Physics 112 Sample Test 5

NAME ____________________

Write out your fundamental relationships.

The wave function

$y(x,t) = A\sin(kx - \omega t + \phi)$

Wave number versus wavelength

$k = \frac{2\pi}{\lambda}$

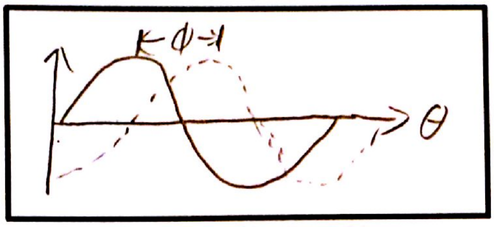
Period versus ~~wavelength~~ angular velocity

$T = \frac{2\pi}{\omega}$

Wave speed

$v = f\lambda$

Interference Phase Picture



Destructive Interference

$\phi = (2n+1)\pi$

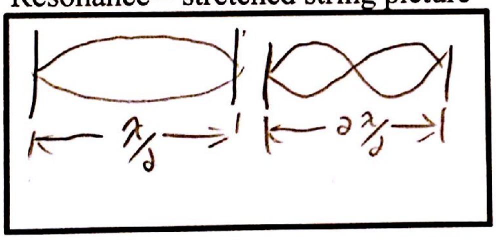
Constructive Interference

$\phi = 2n\pi$

General Phase between waves

$\phi = k\Delta x + \omega\Delta t + m\pi$

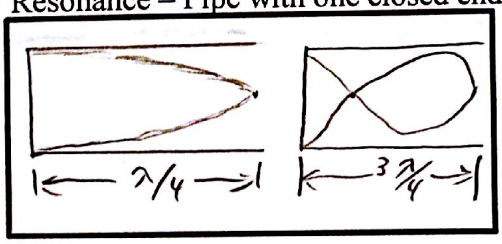
Resonance – stretched string picture



String length versus lambda

$L = n\frac{\lambda}{2}$

Resonance – Pipe with one closed end



Pipe length versus lambda

$\boxed{L = n\frac{\lambda}{4}}$ $n = \text{odd}$

Physics 112 Sample Test 5

Write out your fundamental relationships.

The wave function

$$y(x,t) = A\sin(kx - \omega t + \varphi)$$

Wave number versus wavelength

$$k = \frac{2\pi}{\lambda}$$

Period versus wavelength

$$T = \frac{2\pi}{\lambda}$$

Wave speed

$$v = f\lambda$$

Destructive Interference

$$\varphi = (2n+1)\pi$$

Constructive Interference

$$\varphi = 2n\pi$$

Phase Sources

$$\varphi = k\Delta x + \omega\Delta t + m\pi$$

String length versus lambda

$$L = n\frac{\lambda}{2}$$

Pipe length versus lambda

$$L = n\frac{\lambda}{4}, n = odd$$

Trig IDs

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$$\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

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NAME ______________________________

1. Concepts

The frequency of the fourth harmonic of a string is

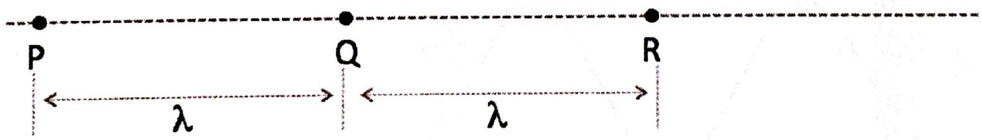
a) one-fourth the frequency of the fundamental
b) one-half the frequency of the fundamental
c) equal to the frequency of the fundamental
d) twice the frequency of the fundamental
e) four times the frequency of the fundamental
f) eight times the frequency of the fundamental

$$L = n\frac{\lambda}{2} \Rightarrow \lambda_n = \frac{2L}{n} \Rightarrow \frac{v}{f_n} = \frac{2L}{n} \Rightarrow f_n = \frac{vn}{2L}$$

In general: $$\frac{f_{n_1}}{f_{n_2}} = \frac{\cancel{v}n_1}{\cancel{2L}} \cdot \frac{\cancel{2L}}{\cancel{v}n_2} = \frac{n_1}{n_2} \Rightarrow \boxed{f_{n_1} = \frac{n_1}{n_2} f_{n_2}}$$

So: $$f_1 = \frac{1}{4} f_4 \Rightarrow f_4 = 4f_1$$

Two point sources, P and Q, emit sound waves of equal wavelengths (λ) and amplitudes (A). If you want to have **fully constructive interference** at point R, what has to be the phase difference between P and Q?



a) 0
b) 1/(2π)
c) 0.5
d) 1/π
e) 1
f) π

Constructive: $\phi = 2n\pi$

$= 0, 2\pi, 4\pi$ etc.

Physics 112 Sample Test 5

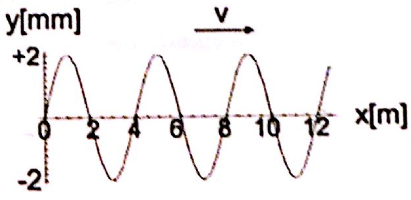
The following is a snapshot at t=0 for a transverse wave traveling ***to the right*** with velocity 2 m/s. Which of the following equations is correct for this wave?

$\boxed{a)\ y(x,t)=2\sin\left[\left(\frac{\pi}{2}\right)x-(\pi)t\right]}$

b) $y(x,t)=2\sin\left[(\pi)x-\left(\frac{\pi}{2}\right)t\right]$

c) $y(x,t)=2\sin\left[\left(\frac{\pi}{2}\right)x-\left(\frac{\pi}{2}\right)t\right]$

d) $y(x,t)=2\sin\left[(\pi)x-(\pi)t\right]$



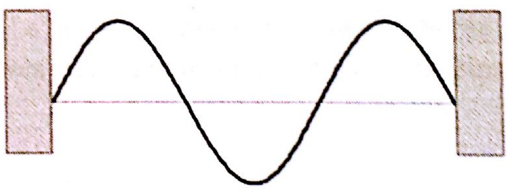
$k = 2\pi/\lambda$, $\lambda = 4\text{m} \Rightarrow k = \frac{2\pi}{4} = \frac{\pi}{2}$

$\omega = \frac{2\pi}{T}$, $T = \frac{\lambda}{v} = \frac{4\text{m}}{2\text{m/s}} = 2\text{s}$

$\omega = \frac{2\pi}{2} = \pi$

A string is vibrating at 300 Hz. Using a strobe light and an ultra fast camera you get a picture of the string as sketched below. The walls are separated by 1 meter. If the string were vibrating in its lowest possible frequency, what would that frequency be?

a) 50 Hz
$\boxed{\text{b) 100 Hz}}$
c) 150 Hz
d) 200 Hz
e) 300 Hz



$n=3$

$f_n = \frac{vn}{2L}$ (see first multiple choice question for proof)

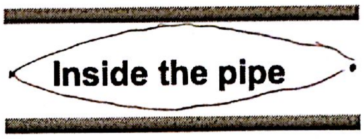
$f_1 = \frac{v}{2L}$, $f_3 = \frac{3v}{2L}$, I know f_3, But I don't know L...

eliminate L by: $\frac{f_1}{f_3} = \frac{\cancel{v}}{\cancel{2L}} \cdot \frac{\cancel{2L}}{3\cancel{v}} \Rightarrow \boxed{f_1 = \frac{1}{3} f_3}$

Physics 112 Sample Test 5

A pipe with two open ends is shown below. The length of the pipe is 1m and the speed of sound is 343 m/s. What is the first harmonic frequency of the sound wave created in this pipe?

a) 343Hz
b) 172Hz
c) 686Hz
d) 1029Hz
e) None of the above

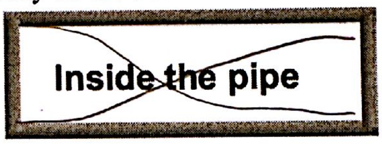


$L = \frac{n\lambda}{2} \Rightarrow f_n = \frac{vn}{2L}$

$f_1 = \frac{343}{2}$

A pipe with two close ends is shown below. The length of the pipe is 1m and the speed of sound is 343 m/s. What is the first harmonic frequency of the sound wave created in this pipe?

a) 343Hz
b) 172Hz
c) 686Hz
d) 1029Hz
e) None of the above



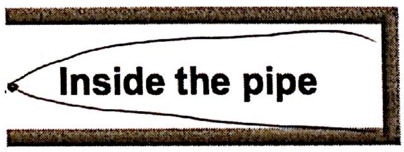
$L = \frac{n\lambda}{2} \Rightarrow f_n = \frac{vn}{2L}$

only odd n is allowed

f_n $\quad f_{n+2}$ $\quad f_{n+4}$

A pipe with one closed end is shown below. 428.75 Hz, 600.25 Hz and 771.75 Hz are three adjacent harmonic frequencies of sound waves created in this pipe. What is the pipe's first harmonic (lowest) frequency?

a) 86Hz
b) 172Hz
c) 343Hz
d) 257Hz
e) None of the above



$L = \frac{n\lambda}{4}$, $n = \text{odd}$

$\Rightarrow f_n = \frac{vn}{4L}$

$f_n = \frac{vn}{4L}$, $f_{(n+2)} = \frac{v(n+2)}{4L}$, I don't know n...

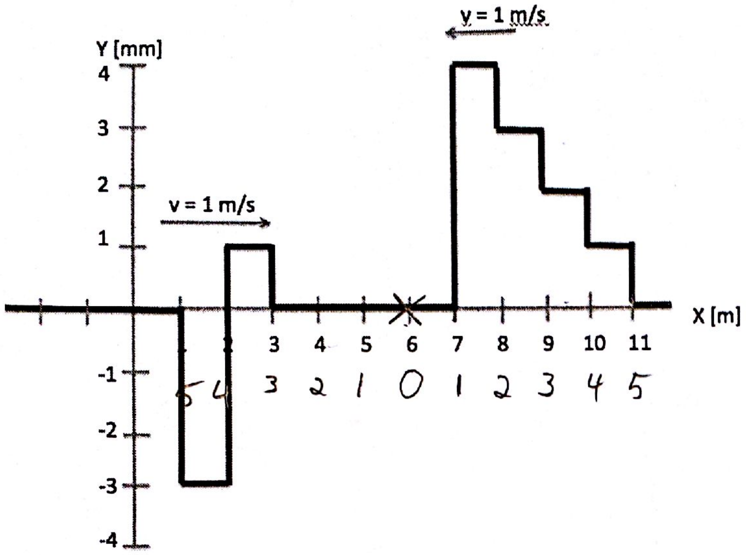
eliminate n by: $f_{n+2} - f_n = \frac{v}{4L}[\cancel{n} + 2 - \cancel{n}] \Rightarrow \Delta f = \frac{v}{2L} \Rightarrow \boxed{L = \frac{v}{2\Delta f}}$

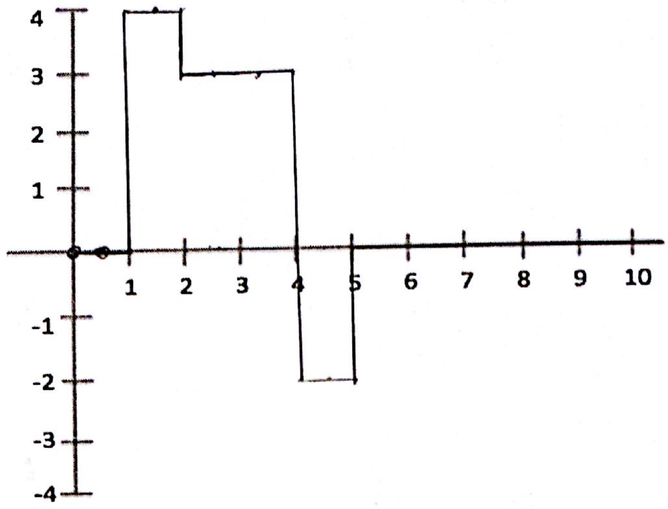
$\Rightarrow f_1 = \frac{v}{4L} \Rightarrow f_1 = \frac{\cancel{v}}{4 2}\cdot\frac{2\Delta f}{\cancel{v}} \Rightarrow \boxed{f_1 = \frac{\Delta f}{2}}$

Physics 112 Sample Test 5

(6)

Below is a snapshot graph of two wave pulses at $t = 0\ s$. The wave pulses are moving towards each other with speeds of $\boldsymbol{1\ m/s}$ as indicated in the graph. At right, draw a history graph of the point at $x = 6\ m$ from $t = 0\ s$ to $t = 10\ s$. Make sure to label the axes.





Physics 112 Sample Test 5

Below are three wave functions that describe transverse traveling waves on strings.

$y_1 = A\sin\left(kx - \omega t + \frac{\varphi}{3}\right)$
$y_2 = A\sin(kx + \omega t + \varphi)$
$y_3 = -A\sin(kx - \omega t + \varphi)$

a) If y_1 and y_2 were put on the same string, would the resulting wave be a traveling wave or a standing wave?

If it's a traveling wave, what is its amplitude? If it is a standing wave, where are the nodes?

b) If y_1 and y_3 were put on the same string, would the resulting wave be a traveling wave or a standing wave?

If it's a traveling wave, what is its amplitude? If it is a standing wave, where are the nodes?

a) $y_{net} = y_1 + y_2$

$= 2A\cos\left[\frac{1}{2}(\cancel{kx} - \omega t + \frac{\phi}{3} - \cancel{kx} - \omega t - \phi)\right]\sin\left[\frac{1}{2}(kx - \cancel{\omega t} + \frac{\phi}{3} + kx + \cancel{\omega t} + \phi)\right]$

$\Rightarrow \boxed{y_{net} = 2A\cos\left(-\omega t - \frac{\phi}{3}\right)\sin\left(kx + \frac{2\phi}{3}\right) \text{ Standing wave.}}$

nodes when: $\sin\left(kx + \frac{2\phi}{3}\right) = 0$

nodes

$\Rightarrow kx + \frac{2\phi}{3} = n\pi \Rightarrow \boxed{x = \frac{1}{k}\left(n\pi - \frac{2\phi}{3}\right)}$

b) $y_{net} = y_1 + y_3 = A\sin\left(kx - \omega t + \frac{\phi}{3}\right) - A\sin(kx - \omega t + \phi)$

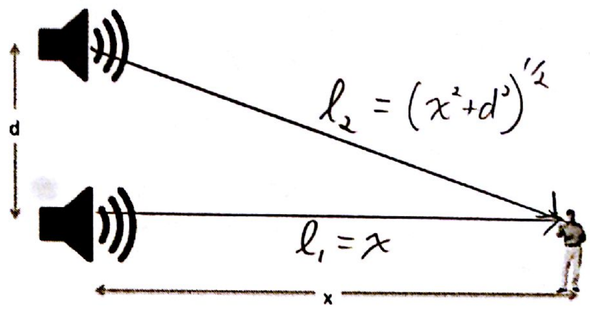
$= 2A\cos\left[\frac{1}{2}(kx - \omega t + \frac{\phi}{3} + kx - \omega t + \phi)\right]\sin\left[\frac{1}{2}(\cancel{kx} - \cancel{\omega t} + \frac{\phi}{3} - \cancel{kx} + \cancel{\omega t} - \phi)\right]$

$\boxed{y_{net} = 2A\cos\left[kx - \omega t + \frac{2\phi}{3}\right]\sin\left[-\frac{\phi}{3}\right] \text{ traveling wave}}$

$\boxed{A_{new} = -2A\sin\left(-\frac{\phi}{3}\right)}$

Physics 112 Sample Test 5

1. The two speakers in the figure are separated by a distance $d = 2.00\ m$ and are in phase. A listener is at a distance $x = 3.75\ m$ directly in front of one speaker. Consider the full audible range for normal hearing, $20\ Hz$ to $20\ kHz$.



a) What is the lowest frequency, f_{min}, that gives destructive interference at the listener's ear?

b) By what number must f_{min} be multiplied to get the second lowest frequency that gives destructive interference?

c) What is the lowest frequency, f_{max}, that gives constructive interference at the listener's ear?

a) $\phi = k\Delta\ell + \cancel{\omega\Delta t} + \cancel{m\pi}$ no time lag or reflections.

want $\phi_0 = (2n+1)\pi \rightarrow$ Destructive

so: $k\Delta\ell = (2n+1)\pi \Rightarrow \frac{2\pi}{\lambda}\Delta\ell = (2n+1)\pi$, $v = \lambda f \Rightarrow \frac{1}{\lambda} = \frac{f}{v}$

$\Rightarrow \frac{f}{v} 2\Delta\ell = (2n+1)$

$$\Rightarrow \boxed{f_n = \frac{v(2n+1)}{2\Delta\ell}}$$

$\Delta\ell = \ell_2 - \ell_1 \Rightarrow \Delta\ell = (x^2+d^2)^{1/2} - x = (3.75_m^2 + 2.0_m^2)^{1/2} - 3.75$

$\Rightarrow \Delta\ell = \frac{1}{2}m$

Does $n=0$ work?

$f_0 = \frac{v}{2\Delta\ell} = \frac{343\,m/s}{2\cdot\frac{1}{2}m} = \boxed{343\ Hz}$ yes, greater than 20 Hz

continued

↓

Sample Test 5, P8 - continued

b) $f_0 = \frac{v}{2\Delta l}$, $f_n = \frac{v(2n+1)}{2\Delta l}$

$$\Rightarrow \frac{f_n}{f_0} = \frac{\cancel{v}(2n+1)}{\cancel{2\Delta l}} \cdot \frac{\cancel{2\Delta l}}{\cancel{v}}$$

$$\Rightarrow \boxed{f_n = (2n+1) f_0}$$

First null at $n=0$, 2^{nd} at $n=1$

$$\boxed{f_1 = 3 f_0}$$

c) Now we want constructi interfe ce

$$k\Delta l = 2n\pi \Rightarrow \frac{\cancel{2\pi}}{\lambda}\Delta l = \cancel{2\pi}n \Rightarrow \frac{f_n}{v}\Delta l = n$$

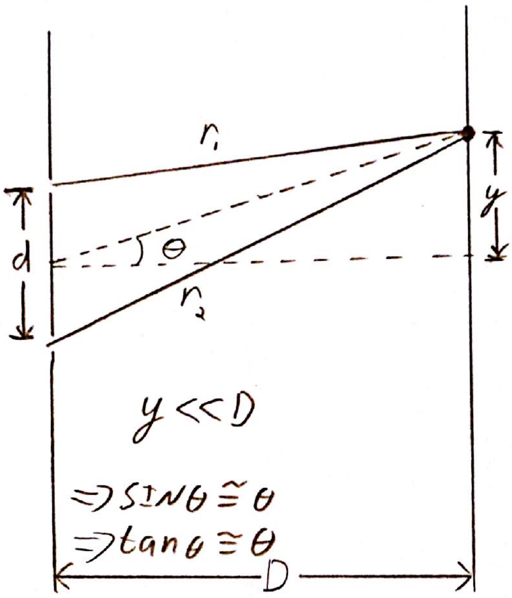
$$f_n = n\frac{v}{\Delta l}$$

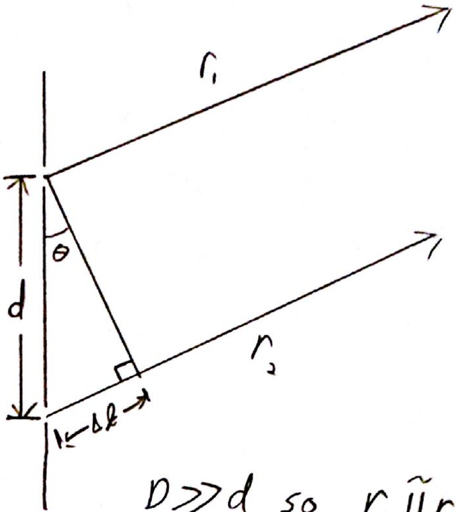
In this case we can't have $n=0$, so $n=1$

$$f_1 = \frac{v}{\Delta l} \Rightarrow f_1 = \frac{343\,m/s}{1/2\,m} = \boxed{171.5\,Hz}$$

Physics 112 Sample Test 5

5. A double-slit experiment is performed with 589-nm light. The distance between the screen and the slits is 2.00 m. The $m = 10$ interference maximum is observed 7.26 mm from the central maximum. How far apart are the slits? (The small-angle approximation is valid here.) *Show all work from scratch rather just plugging into an equation.*





$y \ll D$

$\Rightarrow \sin\theta \cong \theta$

$\Rightarrow \tan\theta \cong \theta$

$D \gg d$ so $r_1 \,\tilde{\parallel}\, r_2$

$\Delta l = d\sin\theta$

constructive

$k\Delta l = 2n\pi$

$\Rightarrow \frac{\cancel{2\pi}}{\lambda} d\sin\theta = \cancel{2\pi} n \Rightarrow \sin\theta = n\frac{\lambda}{d}, \quad \sin\theta \cong \theta$

$\Rightarrow \boxed{\theta \cong n\frac{\lambda}{d}}$

Then: $\tan\theta = \frac{y}{D}, \quad \tan\theta \cong \theta$

$\Rightarrow \theta \cong \frac{y}{D}$ so: $\frac{y}{D} = n\frac{\lambda}{d} \Rightarrow \boxed{d = n\lambda\frac{D}{y}}$

$d = (10)(589\times10^{-9}\,m)\frac{2.0\,m}{7.26\times10^{-3}\,m}$

$\boxed{d = 1.6\times10^{-3}\,m}$

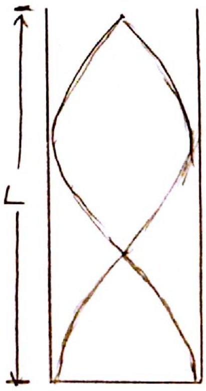
10

Physics 112 Sample Test 5

During an experiment studying the speed of sound in air using an audio oscillator and a tube open at one end and stopped at the other, a particular resonant frequency is found to have nodes roughly 6.94 cm apart. The oscillator's frequency is increased, and the next resonant frequency found has nodes 5.40 cm apart.

(a) What are the two resonant frequencies?
(b) What is the fundamental frequency?
(c) Which harmonics are these two modes?

The speed of sound is 343 m/s.



a) Distance between nodes is: $\Delta x = \frac{\lambda}{2} \Rightarrow \lambda = 2\Delta x$

and $v = \lambda f \Rightarrow v = 2\Delta x f$

$\Rightarrow \boxed{f = \frac{v}{2\Delta x}}$

$\Rightarrow f_n = \frac{343\,m/s}{2 \cdot 6.94\,m} \Rightarrow \boxed{f_n = 24.7\,Hz}$

$\Rightarrow f_{n+2} = \frac{343\,m/s}{2 \cdot 5.4\,m} \Rightarrow \boxed{f_{n+2} = 31.8\,Hz}$

b) In a tube: $L = \frac{\lambda n}{4}$, $n = odd$ $\Rightarrow L = \frac{v}{f}\frac{n}{4} \Rightarrow \boxed{f_n = \frac{vn}{4L}}$

we have: $\boxed{f_n = \frac{vn}{4L} \text{ and } f_{n+2} = \frac{v(n+2)}{4L}}$

We <u>want</u> L but don't know <u>n</u> so eliminate n and Solve for L.

$\Rightarrow f_{n+2} - f_n = \frac{v}{4L}(\cancel{n} + 2 - \cancel{n}) = \frac{v}{2L} \Rightarrow \Delta f = \frac{v}{2L}$

$\Rightarrow \underline{L = \frac{v}{2\Delta f}}$

Now, $f_1 = \frac{v}{4L} \Rightarrow f_1 = \frac{\cancel{v}}{4}\frac{2\Delta f}{\cancel{v}} = \frac{\Delta f}{2}$

continued↓

Sample Test 5, P10 continued

$\Rightarrow \boxed{F_1 = \frac{1}{2}\Delta F}$ where ΔF = distance between any 2 harmonics

So: $F_1 = \frac{1}{2}(31.8\,Hz - 24.7\,Hz) =$

$\Rightarrow \boxed{F_1 = 3.5\,Hz}$

c) $F_n = \frac{v n}{4L} \Rightarrow n = \frac{4L}{v} F_n, \quad L = \frac{v}{2\Delta F}$

$\Rightarrow n = \frac{\cancel{4}^{2} F_n}{\cancel{v}} \cdot \frac{\cancel{v}}{\cancel{2}\Delta F}$

$\Rightarrow \boxed{n = 2\frac{F_n}{\Delta F}}$

$n_1 = 2\frac{24.7}{31.8-24.7} = \boxed{7}$

$n_2 = 2\frac{31.8}{31.8-24.7} = \boxed{9}$