Physics 112 Sample Test 2

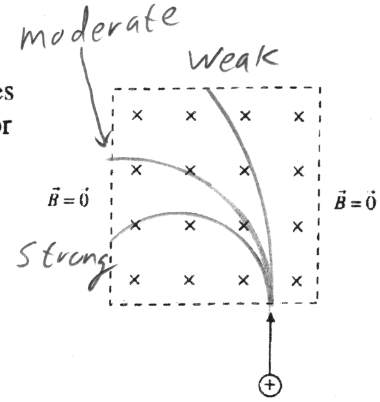
1. Concepts

The magnetic field is constant magnitude inside the dotted lines and zero outside. Sketch and label the trajectory of the charge for

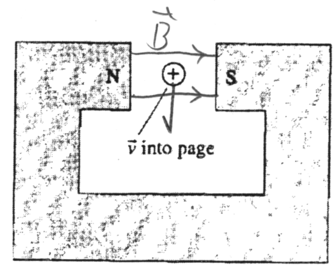
a. A very weak field.

b. A moderate field.

c. A strong field.



A positive ion, initially traveling into the page, is shot through the gap in a magnet. Is the ion deflected up, down, left, or right? Explain.



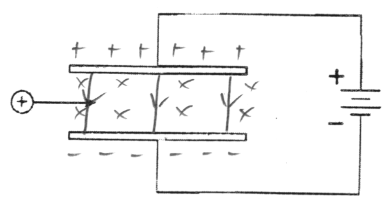
$\vec{F} = q\vec{v} \times \vec{B}$,

RHR gives down the page

A positive ion is shot between the plates of a parallel-plate capacitor.

a. In what direction is the electric force on the ion?

down, in direction of $\vec{E}$



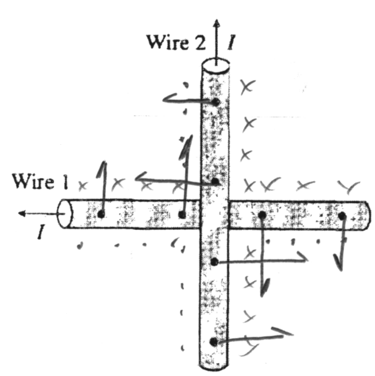
b. Could a magnetic field exert a magnetic force on the ion that is opposite in direction to the electric force? If so, show the magnetic field on the figure.

Yes, B into page will give an upwards force

Two current-carrying wires cross at right angles.

a. Draw magnetic force vectors on the wires at the points indicated with dots.

b. If the wires aren't restrained, how will they behave?



They will rotate until the currents are aligned.

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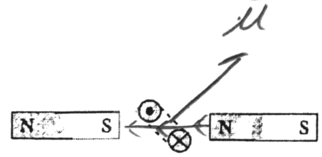
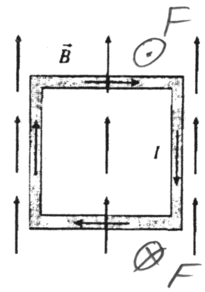


A current loop is placed between two bar magnets. Does the loop move to the right, move to the left, rotate clockwise, rotate counterclockwise, some combination of these, or none of these? Explain.

$\vec{T} = \vec{\mu} \times \vec{B}$, magnetic moment will align with the $\vec{B}$ field. loop will rotate ccw.



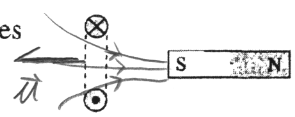
A square current loop is placed in a magnetic field as shown.

a. Does the loop undergo a displacement? If so, is it up, down, left, or right? If not, why not?

Net Force is zero. No displacement.

b. Does the loop rotate? If so, which edge rotates out of the page and which edge into the page? If not, why not?

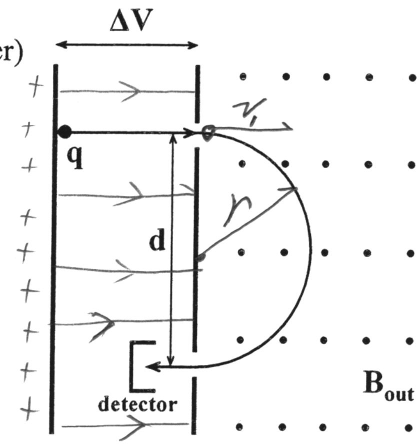
Top edge out of page, bottom edge into page.



The south pole of a bar magnet is brought toward the current loop. Does the bar magnet attract the loop, repel the loop, or have no effect on the loop? Explain.

No effect as long as it isn't perturbed. It is in an unstable equilibrium with $\vec{\mu}$ anti-parallel to the field.

2. A particle of charge q and mass m starts at rest and is accelerated through a potential difference ΔV applied across two parallel plates. It then enters a region of uniform magnetic field (pointing out of the paper) where its trajectory becomes circular. After following a half-circle of diameter d, the ion passes through a small opening and hits a detector.



(a) Is the sign of the ion's charge positive or negative?

(b) Is the sign of charge on the left plate positive or negative?

(c) Find an express for the potential required to hit the detector, ΔV, in terms of B, d, m, and q.

a) Positive. Negative charge would circle counter-clock-wise.

b) Positive to repel and accelerate the positive charge.

c) Part 1: conserve energy to get v_1 of the particle as it passes the right plate

$$\Delta U = \Delta K \Rightarrow q\Delta V = \tfrac{1}{2}mv^2$$

$$\Rightarrow \boxed{v = \left[\frac{2q\Delta V}{m}\right]^{1/2}}$$

Part 2: NSL gets the radius of the particle's orbit

$$F = ma \Rightarrow qvB = m\frac{v^2}{r}$$

subbing in v from part 1:

$$qB = \frac{m}{r}\left[\frac{2q\Delta V}{m}\right]^{1/2}$$

continued

Physics 112 Sample Test 4, P2 continued.

$$d = 2r \Rightarrow r = \tfrac{1}{2}d$$

$$qB = \frac{2m}{d}\left[\frac{2q\Delta V}{m}\right]^{1/2} \Rightarrow q^2B^2 = \frac{4m^{\cancel{2}}}{d^2}\,\frac{2q\Delta V}{\cancel{m}}$$

$$\Rightarrow \boxed{\frac{qB^2d^2}{8m} = \Delta V}$$

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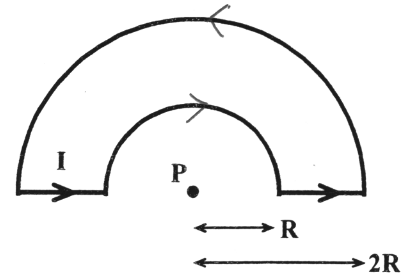
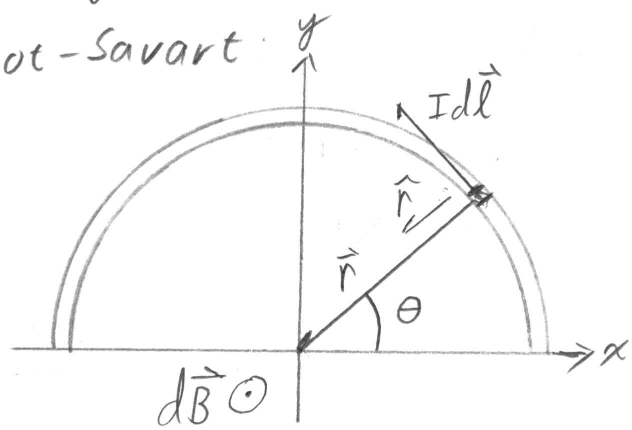


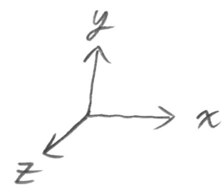
A loop of wire has the shape of two concentric semicircles connected by two radial segments. The loop carries a current I = 420 A in the direction indicated by the arrows on the straight segments. Use the Biot-Savart Law to find the magnitude and direction of the magnetic field at point P, located at the center of both semicircles. Let R = 29 cm.

For full credit, you should derive your equations for magnetic field from first principles. Don't simply write down the results from memory.

Derive general solution for arc of current from Biot-Savart



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$



Let's take a shortcut and get the direction of $d\vec{B}$ from the RHR.

From the diagram, $d\vec{l} \times \hat{r}$ is always in the $\underline{\hat{k}}$.

And: $|I d\vec{l} \times \hat{r}| = I|d\vec{l}||\hat{r}| \sin 90^\circ$

$|\hat{r}| = 1$ by definition

$\sin 90^\circ = 1$

So: $I d\vec{l} \times \hat{r} = I dl\, \hat{k}$ for all angles θ

continued

↓

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\hat{k}$$

* Note that we did not get an extra power of r in the denominator in this case

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi r^2}\int dl\,\hat{k}$$, switching to polar coordinates: $dl = r\,d\theta$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi r^{\cancel{2}}}\int_0^{2\pi}\cancel{r}\,d\theta\,\hat{k} \Rightarrow \vec{B} = \frac{\mu_0 I}{\cancel{4}_2\pi r}\cancel{2}\cancel{\pi}\hat{k}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2r}\hat{k}}$$ → general solution

For the two straight segments, $d\vec{l} \parallel \hat{r} \Rightarrow d\vec{l}\times\hat{r} = 0$

In my coordinate system, ccw current gives $+\hat{k}$ and cw current gives $-\hat{k}$.

So: $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2$

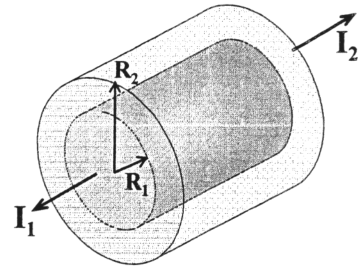
$$\Rightarrow \vec{B}_{net} = \frac{\mu_0 I}{2(2R)}\hat{k} - \frac{\mu_0 I}{2(R)}\hat{k}$$

$$\Rightarrow \vec{B}_{net} = \frac{\mu_0 I}{R}\left[\frac{1}{4} - \frac{1}{2}\right]\hat{k}$$

$$\Rightarrow \boxed{\vec{B}_{net} = -\frac{\mu_0 I}{4R}\hat{k}}$$

Physics 112 Sample Test 2

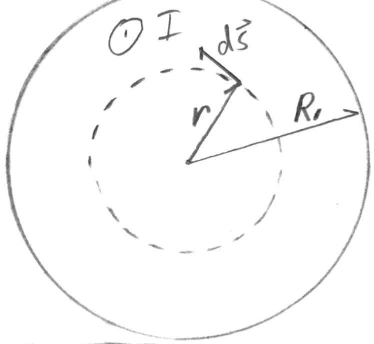
A coaxial cable consists of a thick, solid inner conductor of radius R_1 surrounded by a hollow, very thin outer conductor of radius R_2. The two conductors carry currents I_1 and I_2, respectively, where $\mathbf{I_1 > I_2}$. The currents flow in opposite directions, and the current densities are uniform throughout the conductors.



Use Ampere's law to determine the magnitudes and directions of the magnetic field vectors for these three regions: $r < R_1$, $R_1 < r < R_2$, and $r > R_2$. Draw cross-sectional sketches for each.

For full credit, you should derive your equations for magnetic field from first principles (Ampere's Law). Don't simply write down the results from memory.

$\boxed{r < R_1}$



cross-section of inner conductor

Ampere's Law

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$,

$|\vec{B}|$ is constant wrt $d\vec{s}$

$\vec{B} \parallel d\vec{s}$ at all points around the loop

So: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

$$I_{enc} = \frac{A_r}{A_R} I$$

$$\Rightarrow I_{enc} = \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2} I$$

$$\Rightarrow B \int ds = \mu_0 I_{enc}$$

$$\Rightarrow B \int_0^{2\pi} r\,d\theta = \mu_0 \frac{r^2}{R^2} I$$

$$\Rightarrow B\, 2\pi \cancel{r} = \mu_0 \frac{r^{\cancel{2}}}{R^2} I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}}$$

counter clockwise

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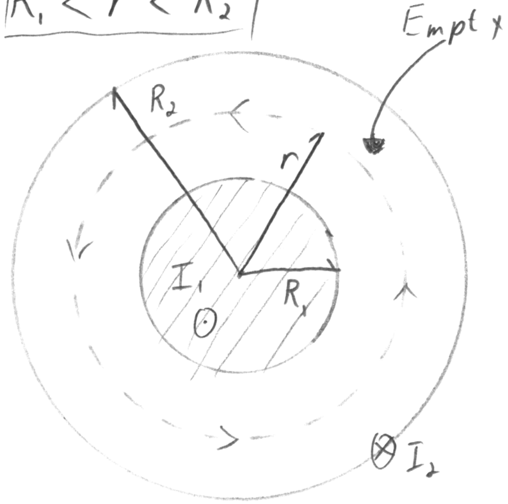
$\boxed{R_1 < r < R_2}$



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \quad , \quad I_{enc} = I_1$$

$$B 2\pi r = \mu_0 I_1$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I_1}{2\pi r}}$$ counter clock wise

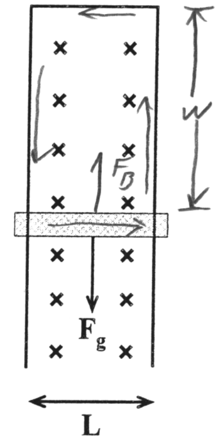
$\boxed{r > R_2}$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \quad , \quad I_{enc} = I_1 - I_2$$

$$\Rightarrow B 2\pi r = \mu_0 (I_1 - I_2)$$

$$\Rightarrow \boxed{B = \frac{\mu_0 (I_1 - I_2)}{2\pi r}}$$ counter clock wise since $I_1 > I_2$

In the figure is a U-shaped conducting rail that is oriented vertically and immersed in a horizontal magnetic field (that points into the paper). The rail has no electrical resistance and does not move. A slide bar with mass *m*, length *L*, and resistance *R* can slide vertically without friction while maintaining electrical contact with the rail. The slide wire is released from rest. It accelerates downward because of its own weight and eventually reaches a terminal velocity v_{term}.



(a) Explain why the bar reaches a terminal velocity instead of accelerating uniformly under the force of gravity.

(b) Find an expression for the terminal velocity using only the given parameters and known constants.

a)
- As the slide bar moves, the loop area increases.
- Increasing loop area causes increasing magnetic flux.
- The loop creates a current (ccw) to oppose the change (velocity dependent)
- The current in the slide bar creates a force in opposition to F_g.
- Velocity increases until $\vec{F}_B + \vec{F}_g = 0$

b) Find current in loop due to moving bar

$$V = -\frac{d\Phi_B}{dt}, \qquad \Phi_B = \vec{B}\cdot\vec{A} \Rightarrow \Phi_B = BLw$$

$$\Rightarrow I = \frac{1}{R}\frac{d\Phi_B}{dt} \qquad \Rightarrow \frac{d\Phi}{dt} = BL\frac{dw}{dt} \Rightarrow \boxed{\frac{d\Phi_B}{dt} = BLv}$$

$$\Rightarrow \boxed{I = \frac{BLv}{R} \text{ ccw}}$$

continued ↓

Phys 112, Sample Test 4, P5 continued.

Find F_B for moving bar

$$\vec{F_b} = I(\vec{L} \times \vec{B}), \quad \vec{L} \perp \vec{B}$$

$$\Rightarrow F_b = ILB$$

$$\Rightarrow F_b = \frac{BLv}{R} \cdot LB$$

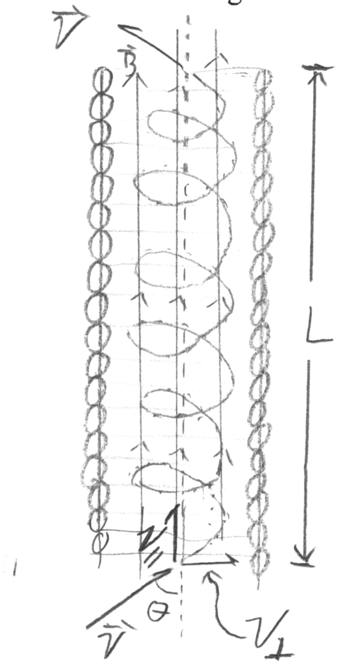
$$\Rightarrow \boxed{F_b = \frac{B^2L^2v}{R} \text{ up page, opposite } F_g}$$

Set $F_B = F_g$ to ensure $a = 0$

$$\frac{B^2L^2v}{R} = mg \Rightarrow \boxed{v_{term} = \frac{mgR}{B^2L^2}}$$

Physics 112 Sample Test 2

A charged particle is fired into the end of a solenoid with a velocity *v* that makes an angle of θ with the solenoid's central axis. The solenoid carries a current *I* and has N turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end?



Strategy

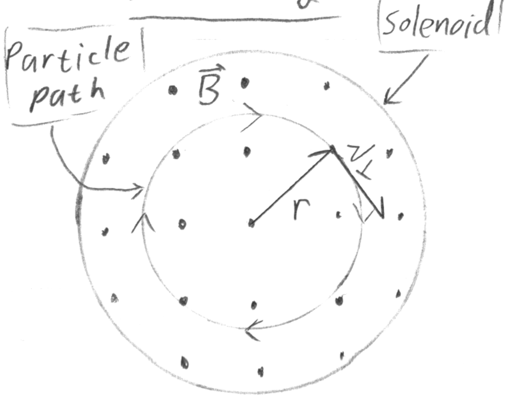
- Find time to get through solenoid, t_1
- Find time for particle to make 1 revolution in the field: t_2
- $\frac{t_1}{t_2}$ = number of revolutions.
- Use Ampere's Law to relate B and I

Find t_1

Time through solenoid depends only on $v_{\parallel}$, the component of velocity parallel to the solenoid axis

$$d = vt \Rightarrow L = v_{\parallel} t \Rightarrow \boxed{t_1 = \frac{L}{v\cos\theta}}$$

Find t_2



* Use NSL to relate $v_\perp$, r, B

$$F = ma$$

$$q\cancel{v}B = m\frac{v_\perp^{\cancel{2}}}{r} \Rightarrow \boxed{qB = \frac{mv_\perp}{r}}$$

continued ↓

Phys 112 Sample Test 4, P6 continued

Period: $d = vt \Rightarrow t = \frac{d}{v}$, t = period and d = circumference

$$\Rightarrow t = \frac{2\pi r}{v_1} \Rightarrow \boxed{\frac{v_1}{r} = \frac{2\pi}{t}}$$

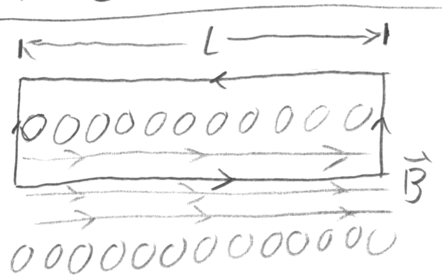
Then: $$qB = \frac{m v_1}{r}$$

$$\Rightarrow qB = \frac{2\pi m}{t_2} \Rightarrow \boxed{t_2 = \frac{2\pi m}{qB}}$$

Find # of revs, n

$$n = \frac{t_1}{t_2} \Rightarrow \boxed{n = \frac{L}{v\cos\theta} \cdot \frac{qB}{2\pi m}}$$

Find B as a fn of N and I



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$\Rightarrow BL = \mu_0 N I \Rightarrow \boxed{B = \frac{\mu_0 N I}{L}}$$

Then: $$n = \frac{\cancel{L}}{v\cos\theta} \frac{q}{2\pi m} \cdot \frac{\mu_0 N I}{\cancel{L}}$$

$$\boxed{n = \frac{\mu_0 q N I}{2\pi m v \cos\theta}}$$