Name: ______________________ **Problems Solved __/ 8**

If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is zero? Explain.

No. $\vec{F}_B = q(\vec{v} \times \vec{B})$ so $\vec{F} = 0$ when $\underline{\vec{v} \times \vec{B} = 0}$

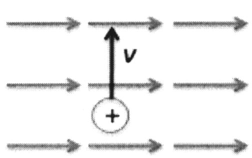
and $\vec{v} \times \vec{B} = 0$ when $\underline{\vec{v} \parallel \vec{B}}$

How can the motion of a charged particle be used to distinguish between a magnetic field and an electric field in a certain region of space?

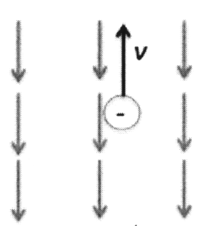
For an $\vec{E}$ Field, $\vec{a}$ is always in the direction of the field

For a $\vec{B}$ Field, $\vec{a}$ is orthogonal to the field <u>and</u> the velocity vector.

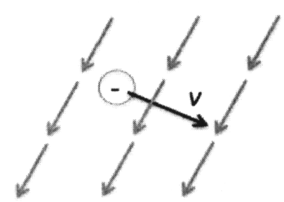
For each of the following, draw the magnetic force vector on the charge or, if appropriate, write "*F* into page," "*F* out of page," or "*F* = 0."



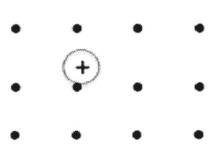
F ⊗ into page



F = 0

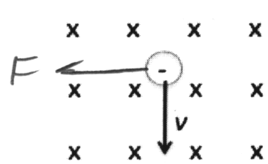


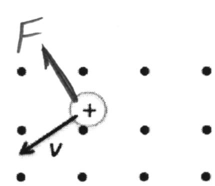
F ⊙ out of page



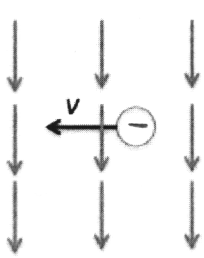
v out of page

F = 0



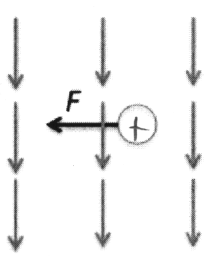


For each of the following, determine the sign of the charge (+ or -).



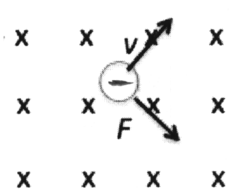
F into page

charge negative

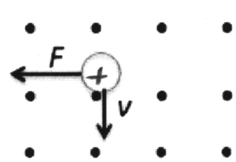


v into page

charge positive



charge negative



Charge positive

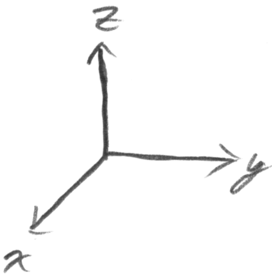
A charge q = -2 nC moves with velocity $\vec{v} = -3.0\times10^{6}\,\hat{\imath}$ m/s. Find the force vector on the charge if the magnetic field is:

(a) $\vec{B} = 0.6\hat{\jmath}$ T (b) $\vec{B} = 0.6\hat{\imath} + 0.6\hat{\jmath}$ T (c) $\vec{B} = 0.6\hat{\imath}$ T (d) $\vec{B} = 0.6\hat{\imath} + 0.6\hat{k}$ T

a) $\vec{F} = q(\vec{v}\times\vec{B}) \Rightarrow \vec{F} = -2\times10^{-9}\,C\left((-3.0\times10^{6}\,m/s)\hat{\imath} \times 0.6\,T\,\hat{\jmath}\right)$

$\Rightarrow \vec{F} = 3.6\times10^{-3}\,N\,(\hat{\imath}\times\hat{\jmath})$

$\Rightarrow \boxed{\vec{F} = 3.6\times10^{-3}\,N\,\hat{k}}$



b) $\vec{F} = 3.6\times10^{-3}\,N\left(\hat{\imath}\times(\hat{\imath}+\hat{\jmath})\right)$

$= 3.6\times10^{-3}\,N\,(\cancel{\hat{\imath}\times\hat{\imath}}^{\,0} + \hat{\imath}\times\hat{\jmath})$

$\boxed{\vec{F} = 3.6\times10^{-3}\,N\,\hat{k}}$

c) $\vec{F} = 3.6\times10^{-3}\,N\left(\hat{\imath}\times(\hat{\imath})\right)$

$\boxed{\vec{F} = 0}$

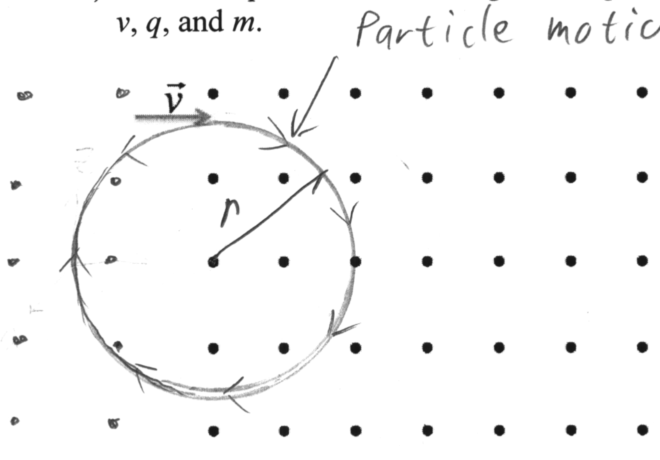
d) $\vec{F} = 3.6\times10^{-3}\,N\left(\hat{\imath}\times(\hat{\imath}+\hat{k})\right)$

$= 3.6\times10^{-3}\,N\,(\cancel{\hat{\imath}\times\hat{\imath}}^{\,0} + \hat{\imath}\times\hat{k})$

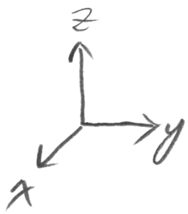
$\boxed{\vec{F} = -3.6\times10^{-3}\,N\,\hat{\imath}}$

If a particle with mass *m,* charge *q,* and velocity *v* moves through a uniform magnetic field, *B* that is perpendicular to the velocity.

a) What is the magnitude and direction of the force on the particle?
b) How does the magnitude of the particle's velocity change?
   (does it increase/decrease/stay the same)? Explain.
c) How does the direction of the particle's velocity change?
   Sketch the particle's motion on the figure.
d) Find an expression for the radius, *r*, of the particle's motion in terms of *B*, *v*, *q*, and *m*.
e) Find an expression for the angular frequency, *ω*, and period, *T*, of the particle's motion in terms of *B*, *v*, *q*, and *m*.



$\vec{B}$ (out of page)



a) With my coordinate system,

$\vec{v} = v\hat{j}$, $\vec{B} = B\hat{i}$

$\vec{F} = q(\vec{v} \times \vec{B})$

$= qvB(\hat{j} \times \hat{i})$

$\boxed{\vec{F} = -qvB\hat{k}}$ → instantaneous direction. – it keeps changing

b) The magnitude of $\vec{v}$ is constant because $\vec{F} = q(\vec{v} \times \vec{B})$, and the cross product gaurentees that $\vec{F} \perp \vec{v}$ at all times. For $|\vec{v}|$ to change, some component of $\vec{F}$ must be parallel to $\vec{v}$.

c) The direction is constantly changing because $\vec{F} \perp \vec{v}$ always. In fact, this leads to circular motion!

continued
↓

Magnetics Set 1 P3 continued

d) in circular motion, $\vec{a} = -\frac{v^2}{r}\hat{r}$

For a charged particle in a magnetic field: $\vec{F} = q(\vec{v} \times \vec{B})$

and $\vec{F} = m\vec{a}$

For our particle, $\vec{F} = -qvB\hat{r}$

So by NSL: $\vec{F} = m\vec{a} \Rightarrow +q\cancel{v}B\cancel{\hat{r}} = +m\frac{v^{\cancel{2}}}{r}\cancel{\hat{r}}$

Then: $\boxed{|\vec{r}| = \frac{mv}{qB}}$

e) Recall that arc-length is: $s = r\theta$

so: $v = r\omega \Rightarrow \omega = \frac{v}{r}$

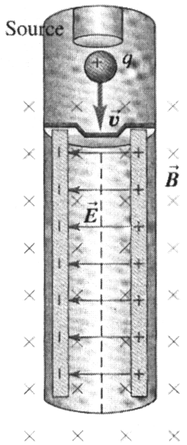
$\Rightarrow \omega = \frac{\cancel{v}}{m\cancel{v}}qB \Rightarrow \boxed{\omega = \frac{qB}{m}}$

Recall also that $T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\frac{m}{qB}}$

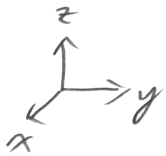
A velocity selector contains perpendicular electric (E) and magnetic (B) fields, as shown in the diagram below. A particle with charge q is shot into the device (perpendicular to both fields) with velocity v.

a) Find an expression for the net force (magnitude and direction) on the particle in terms of q, E, v and B.

b) The only particles that will make it through the tube are those that travel in a straight line. If the particles have charge $+q$, what will be the velocity of particles leaving the tube?



(a)



a) $\vec{F}_{net} = \vec{F}_E + \vec{F}_B$

$= q\vec{E} + q(\vec{v} \times \vec{B})$, $\quad \vec{E} = -E\hat{j}$, $\vec{B} = -B\hat{i}$

$\vec{v} = -v\hat{k}$

So: $\vec{F}_{net} = q\left(-E\hat{j} + (-v\hat{k}) \times (-B\hat{i})\right)$

$= q(-E\hat{j} + vB\hat{j})$

$\Rightarrow \boxed{\vec{F}_{net} = q(vB - E)\hat{j}}$

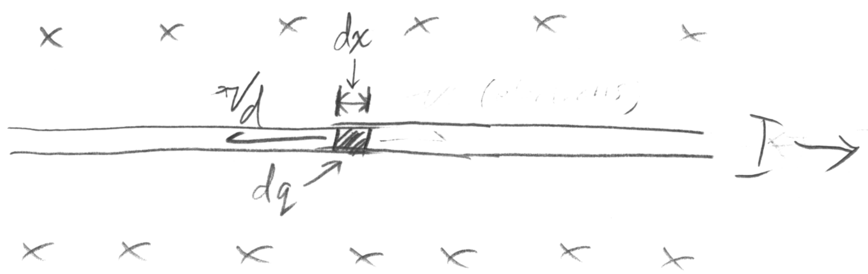
b) For the particles to travel in a straight line,

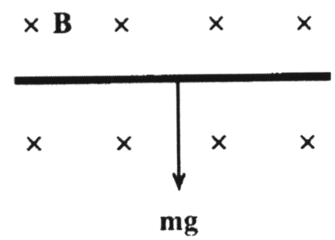
$|\vec{F}_{net}| = 0 \Rightarrow q(vB - E) = 0$

$\Rightarrow vB = E$

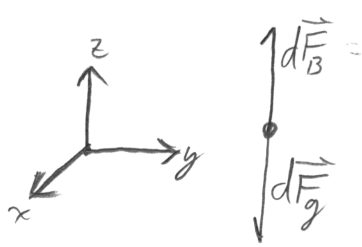
$\Rightarrow \boxed{v = \frac{E}{B}}$

A uniform magnetic field with magnitude $|\vec{B}|$ is pointing into the page. A straight segment of 1.0 mm diameter copper wire is in the plane of the page and perpendicular to the magnetic field. Gravity pulls the wire straight downward, toward the bottom of the page. What current through the wire, and in which direction, will allow the wire to "float" in the magnetic field without moving up or down? The density of copper is 8920 kg/m^3. (Note: you don't need to know the length of the wire.)





FBD for the little chunk of charge dq



Floats when $d\vec{F}_B + d\vec{F}_g = 0$

$$d\vec{F}_B = dq(\vec{v}\times\vec{B})$$

$$d\vec{F}_B = -dq(-v\hat{j}\times(-B)\hat{i})$$, charge is due to electrons

$$dF_B = +dq\frac{dl}{dt}\hat{j}\times(-B)\hat{i}$$

and $\boxed{v = \frac{dl}{dt}}$ → dl is distance dq moves in a time t

But: $I = \frac{dq}{dt} \Rightarrow dq = I\,dt$

$$\Rightarrow dF_B = I\cancel{dt}\frac{dl}{\cancel{dt}}\hat{j}\times -B\hat{i}$$, So, I is in the $+\hat{j}$

continued ↓

Magnetics Set 1, P5 - continued

$$\Rightarrow d\vec{F}_B = IB\,dl\,\hat{k},$$

And integrating over the length of the wire:

$$\vec{F}_B = \int_0^L IB\,dl\,\hat{k} \Rightarrow \boxed{\vec{F}_B = IBL\,\hat{I} \text{ for a long straight wire}}$$

Then, the net gravitational force is:

$$\vec{F}_g = -Mg\hat{k}, \quad M = \rho V = \rho \pi r^2 L$$

$$\vec{F}_g = -\rho \pi r^2 L g \hat{k}$$

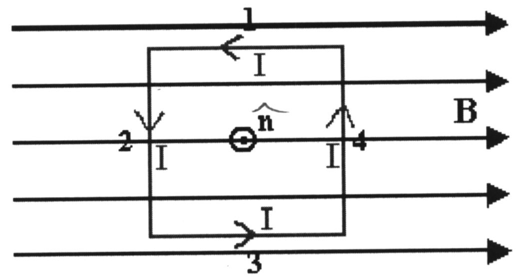
And we want $\vec{F}_B + \vec{F}_g = 0$

$$\Rightarrow IBL - \rho \pi r^2 L g = 0$$

$$\Rightarrow IB\cancel{L} = \rho \pi r^2 \cancel{L} g$$

$$\Rightarrow \boxed{I = \frac{\rho \pi r^2 g}{B}}$$

A square loop of wire with sides of length L is positioned in a magnetic field with field strength B as shown. Sides 1 and 3 are parallel to the field, and sides 2 and 4 are perpendicular to the field. A current, I, runs through the counter clockwise through the wire as shown.



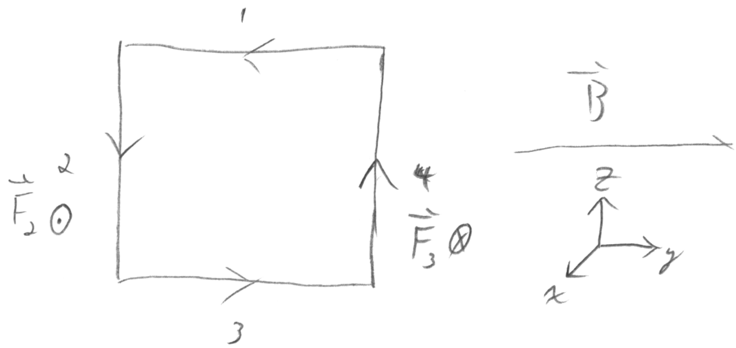
a) Find a vector expression for the *net force* on the loop.

b) Find a vector expression for the *net torque* on the loop.

c) Write your net torque in terms of the area of the loop, A, rather than the length of the sides.

d) What would the net torque be if the B field were perpenducular to the plane of the page?

a)

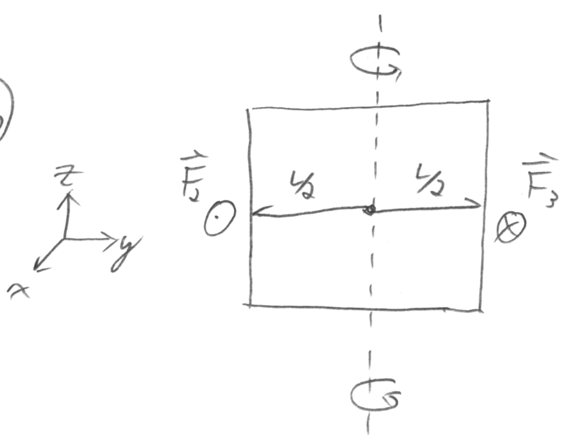


$\vec F = L(\vec I \times \vec B)$

$\vec F_1 = \vec F_2 = 0$ since $\vec I \parallel \vec B$

$\vec F_2 = ILB\hat x$, $\vec F_3 = -ILB\hat x \Rightarrow \boxed{\vec F_2 + \vec F_3 = 0\hat x}$

b)



$\vec T = \vec r \times \vec F$, $\vec r_2 = -\frac{L}{2}\hat y$, $\vec r_3 = \frac{L}{2}\hat y$

$\vec T = \vec T_1 + \vec T_2$

$= -\frac{L}{2}\hat y \times ILB\hat x + \frac{L}{2}\hat y \times ILB\hat x$

$= -\frac{1}{2}IL^2B(\hat y \times \hat x) + -\frac{1}{2}IL^2B(\hat y \times \hat x)$

$\Rightarrow \boxed{\vec T = IL^2B\hat z}$

c) $A = L^2 \Rightarrow \boxed{\vec{T} = IAB\hat{k}}$

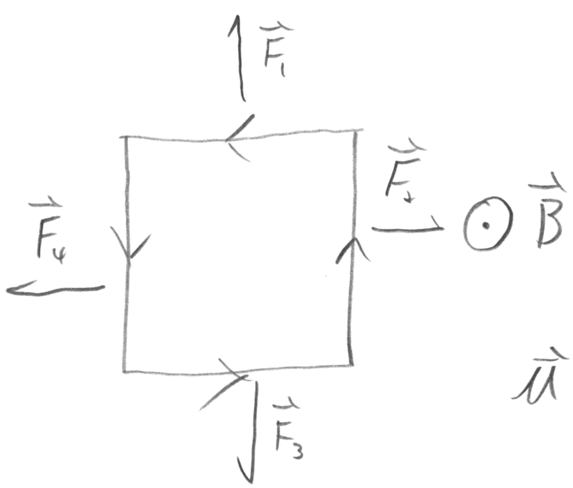
We define: $\vec{\mu} = I\vec{A} \rightarrow$ magnetic moment.

where $\vec{A} = A\hat{n}$ and $\hat{n}$ is the unit normal vector.

Then, $\boxed{\vec{T} = \vec{\mu} \times \vec{B}}$

$\hat{n}$ is given by the right hand rule with fingers curled in the direction of current flow.

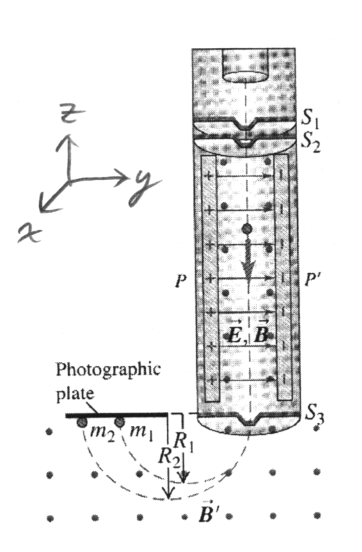
d)



$\boxed{\vec{F}_{net} = 0}$

$\vec{\mu}$ is aligned with the field.

In a **Mass spectrometer**, after the particles pass through a velocity selector, they go through a region with a magnetic field B' and no electric field, as shown in the figure. The particles will move in a semi-circular arc and strike a detector that is parallel to the face of the velocity selector.



Write an expression for the for the distance between the exit point from the velocity selector and the point where the particle strikes the detector.

* Velocity of particle at the exit of the velocity selector:

$$\vec{F}_B + \vec{F}_E = 0 \Rightarrow q(\vec{v} \times \vec{B}) = q\vec{E}$$

$$\Rightarrow q(-v\hat{k} \times B\hat{i}) = qE\hat{j}$$

$$\Rightarrow \boxed{|\vec{v}| = q\frac{E}{B}}$$

* Radius of particle motion in B' field:

$$F = ma \Rightarrow qvB' = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB'}$$

$$\Rightarrow r = \frac{m}{\cancel{q}B'} \cdot \cancel{q}\frac{E}{B}$$

$$\Rightarrow \boxed{r = \frac{mE}{BB'}}$$ independant of charge!

\# Distance from exit: $d = 2r$

$$\Rightarrow \boxed{d = \frac{2mE}{BB'}}$$

An electron accelerates from rest through a potential difference of 350 V. It then leaves the electric field and enters a magnetic field where it travels along a circular path of radius 7.50 cm. (uniform circular motion)

(a) Find an analytic expression for the magnetic field strength in terms of given information.
(b) What are the angular speed ω, orbital frequency *f*, and orbital period *T* of the electron circulating in the magnetic field? Plug in numbers to get numerical answers.
(c) Would your three answers in part (b) increase or decrease if the particle were a proton?

Given

$\Delta V = 350V$

$e =$ ___

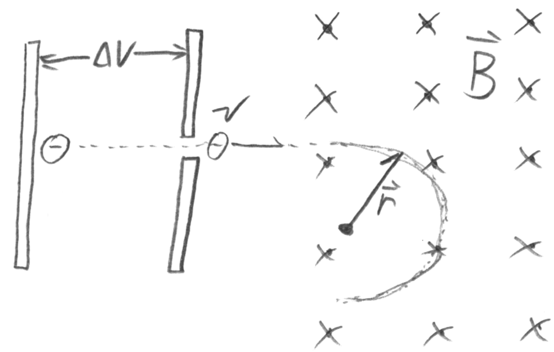
$m_e =$ ___

$r = 7.5cm$

Want

$|\vec{B}|$

a)



* Find v:

$$q\Delta V = \frac{1}{2}mv^2 \Rightarrow e\Delta V = \frac{1}{2}m_e v^2$$

$$\Rightarrow v = \left[\frac{2e\Delta V}{m_e}\right]^{1/2}$$

* Find r in terms of v:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{m_e v}{eB}$$

* Put 'em together:

$$r = \frac{m_e}{eB}\cdot\left[\frac{2e\Delta V}{m_e}\right]^{1/2} \Rightarrow \boxed{r = \left[\frac{2m_e\Delta V}{eB^2}\right]^{1/2}}$$

continued
↓

Magnetics set 1, P8 continued.

b) Recall that: $s = r\theta \Rightarrow$ arc length

$\Rightarrow v = r\omega \Rightarrow$ angular velocity

$\Rightarrow a = r\alpha \Rightarrow$ angular acceleration.

so: $\omega = \frac{v}{r} \Rightarrow \omega = \left[\frac{\cancel{2}e\cancel{\Delta V}}{m_e}\right]^{1/2}\left[\frac{eB^2}{\cancel{2}m_e\cancel{\Delta V}}\right]^{1/2}$

$$\Rightarrow \omega = \left[\frac{e^2B^2}{m_e^2}\right]^{1/2}$$

$$\Rightarrow \boxed{\omega = \frac{eB}{m_e}}$$

Recall that $f = \frac{\omega}{2\pi} \Rightarrow \boxed{f = \frac{eB}{2\pi m_e}}$

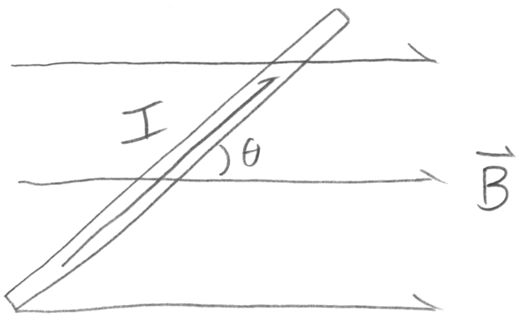
and: $T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\frac{m_e}{eB}}$

c) Protons have the same charge (but positive) But greater mass, so:

$$\boxed{\omega\downarrow,\ f\downarrow,\ T\uparrow}$$

A wire carrying 15 A makes a 25° angle with a uniform magnetic field. The magnetic force per unit length of wire is 0.31 N/m.

(a) What is the magnetic field strength?
(b) What is the maximum force per unit length that could be achieved by reorienting the wire in this field?



Given

$I = 15\ A$

$\theta = 25°$

$F/L = 0.31\ N/m$

a) $\vec{F} = L(\vec{I} \times \vec{B}) \Rightarrow \frac{F}{L} = IBSIN\theta$

$$\Rightarrow B = \frac{F}{L}\frac{1}{ISIN\theta}$$

$$\Rightarrow B = \frac{0.31\ N/m}{15 \cdot SIN(25)} = \boxed{48\ mT}$$

b) Maximum force when $\vec{I} \perp \vec{B}$

$$\frac{F}{L} = IB \Rightarrow \frac{F}{L}_{max} = 15 \cdot 48\ mT = \boxed{0.72\ N/m}$$