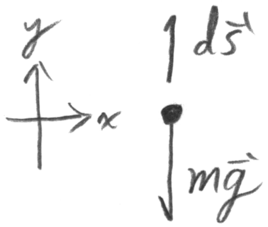
Name: ____________________ **Problems Solved __/ 11**

A weightlifter lifts a mass m a distance d above the ground.

a) Calculate the work done be gravity during the lift by solving the *Work Integral*.
b) What is ΔU, the change in potential energy during the lift?
c) After the lift, the weightlifter drops the weights. Use the *Work Energy Theorem* to calculate the velocity of the weights as they hit the ground assuming that they start from rest and fall a distance d.

a) $\underline{W = \int \vec{F} \cdot d\vec{s}}$ – The work integral



$d\vec{s} = dy\,\hat{j}, \quad \vec{F} = -mg\,\hat{j}$

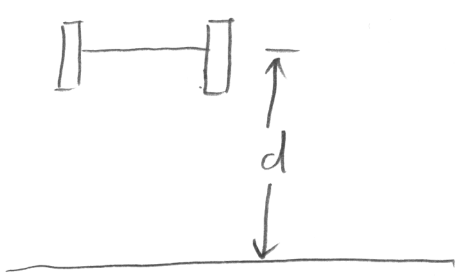
$$\Rightarrow W = \int_0^d (-mg\,\hat{j}) \cdot (dy\,\hat{j}) = -mg \int_0^d dy\,(\hat{j} \cdot \hat{j})$$

limits can start at zero since force is constant in y

$$\Rightarrow \boxed{W = -mgd}$$

b) $\Delta U = -W \Rightarrow \boxed{\Delta U = mgd}$

c)



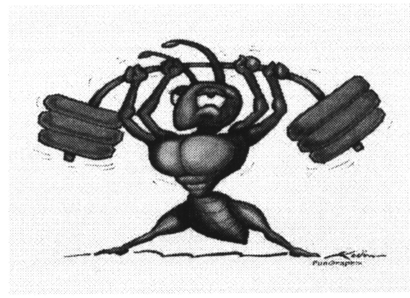
$W_{net} = \Delta K \Rightarrow -\Delta U = \Delta K$

On the trip down, $\Delta U = -mgd$

So: $-(-mgd) = \Delta K \Rightarrow mgd = \frac{1}{2} m V_F^2$

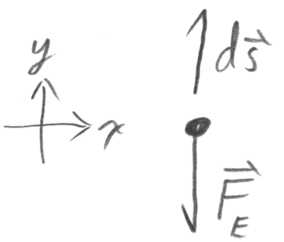
$$\Rightarrow \boxed{V_F = \sqrt{2gd}}$$

A Weight Lifting Space Ant (there is no gravity) is standing on Very Large Charged Plate with surface charge density $+\sigma$ lifting a barbell with a total charge $-q$.



a) Use the Work Integral to calculate the work done by the electric force if he lifts the barbell a distance d.
b) What is ΔU, the change in potential energy during the lift?
c) After the lift, the Weight Lifting Space Ant drops the barbell. Use the *Work Energy Theorem* to calculate the velocity of the barbell as it hits the ground, assuming that it starts from rest and falls a distance d.

a) $W=\int \vec{F}\cdot d\vec{s}$ – The work integral



$\vec{F}_E = -q\vec{E}$, $d\vec{s} = dy\,\hat{j}$, but what's $\vec{E}$?

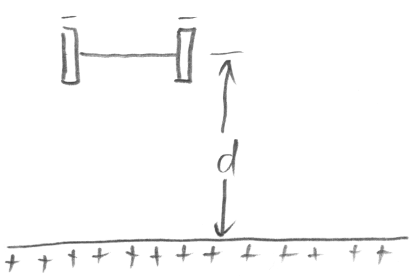
Find E using Gauss's Law

$$\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E\cdot 2\cancel{\pi r^2} = \frac{\sigma \cancel{\pi r^2}}{\epsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{j}$$

$$\Rightarrow W = \int_0^d \left(-q\frac{\sigma}{2\epsilon_0}\hat{j}\right)\cdot\left(dy\,\hat{j}\right) = \frac{-q\sigma}{2\epsilon_0}\int_0^d dy\,(\hat{j}\cdot\hat{j}) \Rightarrow \boxed{W = -\frac{q\sigma}{2\epsilon_0}d}$$

b) $\Delta U = -W \Rightarrow \boxed{\Delta U = \frac{q\sigma}{2\epsilon_0}d}$

c)



$W_{net} = \Delta K \Rightarrow -\Delta U = \Delta K$

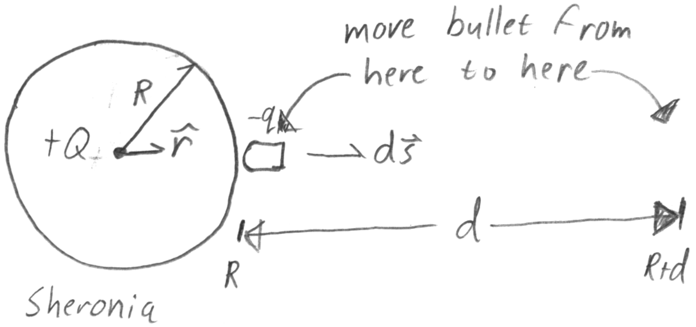
on the trip down, $\Delta U = -\frac{q\sigma}{2\epsilon_0}d$

So: $-\left(-\frac{q\sigma}{2\epsilon_0}d\right) = \Delta K \Rightarrow \frac{q\sigma}{2\epsilon_0}d = \frac{1}{2}mv^2$

$$\Rightarrow \boxed{v = \left[\frac{q\sigma}{m\epsilon_0}d\right]^{1/2}}$$

The Sphericons are negatively charged creatures who live on a positively charged sphere with a radius R and a total charge $+Q$. They are under attack from their evil neighbors, the Cubicons, who are dropping negatively charged bullets on them from a very long way away. Assume that the bullets have a mass m, charge $-q$ and that they start from rest at an effective distance of infinity. Use the Conservation of Energy to determine the velocity of the bullets as they impact the surface of Spheronia.

* What is the change in potential energy of a charged bullet moved infinitely far from Spheronia?



$$W = \int_R^{R+d} \vec{F} \cdot d\vec{s}, \quad \vec{F} = \frac{k(-q)Q}{|\vec{r}|^2}\hat{r}$$

$$d\vec{s} = dr\,\hat{r}$$

Let $d \to \infty$: $W = \int_R^{\infty} \left(\frac{k(-q)Q}{|\vec{r}|^2}\hat{r}\right)(dr\,\hat{r}) = k(-q)Q \int_R^{\infty} \frac{\hat{r} \cdot \hat{r}}{|\vec{r}|^2} dr$

$$W = k(-q)Q\left(-\frac{1}{|\vec{r}|}\right)\Bigg|_R^{\infty} \Rightarrow W = k(-q)Q\left[\underbrace{-\frac{1}{\infty}}_{\text{Zero}} - \left(-\frac{1}{R}\right)\right]$$

$$W = \frac{k(-q)Q}{R} \Rightarrow \boxed{\Delta U = -\frac{k(-q)Q}{R}}$$

continued
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So if the bullets are released from rest at infinity:

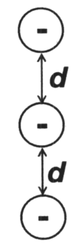
$W_{net} = \Delta K \Rightarrow \underline{-\Delta U = \Delta K}$ where $\underline{\Delta U = \frac{k(-q)Q}{R}}$ on the trip "down"

$$\Rightarrow -\frac{k(-q)Q}{R} = \frac{1}{2}mv_F^2 - \frac{1}{2}mv_I^2, \quad \underline{v_I = 0}$$

$$\Rightarrow \boxed{v_F = \left[\frac{2kqQ}{mR}\right]^{1/2}}$$

Energy – Set 1

Three electrons are spaced 1.0 mm apart along a vertical line. The outer two electrons are fixed in position, and the center one is free to move.



(a) Is the center electron at a point of stable or unstable equilibrium? Why?

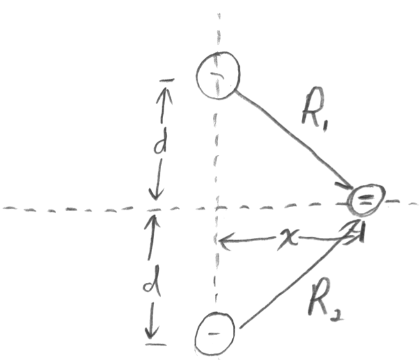
(b) If the center electron is displaced horizontally by an infinitesimal distance, what will be its speed when it is very far away ($r = \infty$)?

a) In general, the potential energy due to a single point charge is: $U = \frac{kqq}{R}$, where R is the charge separation.

And, the potential energy due to a collection of charges can be calculated via super-position.

$$U_{net} = \sum \frac{kqq_i}{R_i}$$

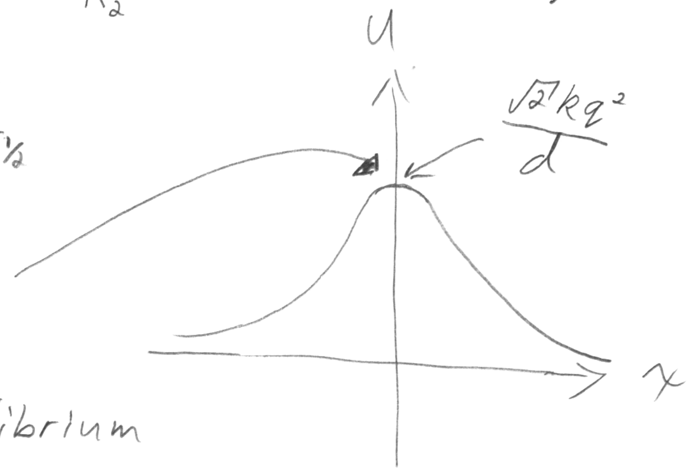
Let's calculate the potential energy of the center electron on the horizontal axis of symmetry



$$U_{net} = \frac{k(-q)(-q)}{R_1} + \frac{k(-q)(-q)}{R_2}, \quad R_1 = R_2 = (x^2+d^2)^{1/2}$$

$$U_{net} = \frac{2kqq}{(x^2+d^2)^{1/2}}$$

$\boxed{\text{Unstable!}}$



Any displacement from equilibrium will result in acceleration away from equilibrium.

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b) Using Conservation of Energy

$U_I = \frac{kqq}{(d^2+d^2)^{1/2}}$ $K_I = 0$ $\Rightarrow \frac{\sqrt{2}\,kq^2}{d} = \frac{1}{2}mV_F^2$

$U_F = 0$ $K_F = \frac{1}{2}mV_F^2$

$$\Rightarrow \boxed{V_F = \left(\frac{2\sqrt{2}\,kq^2}{md}\right)^{1/2}}$$

Two point charges, each of magnitude 2 μC, are located on the x-axis. One is at $x = 1.0$ m, and the other is at $x = -1.0$ m.

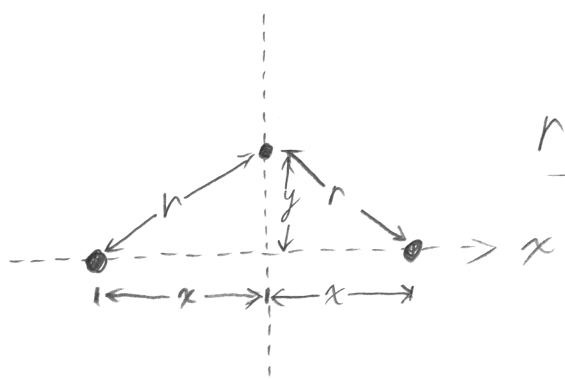
(a) Determine the electric potential on the y-axis at $y = 0.50$ m. Draw a sketch.

(b) Calculate the change in electric potential energy of the system if a third particle of charge -3 μC is brought from infinitely far away and placed on the y-axis at 0.50 m.

a) Electric potential: $V = \frac{kq}{r}$

Given
$q = 2\mu C$
$x = \pm 1.0 m$
$y = 0.5 m$

want
V, U



$r = (x^2 + y^2)^{1/2}$

$$V = V_1 + V_2 \Rightarrow V = \frac{kq}{r} + \frac{kq}{r} = \boxed{\frac{2kq}{r}}$$

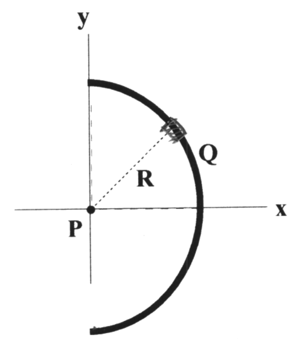
$$\Rightarrow V = \frac{2(9 \times 10^9 \frac{N \cdot m^2}{C^2})(2 \times 10^{-6} C)}{(1^2 + 0.5^2)^{1/2}} = 1.6 \times 10^4 \text{ Volts}$$

$$\text{Volts} = \frac{J}{C}$$

b) $U = qV \Rightarrow U = (-3 \times 10^{-6} C)(1.6 \times 10^4 V) = 0.18 J$

A uniformly charged insulating rod of length 14 cm is bent into the shape of a semicircle. If the rod has a total charge of -7.5 μC, find the electric potential at the center of the semicircle (point *P*).

Hint: The procedure is very similar to the one you followed when finding electric field, but now you don't have to worry about *x*- or *y*-components because *V* is a scalar. Easier! Show all steps.



Given
$Q = -7.5\,\mu C$
$L = 14\,cm$

$V = \int dV$

$= \int \frac{k\,dq}{R}, \quad dq = \lambda\, ds$

$\lambda = \frac{Q}{L}, \quad ds = R\,d\theta$

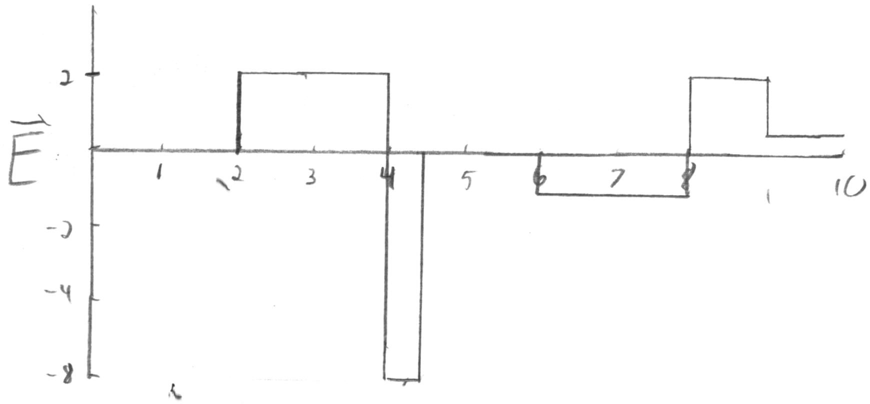
$L = \pi R \Rightarrow R = \frac{L}{\pi}$

$\Rightarrow V = \int_0^{\pi} \frac{k}{R}\frac{Q}{\cancel{L}}\frac{\cancel{L}}{\pi}\,d\theta \Rightarrow V = \frac{kQ}{\pi R}\int_0^{\pi} d\theta$

$\Rightarrow \boxed{V = \frac{kQ}{R}}$

1. Wolfson, Volume II, 2nd Edition, 22.29

$$V = -\int \vec{E}\cdot d\vec{s} \Rightarrow \vec{E} = -\frac{dV}{d\vec{s}}$$



$0 < x < 2;\ dV/dx = 0 \Rightarrow \vec{E} = 0$

$2 < x < 4;\ \frac{dV}{dx} = \frac{-4}{2} = -2 \Rightarrow \vec{E} = 2\hat{\imath}$

$4 < x < 4.25;\ \frac{dV}{dx} = \frac{2}{0.25} = 8 \Rightarrow \vec{E} = -8\hat{\imath}$

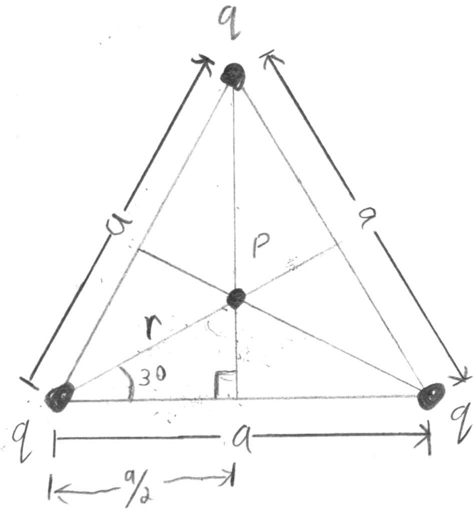
$4.25 < x < 6;\ \frac{dV}{dx} = 0 \Rightarrow \vec{E} = 0$

$6 < x < 8;\ \frac{dV}{dx} = \frac{+2}{2} = +1 \Rightarrow \vec{E} = -1\hat{\imath}$

$8 < x < 9;\ \frac{dV}{dx} = \frac{-2}{1} = -2 \Rightarrow \vec{E} = 2\hat{\imath}$

$9 < x < 10;\ \frac{dV}{dx} = \frac{-0.5}{1} = -\frac{1}{2} \Rightarrow \vec{E} = \frac{1}{2}\hat{\imath}$

2. Wolfson, Volume II, 2nd Edition, 22.50



$$V_p = 3V_q = \boxed{3\frac{kq}{r}}$$

$$r\cos 30 = \frac{a}{2}, \quad \cos 30 = \frac{\sqrt{3}}{2}$$

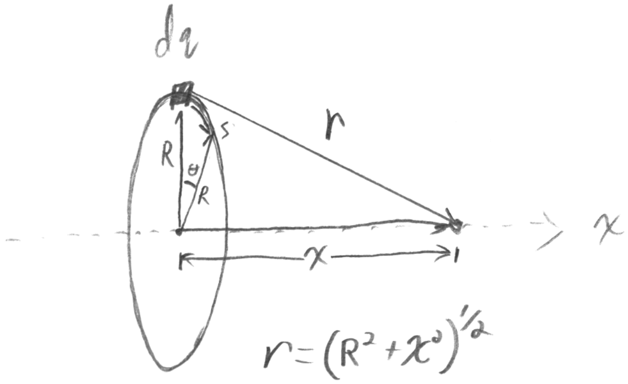
$$\Rightarrow r\frac{\sqrt{3}}{2} = \frac{a}{2} \Rightarrow \boxed{r = \frac{a}{\sqrt{3}}}$$

$$\boxed{V_p = \frac{3\sqrt{3}\,kq}{a}}$$

3. Wolfson, Volume II, 2nd Edition, 22.56

Hints: First draw a sketch of the ring with the *x*-axis as the ring's axis. Next, label the elemental charge *dq* and geometric parameters, and then find a general expression for *V* in terms of *k*, *Q*, *x*, and *R*. The procedure is very similar to the one you followed when finding electric field, but now you don't have to worry about *x*- or *y*-components because *V* is a scalar. Show all steps.

You should get this: $V = \dfrac{kQ}{\sqrt{x^2 + R^2}}$. After you've proven this result, answer the question in the book.



$r = (R^2 + x^2)^{1/2}$

$dq = \lambda ds, \quad \lambda = \frac{Q}{L} = \frac{Q}{2\pi R}$

$ds = R\,d\theta$

$\Rightarrow dq = \frac{Q}{2\pi \cancel{R}} \cancel{R}\, d\theta = \frac{Q}{2\pi} d\theta$

$$V = \int dV = \int \frac{k\,dq}{r}$$

$$= \int_0^{2\pi} \frac{k}{(R^2+x^2)^{1/2}} \frac{Q}{2\pi} d\theta = \frac{kQ}{2\pi (R^2+x^2)^{1/2}} \int_0^{2\pi} d\theta$$

$$\Rightarrow \boxed{V = \frac{kQ}{(R^2+x^2)^{1/2}}}$$

continued

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Energy Set 1, P10 continued

so: $V(x) = \frac{kQ}{(R^2+x^2)^{1/2}}$

and $V(0) = \frac{kQ}{R}$, $V(d) = \frac{kQ}{(d^2+R^2)^{1/2}}$

Given

$V(0) = V_0 = 45\,kV$

$V(d) = V_d = 33\,kV$

$d = 15\,cm$

Coupled equations. Let's solve for R first:

$$\frac{V_0}{V_d} = \frac{\cancel{kQ}}{R} \cdot \frac{(d^2+R^2)^{1/2}}{\cancel{kQ}} \Rightarrow \frac{V_0^2}{V_d^2} = \frac{d^2+R^2}{R^2} \Rightarrow R^2 \frac{V_0^2}{V_d^2} = d^2 + R^2$$

$$\Rightarrow R^2\left(\frac{V_0^2}{V_d^2} - 1\right) = d^2 \Rightarrow R^2\left(\frac{V_0^2 - V_d^2}{V_d^2}\right) = d^2$$

$$\Rightarrow R^2 = \frac{V_d^2}{V_0^2 - V_d^2} d^2 \Rightarrow \boxed{R = \frac{V_d}{(V_0^2 - V_d^2)^{1/2}} d}$$

$$R = \frac{33 \times 10^3\,V}{\left[(45\times10^3\,V)^2 - (33\times10^3\,V)^2\right]^{1/2}} (15 \times 10^{-2}\,m)$$

$$\boxed{R = 16.2\,cm}$$

Then: $V_0 = \frac{kQ}{R} \Rightarrow \boxed{Q = \frac{RV_0}{k}} \Rightarrow Q = \frac{(16.2\times10^{-2}\,m)\cdot(45\times10^3\,V)}{9\times10^9\,N\cdot m^2/C^2}$

$$\boxed{Q = .81\,\mu C}$$