} \Delta V_{eq}' = \frac{1}{2}(226.8 \mu C)(10.8V) = \boxed{1225 \mu J}$$