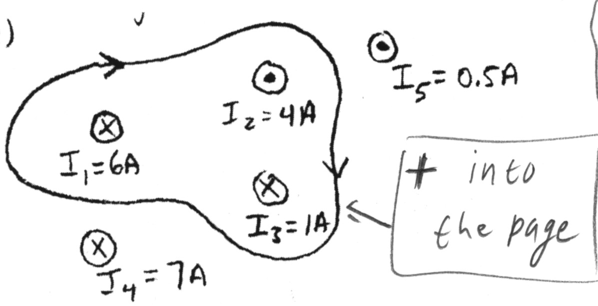
Magnetics – Set 4

Name: ______________________ **Problems Solved __/ 8**

1. (i) Calculate the values of the magnetic field line integrals ($\oint \vec{B} \cdot d\vec{l}$) around the closed paths in the following situations. (You don't have to integrate anything, Just determine what the integral equals by using Ampère's law.)

(a)

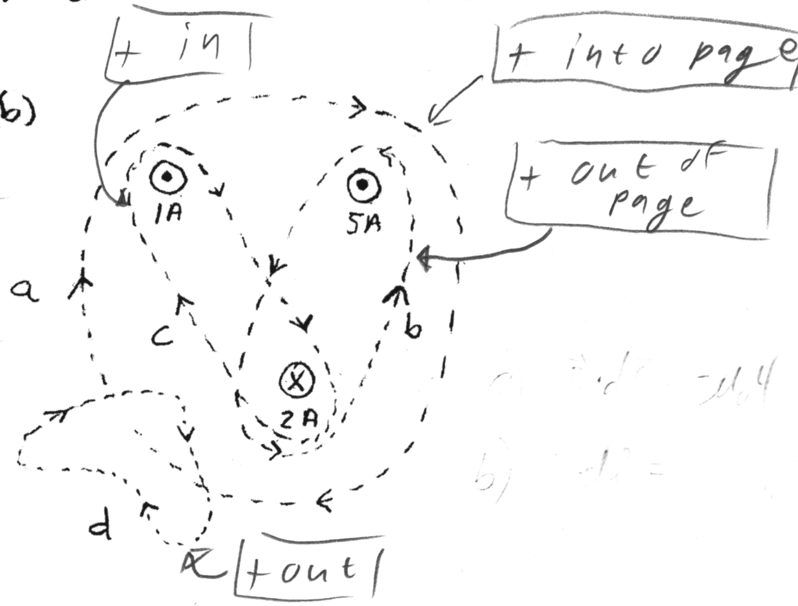


The sign of the current is given by the rhr.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (6A + 1A - 4A)$$

$$\Rightarrow \boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 3A}$$

(b)



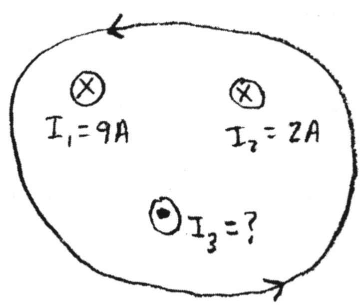
a) $\oint \vec{B} \cdot d\vec{l} = \mu_0(-1 - 5 + 2) = \boxed{-4\mu_0}$

b) $\oint \vec{B} \cdot d\vec{l} = \mu_0(5 - 2) = \boxed{3\mu_0}$

c) $\oint \vec{B} \cdot d\vec{l} = \mu_0(2 - 1) = \boxed{1\mu_0}$

d) $\oint \vec{B} \cdot d\vec{l} = \underline{0}$

(ii) For the system shown, what's the value of I_3 if $\oint \vec{B} \cdot d\vec{l} = 4.92 \times 10^{-6}$ Tm ?



$$\oint \vec{B} \cdot d\vec{l} = (I_3 - I_1 - I_2)\mu_0$$

$$\Rightarrow I_3 = \frac{\oint \vec{B} \cdot d\vec{l}}{\mu_0} + I_1 + I_2$$

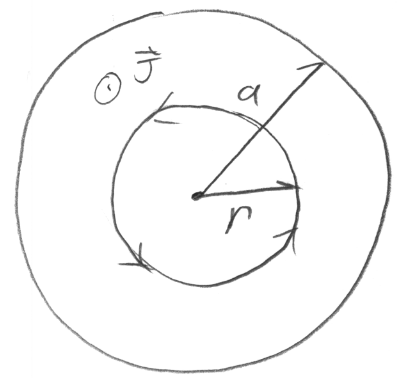
$$I_3 = \frac{4.92 \times 10^{-6}}{1.2 \times 10^{-6}} + 9A + 2A$$

$$\boxed{I_3 = 14.9 A}$$

Consider a long wire (infinite cylinder) with radius a and current density $\vec{J}$.

a) Find the current enclosed by an Amperian Loop that is concentric with the central axis of the wire and whose radius is $r < a$.
b) Find the current enclosed by an Amperian Loop that is concentric with the central axis of the wire and whose radius is $r > a$.
c) Use Ampere's Law to find the magnetic field inside the wire ($r < a$) and outside the wire ($r > a$).

Given: a, $\vec{J}$



a) Want: I_{enc}

$$I_{enc} = \int \vec{J} \cdot d\vec{A} = \boxed{J\pi r^2}$$

b) $$I_{enc} = \int \vec{J} \cdot d\vec{A} = \boxed{J\pi a^2}$$

c) $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc},$$ $\vec{B} \parallel d\vec{s}$ and $|\vec{B}|$ is constant at r

so: $$\boxed{B 2\pi r = \mu_0 I_{enc}}$$

$r<a$: $$B 2\cancel{\pi}\cancel{r} = \mu_0 J \cancel{\pi} r^{\cancel{2}}$$

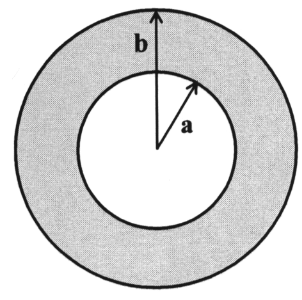
$$\Rightarrow \boxed{B = \frac{\mu_0 J r}{2}}$$

$r>a$: $$B 2\cancel{\pi} r = \mu_0 J \cancel{\pi} a^2$$

$$\Rightarrow \boxed{B = \frac{\mu_0 J a^2}{2r}}$$

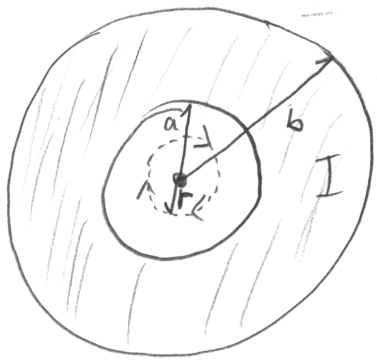
Magnetics – Set 4

A very long conductor is made from a hollow cylinder with an inner radius of **a** and an outer radius of **b**, where **b** = 3**a**. The conductor carries a current **I** uniformly distributed through the material. The current points out of the page. Derive expressions for the magnitude of the magnetic fields (including directions) in all three regions of space (r < a, a < r < b, r > b).



end view

$\underline{r<a:}$

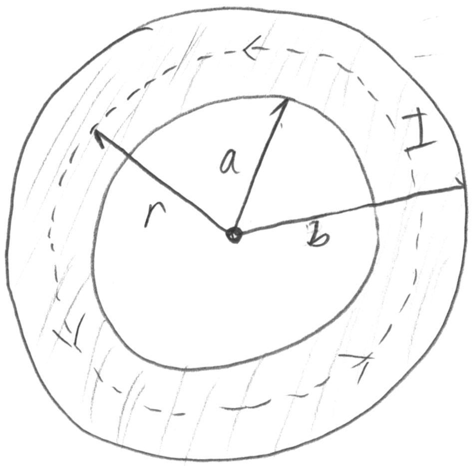


$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

$$\oint \vec{B}\cdot d\vec{s} = 0$$

and, by symmetry, $\boxed{\vec{B} = 0}$

$\underline{a<r<b}$



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

I_{enc} will go as the ratio of the cross sectional areas:

$$I_{enc} = \frac{A_{\phi}}{A_b} I$$

amperian loop

Total area

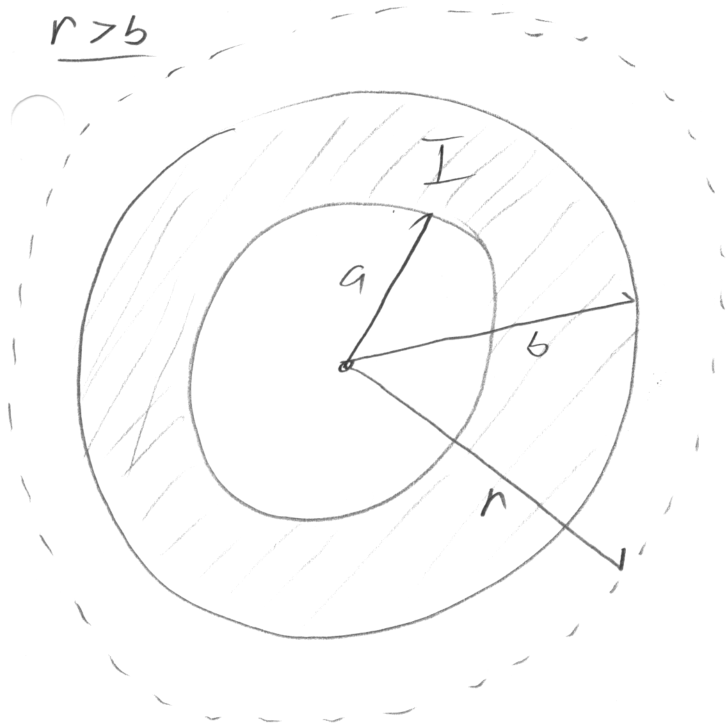
$$\boxed{I_{enc} = \frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2} I}$$

$$\Rightarrow B 2\pi r = \mu_0 \frac{r^2 - a^2}{b^2 - a^2} I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2}}$$

Magnetics Set 4, P3 continued

$r > b$



$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

Here, $I_{enc} = I$

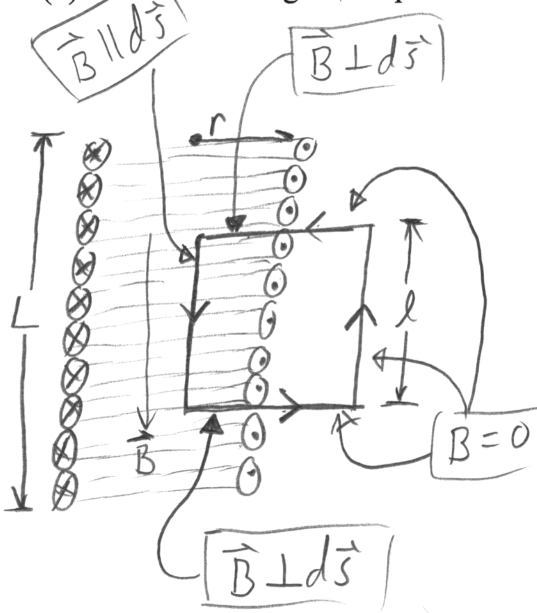
$B 2\pi r = \mu_0 I$

$\Rightarrow \boxed{B = \dfrac{\mu_0 I}{2\pi r}}$

3. A solenoid has a length L, a radius r, a winding of N turns, and carries a current of I.

(a) Calculate the magnitude of the magnetic field inside the solenoid.
(b) What is the magnetic dipole moment of the solenoid?

a)



$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}}$$

(as long as $r \ll L$)

So, $\oint \vec{B} \cdot d\vec{s} = \underline{B\ell}$

I_{enc} will go as I times the number of windings enclosed

$$I_{enc} = nI, \quad n = \frac{\ell}{L} N$$

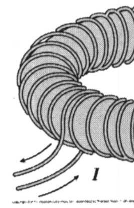
$$\Rightarrow I_{enc} = \frac{\ell}{L} N I$$

where $n = \frac{N}{L}$ (turns/length)

$$\Rightarrow B\cancel{\ell} = \frac{\cancel{\ell}}{L} N I \mu_0 \Rightarrow \boxed{B = \frac{NI}{L}\mu_0} = \boxed{nI\mu_0}$$

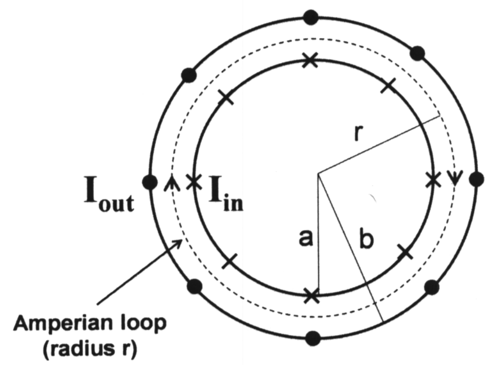
b) $\mu = NIA \Rightarrow \boxed{\mu = NI\pi r^2}$

4. This toroidal coil has a current I passing through its N turns. The inner radius is a, and the outer radius is b.



(a) The magnetic field associated with the toroid is nonzero

(i) only within the "hole" in the donut-shaped coil.
(ii) only within the region bounded by the coils.
(iii) only outside the coils.
(iv) everywhere.



top cross-sectional view

(b) The magnetic field lines must be

(i) straight, and pointing into the page.
(ii) straight, and pointing out of the page.
(iii) straight, and pointing radially.
(iv) clockwise circular (in the top view)
(v) counterclockwise circular (in the top view)

(c) Doubling the total number of turns N in the toroid, without changing the coil's size or the current, will

(i) double the magnetic field.
(ii) quadruple the magnetic field.
(iii) halve the magnetic field.
(iv) not change the magnetic field.

(d) Which is the correct formula for the magnetic field within the coils?

(i) $B = \mu_0 NI$

(ii) $B = \dfrac{\mu_0 NI}{2\pi a}$

(iii) $B = \dfrac{\mu_0 NI}{2\pi b}$

(iv) $B = \dfrac{\mu_0 NI}{2\pi r}$

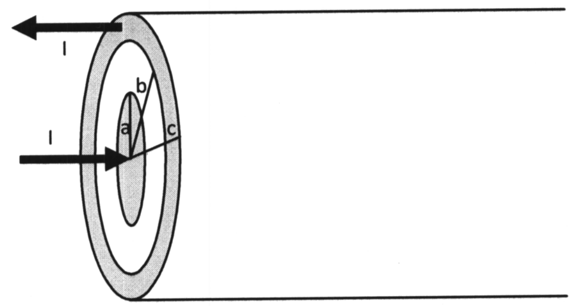
(e) Is a toroidal magnetic field a uniform field? Explain.

No, Because as r increases from a to b,

No additional current is enclosed

A very long solid cylindrical conductor of radius *a* carries a current *I* to the ~~left~~ right. Assume this current is evenly distributed throughout its cross-sectional area. Concentric with this inner cylinder is a cylindrical shell of inner radius *b* and outer radius *c*. It carries an equal current *I* in the opposite direction, evenly distributed throughout the shell. Use Ampère's Law to derive expressions for the magnetic fields for all distances *r* from the center of the system. Your expressions will be in terms of *a*, *b*, *c*, *r*, *I*, and fundamental constants.

It's very helpful to draw front-view cross sections for each region and each Ampèrian loop.



$\boxed{r<a}$

+ into page

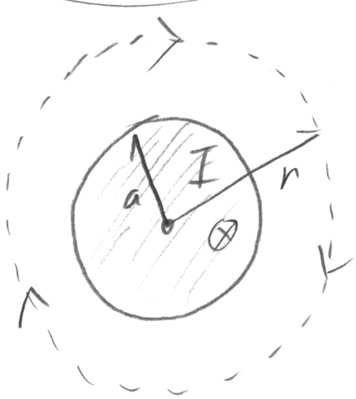
$$\oint \vec{B}\, d\vec{s} = \mu_0 I_{enc}$$

$$I_{enc} = \frac{\pi r^2}{\pi a^2} I \Rightarrow \text{Fraction of current enclosed.}$$

$$\Rightarrow B 2\pi r = \frac{\cancel{\pi} r^2}{\cancel{\pi} a^2} I \mu_0$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi a^2}}$$

$\boxed{a<r<b}$

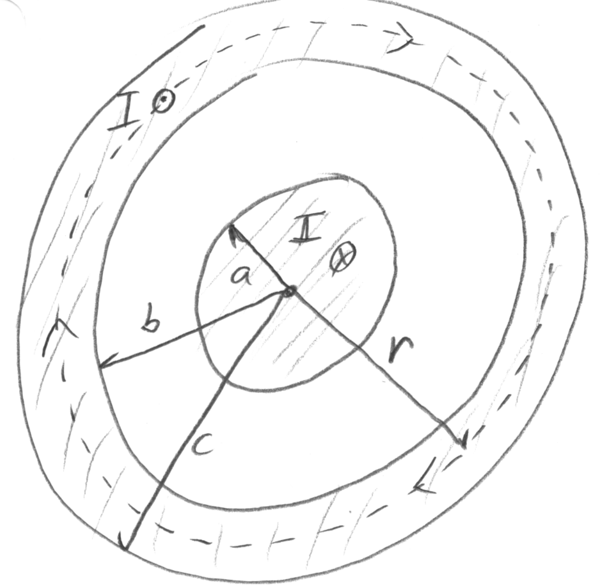


$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}, \quad I_{enc} = I$$

$$\Rightarrow B 2\pi r = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$\boxed{b < r < c}$ ⇒ This is the most complicated



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$I_{enc} = I_{inner} + I_{outer}$$

$I_{inner} = I$ ⇒ current due to inner conductor

$$I_{outer} = -\frac{A_r}{A_c} I$$

A_r = Area of outer shell enclosed within r

A_c = Total area of outer shell

$$\Rightarrow I_{outer} = -\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2} I$$

$$\Rightarrow I_{enc} = I - \frac{r^2 - b^2}{c^2 - b^2} I = \frac{c^2 - r^2}{c^2 - b^2} I$$

$$\Rightarrow B 2\pi r = \mu_0 I \frac{c^2 - r^2}{c^2 - b^2} \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}}$$

$\boxed{r > c}$ Here, $I_{enc} = 0$ so $\oint \vec{B} \cdot d\vec{s} = 0$

and by symmetry, $\boxed{B = 0}$

A gigantic magnetic field of 37.2 T has been achieved at the MIT Francis Bitter National Magnetic Laboratory. Find the current needed to achieve such a field

(a) 2.00 cm from a long straight wire;
(b) At the center of a circular coil of radius 42.0 cm that has 100 turns;
(c) Near the center of a solenoid with radius 2.40 cm, length 32.0 cm, and 40000 turns.

a) $B = \frac{\mu_0 I}{2\pi r}$ for a straight wire.

$$\Rightarrow I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(2\times10^{-2})(37.2)}{1.2\times10^{-6}} = \boxed{8\ \text{~~billion~~ amps}}$$

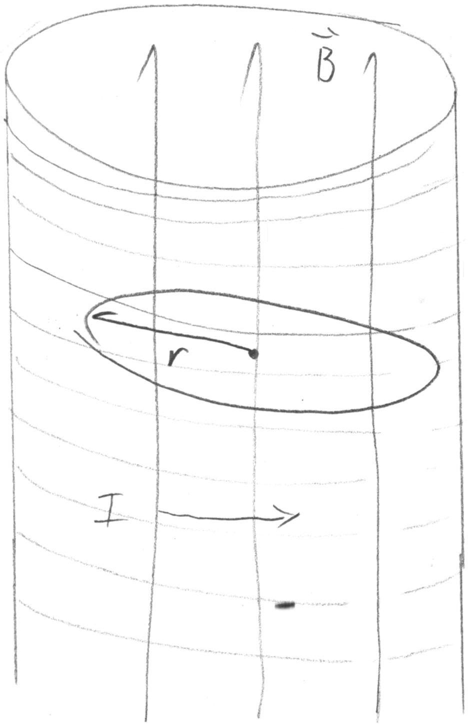
4 million amps

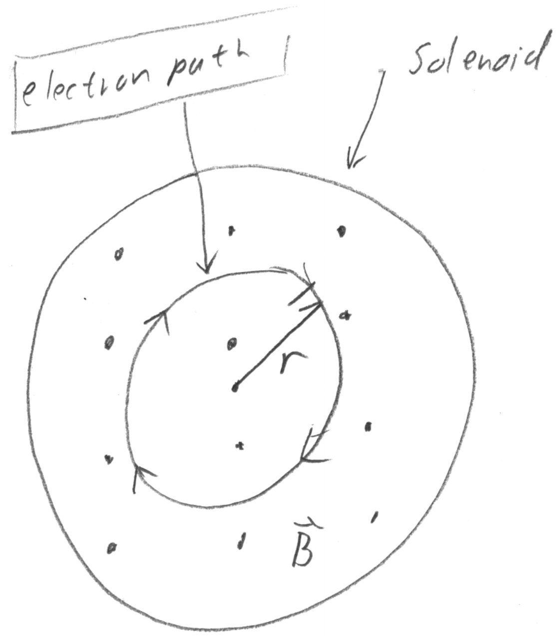
b) $B = \frac{\mu_0 I}{2a} N \Rightarrow I = \frac{2aB}{\mu_0 N} = \frac{2(42\times10^{-2})(37.2)}{(100)(1.2\times10^{-6})}$

$= \boxed{260 \text{ thousand Amps}}$

c) $B = \frac{NI}{L}\mu_0 \Rightarrow I = \frac{BL}{N\mu_0} = \frac{(37.2)(32\times10^{-2})}{(4\times10^{4})(1.2\times10^{-6})} = \boxed{248 \text{ Amps}}$

A long solenoid has N turns per centimeter. An electron moves within the solenoid in a circle of radius a perpendicular to the solenoid axis with a velocity v. Find the current in the solenoid.





Top View

By NSL:

electron charge

$$F = ma \Rightarrow q(\vec{v} \times \vec{B}) = m\frac{v^2}{r}, \quad v \perp B, \; q = e$$

$$\Rightarrow e\cancel{v}B = m\,v^{\cancel{2}}/r$$

$$\Rightarrow r = \frac{mv}{eB}$$

winding density

$$B = N\mu_0 I \quad \Rightarrow \boxed{r = \frac{mv}{eN\mu_0 I}}$$