Physics 112 Sample Test 2

NAME ____________________________

By writing my name above, I affirm that this test represents my work only, without aid from outside sources. In all aspects of this course I perform with honor and integrity.

SHOW YOUR WORK ON ALL OF THE PROBLEMS. YOUR APPROACH TO THE PROBLEM IS AS IMPORTANT AS, IF NOT MORE IMPORTANT THAN, YOUR ANSWER. DRAW **CLEAR AND NEAT PICTURES** SHOWING COORDINATE SYSTEMS AND ALL OF THE RELEVANT PROBLEM VARIABLES. ALSO, **EXPLICITLY** SHOW THE **BASIC EQUATIONS** YOU ARE USING. BE NEAT AND THOROUGH. THE EASIER IT IS FOR ME TO UNDERSTAND WHAT YOU ARE DOING, THE BETTER YOUR GRADE WILL BE.

$$\vec{F} \leftrightarrow \vec{E}: \quad \vec{F} = q\vec{E} = q\left(k\frac{Q}{r^2}\hat{r}\right) \quad \leftrightarrow \quad \vec{E} = \frac{\vec{F}}{q}$$

$$U \leftrightarrow V: \quad U = qV = q\left(k\frac{Q}{r}\right) \quad \leftrightarrow \quad V = \frac{U}{q}$$

$$\vec{F} \leftrightarrow \Delta U: \quad \vec{F}_x = -\frac{dU}{dx}\hat{i} \quad \leftrightarrow \quad \Delta U = -\int_A^B \vec{F} \cdot d\vec{r}$$

$$\vec{E} \leftrightarrow \Delta V: \quad \vec{E}_x = -\frac{dV}{dx}\hat{i} \quad \leftrightarrow \quad \Delta V = -\int_A^B \vec{E} \cdot d\vec{r}$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$U_{cap} = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q(\Delta V)$$

$$\int \frac{x\,dx}{\left(x^2+a^2\right)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$v = v_{0x} + a_x t$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$x = x_0 + \tfrac{1}{2}(v_x + v_{0x})t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x = x_0 + v_x t - \tfrac{1}{2}a_x t^2$$

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1.1) Two charged metal spheres are connected by a copper wire. Note that $r_A > r_B$ and that the spheres are far apart. What quantities must be the <u>same</u> for both spheres?

The conducting wire forces $\Delta V_A = \Delta V_B$

~~a) The total charge.~~
b) The potential.
~~c) The electric field just outside the surface.~~
d) The electric field everywhere inside. ($E = 0$)
~~e) The surface charge density.~~

$$\Delta V = -\int_R^{\infty} \vec{E}\cdot d\vec{s} = -\int_R^{\infty} \frac{kQ}{r^2}\,dr$$

$$\Rightarrow \Delta V_A = \frac{kQ_A}{R_A}, \quad \Delta V_B = \frac{kQ_B}{R_B}$$

Total Charge?

$$\Delta V_A = \Delta V_B \Rightarrow \frac{kQ_A}{R_A} = \frac{kQ_B}{R_B} \Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} \Rightarrow Q_A > Q_B$$

Electric Field?

$$E_A = \frac{kQ_A}{R_A^2}, \quad E_B = \frac{kQ_B}{R_B^2} \Rightarrow \frac{E_A}{E_B} = \frac{kQ_A}{R_A^2}\cdot\frac{R_B^2}{kQ_B}$$

$$\Rightarrow \frac{E_A}{E_B} = \frac{Q_A}{Q_B}\frac{R_B^2}{R_A^2} \Rightarrow \frac{E_A}{E_B} = \frac{R_A}{R_B}\cdot\frac{R_B^2}{R_A^2} \Rightarrow E_A < E_B$$

Surface Charge density?

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} \Rightarrow \frac{4\pi R_A^2 \sigma_A}{4\pi R_B^2 \sigma_B} = \frac{R_A}{R_B} \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A} \qquad \sigma_A < \sigma_B$$

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1.2) Three capacitors, each of identical capacitance C, are combined in a variety of ways (series, parallel, or both) using copper wires. What minimum value of capacitance is found in a combination that uses all three capacitors?

a) C/6
b) C/3
c) C/2
d) C
e) 3C/2

Parallel capacitors add, total capacitance increases.

Series capacitors add inversely, total capacitance decreases

$$\frac{1}{C_{eff}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} \Rightarrow C_{eff} = \frac{C}{3}$$

1.3) The plates of a parallel-plate capacitor have constant charges of +Q and –Q. Which of the following quantities increases as the separation of the plates increases?

a) ~~Electric field.~~
b) Potential difference between the plates.
c) ~~The capacitance.~~

$E = \frac{\sigma}{\epsilon_0}$ ← no d

$\Delta V = \frac{\sigma}{\epsilon_0} d$ ← d↑, ΔV↑

$C = \frac{\epsilon_0 A}{d} \Rightarrow$ d↑, C↓

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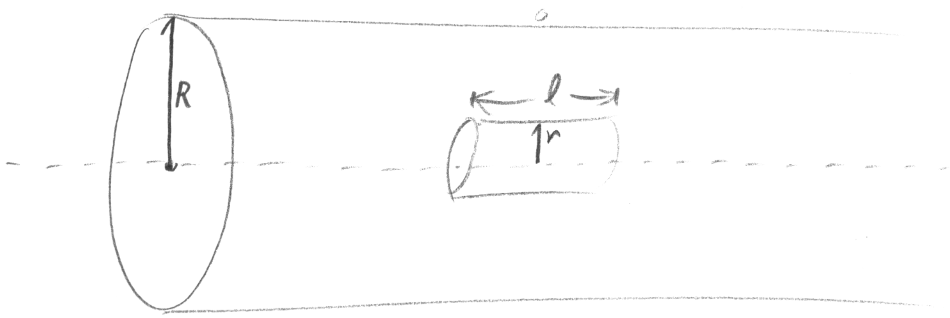
A long solid cylindrical insulator of radius R has a uniform charge density of $+\rho$.

a) Find ΔV along a line perpendicular to the cylinder's central axis from the center of the cylinder to a ~~point infinitely far~~ [eR] away.

b) A particle with mass m and charge +q is placed against the wall of the cylinder and released from rest. What will be its velocity when it is ~~a very long way~~ [eR] away ~~(ie. when $r=\infty$)~~?



a)

$r<R$: $\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E_1 A = \frac{\rho V}{\epsilon_0} \Rightarrow E_1 2\pi r l = \frac{\rho \pi r^2 l}{\epsilon_0}$

$$\Rightarrow \vec{E}_1 = \frac{\rho r}{2\epsilon_0}\hat{r}$$

$r>R$: $\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E_2 A = \frac{\rho V}{\epsilon_0} \Rightarrow E_2 2\pi r l = \frac{\rho \pi R^2 l}{\epsilon_0}$

$$\Rightarrow \vec{E}_2 = \frac{\rho R^2}{2\epsilon_0 r}\hat{r}$$

$$\Delta V = -\int_0^R \vec{E}_1 \cdot d\vec{s} - \int_R^{\infty} \vec{E}_2 \cdot d\vec{s} = -\left[\int_0^R \frac{\rho r}{2\epsilon_0} dr + \int_R^{eR} \frac{\rho R^2}{2\epsilon_0 r} dr\right]$$

$$\Delta V = -\frac{\rho}{2\epsilon_0}\left[\frac{1}{2}R^2 + R^2 \ln(3)\right] \Rightarrow \Delta V = -\frac{\rho R^2}{2\epsilon_0}\left[\frac{1}{2} + \ln(e)\right]$$

$$\Rightarrow \Delta V = -\frac{3}{4}\frac{R^2}{\epsilon_0}$$

Extra Space

b) $-\Delta U = \Delta K$, $\Delta U = q\Delta V = -\frac{3}{4}\frac{qR^2}{\epsilon_0}$

$$\frac{3}{4}\frac{qR^2}{\epsilon_0} = \frac{1}{2}m v_f^2 \Rightarrow \boxed{v_f = \left[\frac{3}{2}\frac{qR^2}{m\epsilon_0}\right]^{1/2}}$$

A solid conducting slab is inserted midway between the plates of a capacitor with capacitance C that charged to initial energy U_I. The slab thickness is 2/3 of the original plate spacing and its area is the same as that of the original plates

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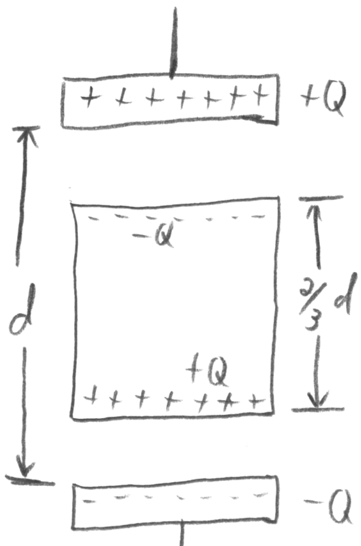
a) Write an expression for the new capacitance in terms of the original capacitance C?

b) Write an expression for the new stored energy in terms of the initial stored energy, assuming the capacitor is not connected to anything?



a) In a parallel plate capacitor:

$$|\vec{E}| = \frac{\sigma}{\epsilon_0}$$

so: $\Delta V = -\int_0^d \vec{E}\cdot d\vec{s} = -\frac{\sigma}{\epsilon_0} d$

and: $C = \frac{|Q|}{|\Delta V|} = \frac{\sigma A}{\sigma d}\epsilon_0 = \boxed{\frac{\epsilon_0 A}{d}}$

$\Rightarrow \boxed{C = \frac{\epsilon_0 A}{d},}$ before the conductor is inserted

* After the conductor is inserted, the charge Q stays the same so σ doesn't change and $\vec{E}$ is the same.

But since $E = 0$ inside the conductor, $\Delta V_{new} = -\frac{\sigma}{\epsilon_0}\frac{1}{3}d$

Then: $C_{new} = \frac{|Q|}{|\Delta V_{new}|} = \frac{\sigma A \epsilon_0 3}{\sigma d} \Rightarrow \boxed{C_{new} = 3\frac{A\epsilon_0}{d} = 3C}$

b) $U_I = \frac{Q^2}{2C}$, $U_{new} = \frac{Q^2}{2C_{new}} = \frac{Q^2}{6C} \Rightarrow \frac{U_{new}}{U_I} = \frac{Q^2}{6C}\cdot\frac{2C}{Q^2}$

$$\boxed{U_{new} = \frac{1}{3}U_I}$$

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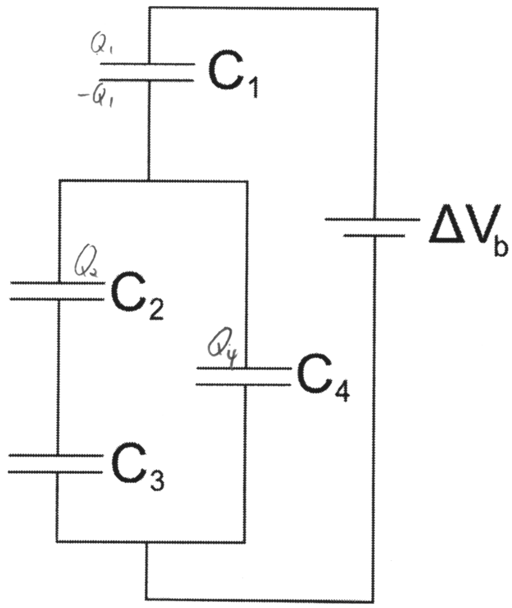
$C = \frac{Q}{V}$ $\quad Q_1 = Q_2 + Q_4 = C_{eff}\Delta V_B$

In the capacitor network at the right, all capacitors have the same capacitance C and the battery provides a potential difference ΔV_b.



Find an expression for the potential difference, ΔV_3, and the charge, Q_3, on C_3 in terms of C and ΔV_b.

Find Ceff, Then we can get Q_1

$$\frac{1}{C_{23}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow \boxed{C_{23} = \frac{C}{2}}$$

$$C_{234} = C_{23} + C = \tfrac{1}{2}C + C$$

$$\Rightarrow \boxed{C_{234} = \tfrac{3}{2}C}$$

$$C_{eff} = C_{1234} = \frac{1}{C} + \frac{1}{C_{234}} = \frac{1}{C} + \frac{2}{3}\frac{1}{C} = \frac{5}{3}\frac{1}{C}$$

$$\Rightarrow \boxed{C_{eff} = \tfrac{3}{5}C}$$

Find Q_1 and ΔV_1, which will lead to ΔV_{234}

$$Q_1 = C_{eff}\Delta V_b \Rightarrow \boxed{Q_1 = \tfrac{3}{5}C\Delta V_b}$$

$$\Delta V_1 = \frac{Q_1}{C_1} \Rightarrow \Delta V_1 = \frac{\tfrac{3}{5}C\Delta V_b}{C} \Rightarrow \boxed{\Delta V_1 = \tfrac{3}{5}\Delta V_b}$$

$$\Delta V_{234} = \Delta V_b - \Delta V_1 = \Delta V_b - \tfrac{3}{5}\Delta V_b \Rightarrow \boxed{\Delta V_{234} = \tfrac{2}{5}\Delta V_b}$$

Now we can find Q_3 and ΔV_3

$$\Delta V_{234} = \Delta V_{23} = \Delta V_4, \quad Q_2 = Q_3 = C_{23}\Delta V_{23} = \tfrac{1}{2}C\,\tfrac{2}{5}\Delta V_b \Rightarrow \boxed{Q_3 = \tfrac{1}{5}C\Delta V_b}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{\tfrac{1}{5}C\Delta V_b}{C} \Rightarrow \boxed{\Delta V_3 = \tfrac{1}{5}\Delta V_b}$$